Solving Constrained Via Minimization by Compact Linear Programming

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Abstract

Via minimization is an important problem in integrated circuit layout and printed circuit board design. In this paper, a linear (non-integral) programming approach to two-layer constrained via minimization (CVM) is presented. The approach finds optimum solutions for routings containing no more than three way splits, and guarantees *provably good* results for the general case. Most importantly, the size of linear programming formulation is *polynomial* in terms of the size of the CVM problem. The significance of our work lies in three aspects. First, since linear programming can be solved in polynomial time, our work thus provides, for the first time, a mathematical programming solution with computational efficiency comparable to known combinatorial CVM algorithms. Second, our compact linear programming approach is provably good and natural for general CVM, while previous restricted CVM algorithms are difficult to be extended to the general case. Third, our approach can handle additional constraints in a unified manner, and thus provides an efficient method for performance-driven layer assignment. Our approach is based on some new graph-theoretic and polyhedron-combinatorial results presented in this paper on the structure of the CVM problem.

I. Introduction

Minimizing the number of vias used to connect wires on different routing layers is an important problem in the design of integrated circuits (ICs), printed circuit boards (PCBs), and multichip modules (MCMs). This is because that the increase of vias usually causes the decrease of circuit yield and reliability, the increase of the manufacturing cost, and the decrease of circuit performance. The problem is even more important for deep submicron ICs, high speed PCBs and MCMs, since complex-structured vias introduce many signal integrity problems [6]. For instance, an IBM thermal conduction module may contain up to 350 000 vias. Vias cause signal distortion and reflections as well as severe degradation in the high frequency components. In multilayered digital circuits, vias constitute one of the most commonly used class of interconnects. This physical discontinuities or nonuniformalities in the connections may cause severe reflections when they can no longer be considered as conducting wires, but behavior as transmission lines and/or waveguides. Moreover, most existing routing tools generate a large number of unnecessary vias, due to the fact that they usually assign all vertical wire segments to one layer and all horizontal wire segments to the other layer.

In this paper, we consider two-layer constrained via minimization (CVM). The problem itself is not only practically relevant, but also forms a basis for the multi-layer problem. The two-layer CVM can be described using Fig. 1. It is assumed that physical placement and routing have been completed, and we have a *partial* routing that consists of

- a set of horizontal and vertical lines
- a set of *potential vias*, denoted by solid bullets
- and a set of *cross-overs*, denoted by small circles.

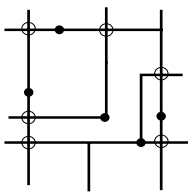

Fig. 1: A partial routing.

The set of potential vias partitions a partial routing into a set of *wire segments*. The pattern of wire segments joining at a potential via is called a *(wire) split*. A *valid layer assignment* is an assignment of all the wire segments into two layers such that no two wire segments that meet at a cross over are assigned to the same layer. We assume that potential vias are given such that a valid layer assignment always exists. A *via* is needed if wire segments joining at a potential via are assigned to more than one layer. The *constrained via minimization (CVM) problem* is to find a valid layer assignment of all wire segments so as to minimize the number of vias needed. If a partial routing contains only two-way and/or three-way splits, the problem is called *restricted CVM*.

The CVM problem is a classical problem in physical layout. It was first addressed by Hashimoto and Stevens [8] and has attracted a lot of attentions in the

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past two decades [10, 12, 14, 15]. Previous work can be classified into two categories: combinatorial algorithms and mathematical programming. Polynomial *combinatorial* algorithms have been developed for restricted CVM [3, 10]. The general problem has been proven to be NP-complete [1]. Mathematical programming approaches were developed in [4, 13, 15]. They are especially interesting for performance-driven layer assignment. It has been shown recently that timing constraints for performance-driven layer assignment can be represented by a set of linear inequalities [13]. This set of inequalities can be handled naturally within the framework of mathematical programming. Unfortunately, all the current mathematical programming approaches are based on integer programming. It is well known that integer programming is NP-complete, and branch and bound has to be employed. This implies that mathematical programming may take exponential time even for restricted CVM.

