An Efficient Algorithm for 3-D Reluctance Extraction Considering High Frequency Effect

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Abstract – As shown in literatures, partial reluctance based circuit analysis is efficient in capturing on-chip inductance effect, because the partial reluctance exhibits much better locality than partial inductance. However, most previous works on reluctance extraction did not take high frequency effect into account and were not efficient enough for 3-D complex structure. In this paper, a new reluctance extraction algorithm is proposed considering the high frequency effect. Numerical experiments demonstrate that our algorithm can handle complex 3-D interconnect structures while exhibiting high accuracy and a speed-up ratio of several tens to hundreds over FastHenry.

I. Introduction

The industry of modern VLSI circuits is advancing toward ultra deep sub-micron technology or even nano technology, and the operating frequency of circuit has reached multiple giga-hertz (GHz). Currently, the interconnect delay and coupling has become critical for the design of high performance circuits, such that the conventional RC model of interconnect is not enough for accurate circuit analysis. Therefore, accurate and efficient modeling of on-chip inductance effect becomes very important. Moreover, the reduction of resistance by copper and capacitance by low- κ dielectric, denser geometries and growing complexity of interconnect design also make on-chip inductance indispensable [1][2].

Modeling the on-chip inductance effect is difficult because of the unknown circuit return path prior to the extraction and simulation. Thus, on-chip inductance extraction and the circuit return path becomes a “chicken-egg” paradox. With the partial element equivalent circuit (PEEC) model [3], the return path problem is solved by defining the concept of partial inductance whose return path is at infinity. However, since the coupling of partial inductance is among all the conductor segments, the resulting partial inductance matrix is a dense one. Although faraway terms in the partial inductance matrix maybe small, simply truncating them may lead to the system unstable [4].

In order to overcome this problem, a new concept ‘partial reluctance’ was first proposed in [5], it was also called K element. The partial reluctance matrix is denoted by $\boldsymbol{K}$, and its definition is the inversion of partial inductance matrix, i.e.:

$$\boldsymbol{K} = \boldsymbol{L}^{-1} \qquad (1)$$

where $\boldsymbol{L}$ is the partial inductance matrix. The partial reluctance has locality similar to capacitance, therefore circuit analysis based on it is much more efficient than that based on partial inductance. Experiments in later papers like [6] showed that circuit simulation based on partial reluctance also has great advantage over partial inductance based simulation both in speed and accuracy. Ref. [7] proved that by ignoring small elements in matrix, the sparsified partial reluctance matrix is positive definite and the circuit simulation based on it is stable.

As mentioned before, the operating frequency of current VLSI circuit has reached several GHz, so high frequency effect becomes more and more important. However, among the existing literatures about partial reluctance extraction, only [8] and [11] considered the high frequency effect. Ref. [8] demonstrated the necessity of considering high frequency effect through simulation experiments, and presented an algorithm for extracting frequency-dependent reluctance. But the inversion of matrices in the algorithm prohibits its performance especially when the structure of input circuit is complex. Besides, [11] proposed an impedance extraction algorithm based on the idea of K element. However, the algorithm in [11] extended the locality of reluctance to the admittance (reciprocal of impedance), which actually only holds for very high frequency. In this paper, we propose a new algorithm for frequency-dependent reluctance extraction. This algorithm considers the physical meaning of partial reluctance, and extract the reluctance directly without any matrix inversion. This is superior to the algorithms in [8] and [11], because there is no expensive operation of matrix inversion in the proposed algorithm. Numerical experiments are performed on interconnect structure generated randomly and some practical structures, and their results demonstrate the efficiency of our new algorithm.

The rest of this paper is organized as follows. Section II discusses how to extract the partial reluctance considering high frequency effect. In Section III, the details of our new algorithm which is called direct reluctance extraction is introduced. The experiment results are given in Section IV, where both interconnect structure generated randomly and practical structures from industry are tested. Finally, Section V is the conclusion.

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II. Partial Reluctance Extraction Considering High Frequency Effect

A. Capturing High Frequency Effect

It is well known that as the frequency goes high, the current on a conductor is no long evenly distributed. In order to capture such high frequency effect, the conductors need to meshed into filaments along the current direction. With the magneto quasi static (MQS) assumption, the current can be considered to flow parallel to conductor surface, and the current is distributed evenly in each filament.

Since the current density near the surface of the conductor is larger than that far from the surface because of the skin effect, we can mesh the cross section of a conductor segment non-uniformly. How many filaments should be meshed for a conductor mainly depends on the operating frequency and the size of the conductor. Fig. 1 shows an example of two parallel conductors which are mashed into 5×3 filaments each.