In this paper, a linear (non-integral) programming approach to constrained via minimization is presented. The approach is *exact* for restricted CVM, and *provably good* for general CVM. Most importantly, the size of linear programming formulation is *polynomial* in terms of the size of the CVM problem. Since linear programming can be solved in polynomial time [9], our work, for the first time, provides a mathematical programming solution that is theoretically comparable to best-known combinatorial CVM algorithms [10].

The paper is organized as follows: Section II overview briefly how the CVM problem can be formulated as the maximum balance problem in a planar signed hypergraph, which we shall refer to as the prime formulation. In Section III, we introduce the concepts of planar dual and hypergraph T-join, and prove that the maximum balance problem in a planar signed hypergraph is equivalent to the minimum T-join problem in a planar marked hypergraph. In Section IV, we derive a polynomial-size linear program for partial routings that contain only two-way splits. An illustrative example is presented in Section V. Section VI concludes the paper.

II. Prime Formulation

In this section, we review the signed hypergraph formulation of CVM [11] using the partial routing in Fig. 1. As illustrated in Fig. 2, we are given a set P of *points* ($t_1, \ldots, t_{11}$ and $e_1, \ldots, e_5$ in the figure) in a plane, which are either *terminals* (t_i) or *potential vias* (e_i). We are also given a set N of *(wire) segments*, each segment being a wire in the plane with multiple end-points and connecting some points from P. For example, $\{t_7, t_9, e_4\}$ represents a segment. Two segments may *cross* each other (e.g., $\{t_7, t_9, e_4\}$ and $\{t_8, e_1\}$); this defines a *crossing relation* X on N. The CVM problem is to assign segments to the two layers in such a way that (1) no crossing segments appear in the same layer, and (2) the number of vias that connect segments assigned to different layers is minimized. As is usually done in practice, *we assume that there does exist a solution that satisfies condition (1).*

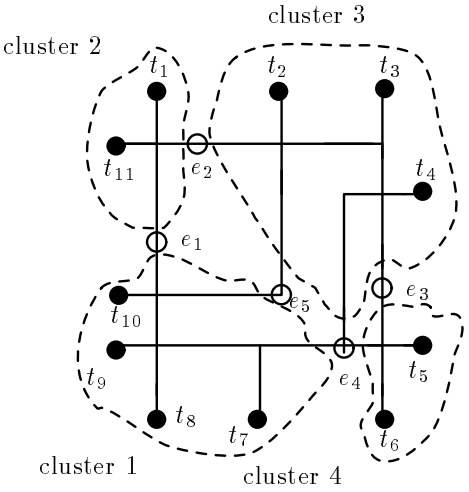

Fig. 2: A partition of clusters.

Two wire segments are said to be *assignment-constrained*, if the layer assignment of one wire segment determines the layer assignment of the other. In particular, two wire segments that cross over each other are assignment-constrained (to different layers); two wire segments that cross over a common wire segment are assignment-constrained (to the same layer). The maximal set of wire segments that are assignment-constrained forms a *cluster*. In our example, four clusters (disjoint sets of wire segments) are illustrated in Fig. 2. Segments in each cluster can be partitioned to two groups; each group must be assigned to one layer. Thus, we arbitrarily choose, for each cluster, one group of wire segments as the *reference wire segments*, the other group of wire segments as the *non-reference wire segments*. Then we can represent the CVM problem as follows.

1. Represent each cluster by a vertex.
2. Represent each potential via by an edge.
3. If a cluster joins a potential via through a reference (non-reference) wire segment, then the corresponding vertex is incident with the corresponding edge through a positive (negative) sign.

Since a potential via may connect more than two clusters, this leads to a signed hypergraph. In our example, we choose segments $\{t_7, t_9, e_4\}$ and $\{t_{11}, e_5\}$ as the reference wire segments for cluster 1, $\{t_{12}, e_2\}$ for cluster 2, $\{t_2, e_5\}$ and $\{t_4, e_4\}$ for cluster 3, and $\{t_6, e_3\}$ for cluster 4. Figure 3 shows the resulting signed hypergraph. It is planar.

Formally, a *signed hypergraph* H is an ordered triple (V, E, ψ) consisting of a set V of vertices, a set E of edges, and an incidence function $\psi : V \times E \to \{-1, 0, 1\}$. If each edge connects two vertices (a), a signed hypergraph

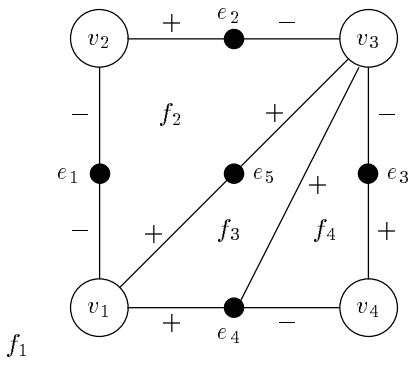

Fig. 3: A planar signed hypergraph H.

degenerates to a signed graph [7]. If -1 does not appear in the incidence matrix (b), it degenerates to a hypergraph. It degenerates to a graph, if both (a) and (b).

In terms of the resulting signed hypergraph, assigning wire segments to the two layers corresponds to partitioning vertices into two groups. The condition for not requiring a real via at the position of a potential via is that all the wire segments joined at that potential via are assigned to the same layer. That is, all the vertices joined positively at an edge must be assigned to one group, and all the vertices joined negatively at that edge must be assigned to the other group; Formally, we say that the edge is *balanced* by that bipartition. Then CVM amounts to partitioning all the vertices into two groups, so as to maximize the number of balanced edges. This is called *the maximum balance problem* in a signed hypergraph. The set of unbalanced edges in a maximum balance bipartition is called a *maximum balance cut*.

III. Dual Formulation

For a signed hypergraph, we introduce the sign of a cycle as the product of all the incidences (1 or -1) involved in the cycle. Then we have the following key observation.

Theorem 1 (Structure Theorem) *There exists a bipartition that balances all the edges in a signed hypergraph H if and only if H is free of negative cycles.*

From Theorem 1, the maximum balance problem amounts to removing a minimum set of edges such that the remaining signed hypergraph is free of negative cycles. This motivates us to introduce the notion of planar dual and hypergraph T-join in this section.

A. Planar Duals and Marked Hypergraphs

Consider a planar signed hypergraph embedded in the plane[†]. Such a planar signed hypergraph partitions the plane into a number of connected regions, called *faces*. Figure 3 shows a planar signed hypergraph H with four faces, f_1, f_2, f_3, and f_4. We also refer to the cycle that forms the boundary of a given face as a face. For example, f_2 in Fig. 3 corresponds to the cycle $v_1e_1v_2e_2v_3e_5v_1$.

A *positive (negative) face* is a face (i.e., a cycle) with positive (negative) sign. In Fig. 3, faces f_1 and f_2 are negative, whereas faces f_3 and f_4 are positive.

Each planar signed hypergraph has exactly one unbounded face, called the *exterior face*; in Fig. 3, f_1 is the exterior face. All other faces are called *interior faces*. the exterior face *encloses* a set of interior faces. Similarly, we also say that a cycle encloses a set of faces. For example, cycle $v_1e_1v_2e_2v_3e_4v_1$ encloses faces f_2 and f_3.

Proposition 1 *For a planar signed hypergraph H, the sign of a cycle is equal to the product of the signs of all the faces that are enclosed by the cycle.*

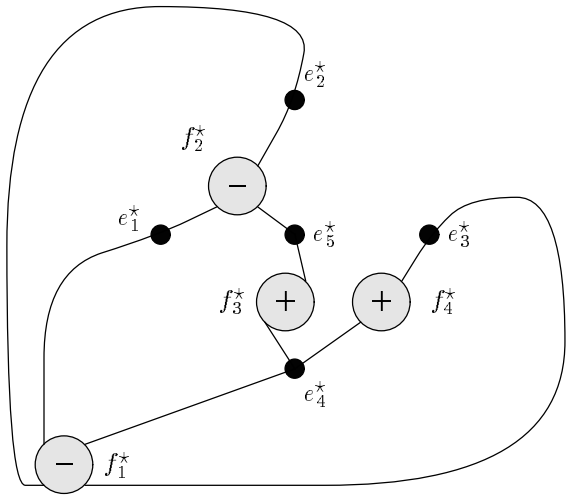

Fig. 4: Planar dual of a planar signed hypergraph H.