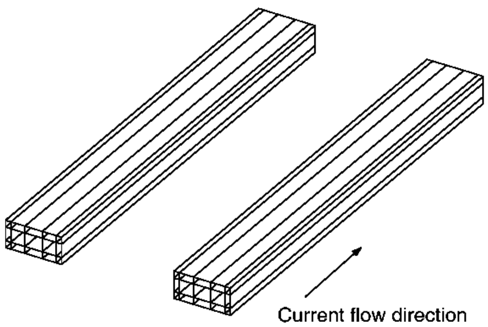

Fig. 1. An example of two parallel conductors which are mashed into 5×3 filaments non-uniformly.

B. Frequency-Dependent Reluctance Extraction

The general process of frequency-dependent reluctance extraction can be divided into the following steps. First, for each conductor, a proper window is selected for it using some window selection strategy. We will discuss it in detail in next subsection. Second, in order to capture the high frequency effect, each conductor is meshed into some filaments as described in last subsection. After filaments meshing, the local reluctance matrix for each window is calculated using some methods. Finally, the local reluctance matrices are combined into a big one, which is the whole reluctance matrix of the circuit.

The differences between different methods of frequency dependent reluctance extraction are mainly focused on the second step, i.e. how to extract the local reluctance matrix for each window. The second step of the algorithm presented in [8] can be summarized as follows. For each window, It first calculate the impedance matrix of all filaments in it. Then it sets a conductor as the aggressor, i.e. the voltage on it is set to 1 while others are 0 and the filaments current distribution is calculated. Adding the currents in the same conductor together, the result is equal to one column of the local admittance matrix. The above process is repeated again and again until all conductors in the window are set as the aggressor one by one. Now, it gets the local admittance matrix, by inversing it, the local impedance matrix of conductors is obtained. Select the imaginary part of the local impedance matrix and inverse it again, the resulting matrix is the local reluctance matrix.

From the above summary, we know the algorithm in [8] needs two times matrix inversion for each window. It is clear that the matrix inversion work is costly especially when the structure is complex and it has a lot of windows. So developing an extraction method for directly extracting partial reluctance without the matrix inversion would be valuable. We propose such a novel algorithm in this paper. Actually, in our algorithm, we only need to slove an equation for each window, the details will be discussed in Section III.

C. Window Selection Strategy

How to select the coupling window is the key point to assure the accuracy of reluctance extraction. Too large window size will bring high accuracy as well as long running time, and too small window size may result in less accuracy. A window selection algorithm has been proposed based on the shield effect [10], which however was applied to 2-D simple structures. Based on some definitions and tips in it, a more complicated window selection method dealing with 3-D complex structures was proposed in [11]. This method considers both shield effect and distance among conductors, and is used in our frequency-dependent reluctance extraction.

To select the coupling window is actually to choose some nearby conductors for the specified aggressor conductor (the rest are usually called victim conductors). In [11], a coupling level is defined for each victim according to its position relation with the aggressor. Then all victims whose coupling level is less than a specified "max coupling level" form the window.

For a general 3-D structure, the procedure of determining the coupling levels of the victims is performed on the conductor projections in XOY, ZOX, and YOZ planes, respectively. In each coordinate plane, the projections are sorted in two directions at first to get two queues of projections. For each queue, the projection is set as aggressor in turn, then the coupling level of other projection is calculated according to the shield level and their distance. After implementing the above procedure for all the six projection queues, the minimum of the levels of conductor i and j is used as the final result. For more detail, please refer to Section III of [11].

III. Direct Extraction of Frequency-Dependent Reluctance

In this section, we propose a novel algorithm which extract $\boldsymbol{K}$ matrix directly without any matrix inversion work.

A. Details of Direct Partial Reluctance Extraction

With the sinusoidal steady-assumption, for a system with n conductors at angular frequency ω, the relation of current vector $\boldsymbol{I} \in \mathbb{C}^n$ and voltage vector $\boldsymbol{V} \in \mathbb{C}^n$ on the conductors can be expressed as:

$$(\boldsymbol{R}+j\omega\boldsymbol{L})\boldsymbol{I}=\boldsymbol{V} \qquad (2)$$

where $\boldsymbol{R}$, $\boldsymbol{L}\in\mathbb{R}^{n\times n}$ are the resistance matrix and the partial inductance matrix of the system respectively. Transforming Eq. (2) and combining it with the definition of partial reluctance matrix $\boldsymbol{K}$ in Eq. (1), we have:

$$j\omega\boldsymbol{I}=\boldsymbol{K}(\boldsymbol{V}-\boldsymbol{RI}) \qquad (3)$$

Notice that ***V-RI*** is an $n\times 1$ vector, if we set entry i of ***V-RI*** to $j\omega$ and other entries to zero, then the resulting current distribution $\boldsymbol{I}$ would be equal to the ith column of $\boldsymbol{K}$.