For example, in Fig. 3, cycle $C = v_1e_1v_2e_2v_3e_4v_1$ encloses two faces f_2 and f_3. It is easy to verify that the sign of C is equal to the product of the signs of f_2 and f_3, which is negative. The sign of f_1 is equal to the product of the signs of f_2, f_3 and f_4. In general,

Proposition 2 *A planar signed hypergraph has an even number of negative faces.*

A face is said to be *incident* with the vertices and edges in its boundary. When an edge is incident with more than one face, we say that the edge *separates* the faces incident with it. For example, in Fig. 3, edge e_1 separates faces f_1 and f_2, and edge e_4 separates faces f_1, f_3 and f_4. It can be seen that the number of faces separated by an edge is less than or equal to the degree of the edge.

Let H be a planar signed hypergraph; the *planar dual* $H^\star$ of H is defined as follows: corresponding to each face f of H there is a vertex $f^\star$ of $H^\star$; Corresponding to edge e of H that separates two or more faces, there is an edge $e^\star$

[†]In this paper, when we speak of a planar signed hypergraph, we mean a planar signed hypergraph embedded in the plane.

of $H^\star$; a set of vertices in $H^\star$ are joined by edge $e^\star$ if and only if their corresponding faces are separated by edge e in H. Associated with each vertex $f^\star$ in $H^\star$ is a sign that is the sign of its corresponding face f in H. For example, the planar dual $H^\star$ of the planar signed hypergraph H in Fig. 3 is shown in Fig. 4.

The planar dual of a planar signed hypergraph is a hypergraph in which each vertex is associated with either a positive sign or a negative sign. Such hypergraphs are called *marked hypergraphs*, following the convention of Beineke and Harary [2], who invented the notion of *marked graphs* in the modeling of relations between persons in psychology. In a marked hypergraph, vertices associated with a positive (negative) sign are called *positive (negative) vertices*.

A marked hypergraph that is the planar dual of a planar signed hypergraph has several properties. First, it is planar. Second, it is connected, i.e., for every pair of vertices u and v, there exists a path with u and v as end-vertices. Third, it has an even number of negative vertices.

B. Concept of Hypergraph T-Joins

Let $G = (V, E, \psi)$ be a graph and let $T \subseteq V$ be of even cardinality. Recall that a *graph T-join* of $\langle G, T\rangle$ is defined to be a subset of the edges that meets each vertex of T an odd number of times and that meets each vertex of $V - T$ an even number of times. We note that, if we are given a graph $G = (V, E, \psi)$ and $T = V$, then a T-join of $\langle G, T\rangle$ that meets each vertex exactly once is known as *perfect matching*—a concept of fundamental importance in graph theory and combinatorial optimization.

We now consider how to generalize the notion of T-join to marked hypergraphs. Let $H = (V, E, \psi_H)$ be a marked hypergraph with $T \subseteq V$ of even cardinality, and $G = (V \cup E, F, \psi_G)$ be its underlying bipartite graph. A graph T-join of $\langle G, T\rangle$ is a set $F_1 \subseteq F$ of edges that meets each vertex of T in G an odd number of times and that meets each vertex of $(V \cup E) - T$ in G an even number of times. A *hypergraph T-join* of $\langle H, T\rangle$ is defined to be $E \cap V(G(F_1))$, i.e., the subset of edges in H that are "used" by a T-join in the underlying graph of H. For example, in the marked hypergraph in Fig. 4, all hypergraph T-joins are $\{e_1^\star\}$, $\{e_2^\star\}$, $\{e_4^\star, e_5^\star\}$, $\{e_3^\star, e_4^\star, e_5^\star\}$, $\{e_1^\star, e_2^\star, e_4^\star, e_5^\star\}$, and $\{e_1^\star, e_2^\star, e_3^\star, e_4^\star, e_5^\star\}$.

Note that hypergraph T-join degenerates graph T-join when H is a graph. On the other hand, a hypergraph T-join may meet a vertex in T an even number of times: For example, $\{e_3^\star, e_4^\star, e_5^\star\}$ is a hypergraph T-join, which meets $f_1^\star$ twice. It may also meet a vertex in $V - T$ an odd number of times: For example, $\{e_4^\star, e_5^\star\}$ is a hypergraph T-join that meets $f_4^\star$ once.

C. Duality of Max Balance Cut and Min T-Join

The first main result of this paper is as follows:

Theorem 2 (Duality Theorem) *A maximum balance cut in a planar signed hypergraph is a minimum hypergraph T-join in its planar dual.*

For example, consider the marked hypergraph $H^\star$ in Fig. 4 with $T = \{f_1^\star, f_2^\star\}$. Sets $\{e_1^\star\}$ and $\{e_2^\star\}$ are two minimum hypergraph T-joins. The two corresponding maximum balance cuts in H are $\{e_1\}$ and $\{e_2\}$.

D. Marked Hypergraph Formulation of CVM

Given a hypergraph $H = (V, E, \psi)$ and $T \subseteq V$ of even cardinality, the *minimum hypergraph T-join* problem is to find a hypergraph T-join having minimum cardinality. Theorem 2 states that the general CVM problem can be formulated as the minimum hypergraph T-join problem. In fact, we can formulate the hypergraph T-join problem for the CVM problem directly from a given partial routing: construct the clusters, represent each face (separated by clusters) by vertices, and represent each potential via by an edge. The sign of a vertex is determined by the number of cross-overs in the cycle that immediately encloses the corresponding face: positive if it is even, and negative if it is odd.

IV. Compact Linear Program for Restricted CVM

In this section, we derive a compact linear programming formulation for restricted CVM. This is done by studying the maximum balance problem in planar signed graphs. We apply the duality theorem in the previous section and a fundamental theory of polyhedral combinatorics. For the general CVM, a mathematical programming method has been proposed to find best possible planar signed graph approximation of a planar signed hypergraph [11].

Consider a signed graph $G = (V, E, \psi)$. Suppose that $\pi = (V^+, V^-)$ is a bipartition, and $C \subseteq E$ is the corresponding cut, i.e., a set of edges such that each edge joins a vertex in V^+ with a vertex in V^-. We associate a *cut* variable z_e with each edge $e \in E$ as follows:

$$z_e = \begin{cases} 1 & \text{if } e \in C, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

A feasible solution to the maximum balance problem corresponds to a vector $\mathbf{z}$ in $R^{|E|}$ that defines a cut, where R is the set of real numbers. Let $\mathcal{Q}$ be the set of all the cycles in G; then vector $\mathbf{z} \in R^{|E|}$ defines a cut, iff

$$z_e \in \{0, 1\}, \tag{2}$$

for all $e \in E$, and

$$\sum_{e \in Q} z_e \equiv 0 \pmod 2. \tag{3}$$

for all $Q \in \mathcal{Q}$. Here (3) states that each cycle must be cut by a bipartition an even number of times.

In a signed graph, a positive edge is an edge with either two positive vertex-edge incidences, or two negative incidences; a negative edge is an edge with one positive vertex-edge incidence and one negative incidence. Let E^+ (E^-) denote the set of positive (negative) edges in E; then edge e is balanced by a bipartition if and only if

$$z_e = 0 \qquad \text{for } e \in E^+,$$
$$z_e = 1 \qquad \text{for } e \in E^-.$$

The maximum balance problem is to find $\mathbf{z} \in R^{|E|}$ for an integer linear program defined by

$$\textit{maximize} \qquad \sum_{e \in E^+} w_e(1 - z_e) + \sum_{e \in E^-} w_e z_e,$$

subject to (2) and (3), where w_e is a real weight associated with edge e.

From Theorem 2, the maximum balance problem in a planar signed graph reduces to the minimum T-join problem in its planar dual. Then, according to Edmonds and Johnson [5], and noticing that cycles and cuts are exchangeable under planar duality, (2)-(3) can be replaced by the following set of linear inequalities:

$$\sum_{e \in Q-U} z_e + \sum_{e \in U} (1 - z_e) \geq 1, \tag{4}$$

where Q is a cycle in $\mathcal{Q}$, $U \subseteq Q$ and $|U|$ is odd, and $Q - U$ denotes edges in Q but not in U. The inequalities in (4) are called *blossom inequalities*.

In summary, the maximum balance problem in a planar signed graph is formulated as the following linear program:

$$\textit{maximize} \qquad \sum_{e \in E^+} w_e(1 - z_e) + \sum_{e \in E^-} w_e z_e,$$

subject to

$$z_e \geq 0, \tag{5}$$

and

$$\sum_{e \in Q-U} z_e + \sum_{e \in U} (1 - z_e) \geq 1, \tag{6}$$

where $Q \in \mathcal{Q}$, $U \subseteq Q$ and $|U|$ is odd.