Now we talk about the physical meaning of it. According to the definition of magnetic vector potential A and Faraday's law, we have:

$$E=-j\omega A-\nabla\Phi \qquad (4)$$

where E is the electric field, Φ is the scalar potential. Integrating both side of Eq. (4) along the length direction of the conductor from one end a to the other b, it becomes:

$$V_{ab}=-(\Phi_b-\Phi_a)=j\omega\int_b^a Adl+El_{ab} \qquad (5)$$

As E contributes to the resistance potential drop from $J=\sigma E$, so:

$$El_{ab}=RI \qquad (6)$$

Combining Eq. (5) and Eq. (6), we have:

$$V_{ab}-RI=j\omega\int_b^a Adl \qquad (7)$$

Eq.(7) tells us that setting entry i of ***V-RI*** to $j\omega$ equals to setting entry i of the magnetic vector potential drop $\int Adl$ to 1. Now, the physical meaning of partial reluctance is clear. Actually, the ith column of partial reluctance matrix $\boldsymbol{K}$ is the current distribution on the conductors when we set the ith entry (corresponding to the aggressor) of the vector potential drop to 1 and other entries (corresponding to the victims) to 0. The physical meaning of partial reluctance makes it possible to be extracted directly instead of inverse the partial inductance $\boldsymbol{L}$.

In order to capturing the high frequency effect, we have already meshed each conductor into several filaments as discussed in Section II. Assuming each filament is thin enough that the current can be approximately uniformly distributed in it. We note the current and voltage of filament i as $\hat{I}_i$ and $\hat{V}_i$ respectively. The relation between them can be expressed as:

$$\hat{R}_{ii}\hat{I}_i+j\omega\sum_{j=1}^{m}\hat{L}_{ij}\hat{I}_j=\hat{V}_i \qquad (8)$$

where m is the number of filaments. The filament's DC resistance $\hat{R}_{ii}$ can be obtained by:

$$\hat{R}_{ii}=l_i/\sigma\hat{a}_i \qquad (9)$$

where σ is the conductivity of the conductor, l_i is the length of it and $\hat{a}_i$ is the cross-section area of the filament. The mutual partial inductance between the filaments can be obtained by [3]:

$$\hat{L}_{ij}=\frac{\mu_0}{4\pi\hat{a}_i\hat{a}_j}\left[\int_{a_i}\int_{a_j}\int_{l_i}\int_{l_j}\frac{d\vec{l}_i d\vec{l}_j}{\|\vec{r}_i-\vec{r}_j\|}d\hat{a}_i d\hat{a}_j\right] \qquad (10)$$

where μ_0 is the magnetic permeability. Put Eq. (8) of each filaments together, we get the matrix form of them like:

$$(\hat{\boldsymbol{R}}+j\omega\hat{\boldsymbol{L}})\hat{\boldsymbol{I}}=\hat{\boldsymbol{V}} \qquad (11)$$

where $\hat{\boldsymbol{R}}$ is an $m\times m$ diagonal matrix and $\hat{\boldsymbol{L}}$ is an $m\times m$ matrix. They can be easily obtained according to Eq. (9) and Eq. (10). $\hat{\boldsymbol{I}}\in\mathbb{C}^m$ is the vector of current on the m filaments, which is what we want to know and $\hat{\boldsymbol{V}}\in\mathbb{C}^m$ is the vector of potential drop on the m filaments.