Any linear program can be solved in polynomial time in the size of the program [9]. However, the number of blossom inequalities in the formulation above may be *exponential* in terms of the graph size, as seen from the following proposition.

Proposition 3 *For a cycle of length k, the number of blossom inequalities in (6) is 2^{k-1}.*

We can show that the number of inequalities in the formulation above can be reduced so that it is only *polynomial* in the size of the CVM problem. The reduction is accomplished in three steps. First, reduce the size of a signed graph resulting from CVM by pre-processing self-loops, parallel edges, series edges, and cut edges. Second, triangulate a given planar signed graph, i,e., adding certain zero-weighted edges so that each face is enclosed by exactly three edges. Third, list only blossom inequalities for bounded faces. Now, we are ready to present the second major result of this paper:

Theorem 3 *For a connected planar signed graph with n vertices and an exterior of length k, a linear programming formulation of the maximum balance problem can be constructed that consists of $3n - k - 3$ variables, $8n - 4k + 8$ blossom inequalities, and $32n - 16k + 32$ nonzero elements.*

Graph triangulation makes a graph dense; however, we can prove that the resulting linear program for a triangulated planar signed graph is always sparser than that of the original planar signed graph.

V. An Illustrative Example

We demonstrate how to use the compact linear programming approach to solve the CVM problem in Fig. 1. As shown in Figs. 5(a) and (b), a hyperedge with degree 3 can be represented *exactly* by a weighted planar signed graph [11]. One can verify that the cost function is $0.5(1 - z_5) + 0.5z_3 + 0.5z_4 - 0.5$. Substitute this transformation into the original signed hypergraph, and apply graph simplification techniques, we can obtain a weighted triangled planar signed graph, shown in Fig. 5(d).

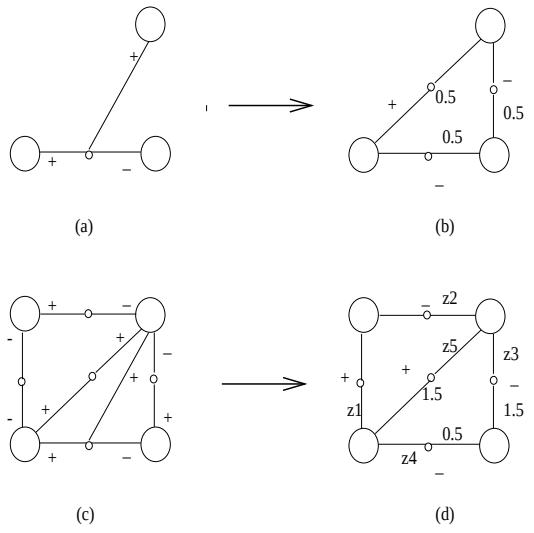

Fig. 5: Modeling a planar signed hypergraph.

To set up our compact linear programming formulation, let us consider how to formulate the objective function. Edges e_1 and e_5 are positive edges (E^+), whereas edges e_2, e_3, and e_4 are negative edges. This leads to the following

objective function:

$$maximize \quad (1-z_1)+1.5(1-z_5)+z_2+1.5z_3+0.5z_4-0.5$$

Now we consider the constraints. First, we have

$$z_1, \quad z_2, \quad z_3, \quad z_4, \quad z_5 \geq 0$$

There are two bounded faces. Consider the face that encloses edges e_2, e_5 and e_1. The all odd sets U of edges are e_2, e_5, e_1, and e_2, e_5, e_1. So we can set up the following constraints:

$$\begin{aligned} z_2 + z_5 + (1 - z_1) &\geq 1 \\ z_5 + z_1 + (1 - z_2) &\geq 1 \\ z_1 + z_2 + (1 - z_5) &\geq 1 \\ (1 - z_1) + (1 - z_2) + (1 - z_5) &\geq 1 \end{aligned}$$

Similarly, the face that encloses e_5, e_3 and e_4 introduces the following constraints:

$$\begin{aligned} z_3 + z_4 + (1 - z_5) &\geq 1 \\ z_4 + z_5 + (1 - z_3) &\geq 1 \\ z_5 + z_3 + (1 - z_4) &\geq 1 \\ (1 - z_3) + (1 - z_4) + (1 - z_5) &\geq 1 \end{aligned}$$

The solution to the problem above is

$$z_1 = z_2 = z_5 = 0, \quad z_3 = z_4 = 1,$$

with the objective function being 4. That is, four edges are balanced and one edge (e_2) is unbalanced. This implies that one via is needed at potential via e_2. The resulting optimum CVM solution is illustrated in Fig. 6.