Because there is no transverse current between filaments, the potential drop on different filaments in the same conductor are equal, and the current of a conductor is equal to the sum of the filaments current in the same conductor. For convenience, we define a mesh incidence matrix $\boldsymbol{M}\in\mathbb{R}^{n\times m}$ as:

$$M_{ij}=\begin{cases}1 & \textit{when filament } j \textit{ is in coductor } i\\ 0 & \textit{otherwise}\end{cases} \qquad (12)$$

Then the above relation can be expressed as:

$$\hat{\boldsymbol{V}}=\boldsymbol{M}^t\boldsymbol{V}\,,\quad \boldsymbol{I}=\boldsymbol{M}\hat{\boldsymbol{I}} \qquad (13)$$

Since the partial reluctance matrix $\boldsymbol{K}$ is real, the resulting current $\boldsymbol{I}$ must be real when we set the vector potential drop $\int Adl$ along the aggressive conductor to 1 and others to 0, i.e.,

$$\boldsymbol{I}_{im}=0 \qquad (14)$$

Rewrite Eq. (11) as:

$$\begin{cases}\hat{\boldsymbol{R}}\hat{\boldsymbol{I}}_{re}-\omega\hat{\boldsymbol{L}}\hat{\boldsymbol{I}}_{im}=\hat{\boldsymbol{V}}_{re}=\boldsymbol{M}^t\boldsymbol{V}_{re}\\ \omega\hat{\boldsymbol{L}}\hat{\boldsymbol{I}}_{re}+\hat{\boldsymbol{R}}\hat{\boldsymbol{I}}_{im}=\hat{\boldsymbol{V}}_{im}=\omega\boldsymbol{M}^t\int Adl\end{cases} \qquad (15)$$

Put Eq. (14) and Eq. (15) together into a matrix form, we have:

$$\begin{bmatrix} \hat{R} & -\omega\hat{L} & -M^t \\ \omega\hat{L} & \hat{R} & 0 \\ 0 & M & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ \hat{I}_{im} \\ V_{re} \end{bmatrix} = \begin{bmatrix} 0 \\ \omega M^t \int Adl \\ 0 \end{bmatrix} \tag{16}$$

By solving this linear equation, we can get $\hat{I}$. Then, the current on conductors I can be got by $I = M\hat{I}$. Suppose we set conductor i to be the aggressor, then I is equal to the ith column of partial reluctance matrix K. In this way, we can obtain the whole partial reluctance matrix K, by setting each conductor to be the aggressor one by one and repeat this process.

B. Condensing the Coefficient Matrix

The dimension of linear system (16) is $2m+n$, where m is the number of filaments and n is the number of conductors in a window. Actually, we do not need to solve linear system (16), just solving a linear system with dimension $m+n$ is enough. Since m is much larger than n, reducing the problem dimension from $2m+n$ to $m+n$ would cut the equation solving time by nearly 7/8. Notice Eq. (14) shows that when we set the vector potential drop along the aggressor to 1 and others to 0, the resulting current vector is real. That means we only need to know $\hat{I}_{re}$ without caring $\hat{I}_{im}$. In the following, we will show how to condense the coefficient matrix.

From the second equation of Eq. (16) we have:

$$\hat{I}_{im} = \omega\hat{R}^{-1}M^t \int Adl - \omega\hat{R}^{-1}\hat{L}\hat{I}_{re} \tag{17}$$

Replace $\hat{I}_{im}$ in the first and the third equation of Eq. (16) with Eq. (17), the resulting two equations are:

$$(I + (\omega\hat{R}^{-1}\hat{L})^2)\hat{I}_{re} - \hat{R}^{-1}M^tV_{re} = \omega^2\hat{R}^{-1}\hat{L}\hat{R}^{-1}M^t \int Adl \tag{18}$$

$$M\hat{R}^{-1}\hat{L}\hat{I}_{re} = M\hat{R}^{-1}M^t \int Adl \tag{19}$$

where I is the indentity matrix. Combining Eq. (18) and Eq. (19), we get the condensed form of the linear system:

$$\begin{bmatrix} I+(\omega\hat{R}^{-1}\hat{L})^2 & -\hat{R}^{-1}M^t \\ M\hat{R}^{-1}\hat{L} & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ V_{re} \end{bmatrix} = \begin{bmatrix} \omega^2\hat{R}^{-1}\hat{L}\hat{R}^{-1}M^t \int Adl \\ M\hat{R}^{-1}M^t \int Adl \end{bmatrix} \tag{20}$$

After the above process, the dimension of equation is reduced to $m+n$.

It should be pointed out that the computational expense of this condensing procedure is very little, because the $\hat{R}$ in Eq. (20) is a diagonal matrix whose inverse is trivial. Besides, the M matrix is very sparse, taking it into account also saves computing time.

C. Algorithm Description

Given a circuit, for each conductor in it, our partial reluctance extraction algorithm first selects a window for the conductor using the window selection strategy presented in Section II. When extracting the partial reluctance, we only consider the conductors in the same window, this is according to the excellent locality of partial reluctance. For each conductor, we set it as the only aggressor in its window, and extract the column of partial reluctance of the whole partial reluctance matrix corresponding to it. The proposed reluctance extraction algorithm considering high frequency effect is summarized as follows.