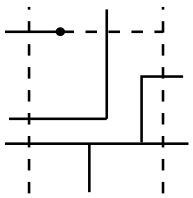

Fig. 6: An optimum CVM solution to Fig. 1.

VI. Conclusion

This paper presented a planar dual formulation for two-layer CVM. A compact linear program characterization of the restricted CVM problem was derived for the first time in this paper. Using polynomial algorithms for linear programming [9], we have an exact and polynomial algorithm for restricted CVM. Combined with optimum approximation of hyperedges via mathematical programming [11], provably good solutions of the general CVM can be obtained in polynomial time. A compact linear program characterization is especially interesting for performance-driven routing.

References

[1] K. Ahn and S. Sahni, "Constrained via minimization", *IEEE Trans. on Computer-Aided Design*, vol. 12, no. 2, pp. 273–282, Feb. 1993.

[2] L. W. Beineke and F. Harary, "Consistent graphs with signed points", *Riv. di Mat. per le Sci. Econ. e Soc.*, vol. 1, pp. 81–88, 1978.

[3] R. W. Chen, Y. Kajitani, and S. P. Chan, "A graph-theoretic via minimization algorithm for two-layer printed boards", *IEEE Trans. on Circuits and Systems*, vol. CAS-30, no. 5, pp. 284–299, May 1983.

[4] M. J. Ciesielski and E. Kinnen, "An optimum layer assignment for routing in ICs and PCBs", in *Proc. of IEEE/ACM Design Automation Conf.*, 1981, pp. 733–737.

[5] J. Edmonds and E. L. Johnson, "Matching, Euler tours and the Chinese postman", *Math. Programming*, vol. 5, pp. 88–124, 1973.

[6] R. Goyal, "Managing signal integrity", *IEEE Spectrum*, vol. 31, no. 3, pp. 54–58, March 1994.

[7] F. Harary, "On the notation of balance of a signed graph", *Michigan Math. J.*, vol. 2, pp. 143–146, 1953-54.

[8] A. Hashimoto and J. Stevens, "Wire routing optimizing channel assignment within large aperture", in *Proc. of 8th Design Automation Workshop*, June 1971, pp. 155–169.

[9] N. Karmarkar, "A new polynomial-time algorithm for linear programming", *Combinatorica*, vol. 4, no. 4, pp. 373–395, 1984.

[10] Y. S. Kuo, T. C. Chern, and W. K. Shih, "Fast algorithm for optimal layer assignment", in *Proc. of IEEE/ACM Design Automation Conf.*, 1988, pp. 554–559.

[11] C.-J. Shi, "A signed hypergraph model of constrained via minimization", pp. 159-166 in *Proc. Great Lakes Symp. on VLSI*, 1992.

[12] C.-J. Shi, "Constrained via minimization and signed hypergraph partitioning", pp. 337-356 in *Algorithmic Aspects of VLSI Layouts*, D. T. Lee and M. Sarrafzadeh (eds.), World Scientific Publishing Company, 1993.

[13] C.-J. Shi, A. Vannelli and J. Vlach, "Performance-driven layer assignment for printed circuit boards and integrated circuits", pp. 171-174 in *Proc. IEEE International ASIC Conf.*, Rochester, N.Y., Sept. 1996.

[14] K. R. Stevens and W. M. vanCleemput, "Global via minimization in generalized routing environment", in *Proc. IEEE Int. Symp. on Circuits and Systems*, 1979, pp. 689–692.

[15] X. M. Xiong and E. S. Kuh, "A unified approach to the via minimization problem", *IEEE Trans. on Circuits and Systems*, vol. CAS-36, no. 2, pp. 190–204, Feb. 1989.