<u>Direct partial reluctance extraction algorithm</u>

1. For each conductor in the given circuit, select a proper window for it using the window selection strategy in Section II.
2. Mesh each conductor into several filaments non-uniformly, the number of filaments mainly depends on the frequency and the size of the conductor.
3. Suppose there are N conductors in the circuit, for conductor i (i from 1 to N) do:
   (a) Pick out all the conductors in conductor i's window, suppose there are n_i conductors and m_i filaments.
   (b) Form the resistance matrix R, mutual inductance matrix L of filaments and the mesh incidence matrix M of the conductors in this window using Eq. (9) and Eq. (10).
   (c) Set the vector potential drop along the aggressor to 1 and others to 0.
   (d) Form the condensed form of the linear system for this window, whose dimension is m_i+n_i.
   (e) Solve the linear system to get $\hat{I}_{re}$. Since the dimension is not very large, we use direct method based on LU decomposition to solve it.
   (f) Calculate the current of the conductors in the window by $I = M\hat{I}_{re}$.
   (g) Put the items of vector I into the corresponding positions of the whole partial reluctance matrix K_{asym}. These items form the column of K_{asym} that corresponding to conductor i. Other positions in this column are filled by 0.
4. Make the K symmetric by $K = (K_{asym} + K^t_{asym})/2$.

IV. Numerical Results

The proposed algorithm has been implemented in C language as a software prototype for 3-D interconnect partial reluctance extraction. With it, we did a series of experiments and compared our results with famous frequency-dependent inductance extraction tool: FastHenry [9]. All the experiments are run on our Sun Fire V880 server with a 750MHz CPU. FastHenry is used to extract the impedance matrix and the imaginary parts of it elements are the partial inductances. The result of our algorithm is partial reluctance matrix. In order to compare the result with FastHenry, it needs to be inversed to get the partial inductance matrix. Finally, we compute the loop inductance of each pair of conductors, and compare it with that from the results of FastHenry.

The first example is a structure generated randomly, including 300 conductors. The conductors in the circuit have unequal length and are placed parallel, with width and height set to 1.0 μm. The conductivity is used copper conductivity as default. The second and the third examples are real package structure from industry, obtained from the webpage of FastHenry. The second example is a 30 pins structure, it contains 260 conductor segments, as shown in Fig. 2. The third one is a 35 pins structure and contains 175 conductor segments, as shown in Fig. 3. In our experiment, the operating frequency of all the three examples are set to 10 GHz. For the three examples, each conductor is partitioned into 2×2, 3×3 and 3×5 filaments, respectively. And, the same filament meshing scheme is used in our algirithm and FastHenry.

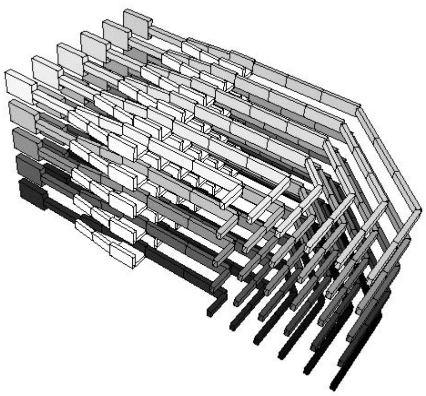

Fig. 2. A real industry package structure with 30 pins.

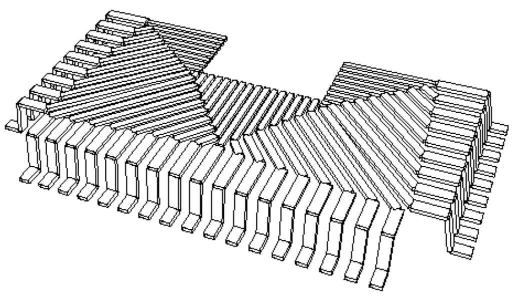

Fig. 3. A real industry package structure with 35 pins.

The results of our experiments are shown in Table I and Table II. Table I shows the error distribution of loop inductance for the three examples while using our algorithm. Table II shows the CPU time of the proposed algorithm and FastHenry. From the results, it is clear that the proposed algorithm has several tens to several hundreds speedup over FastHenry, while maintaining good accuracy. For the random interconnect structure, the errors of loop inductance are mainly less than 6%. Actually, the speed and the accuracy is a tradeoff, we can decrease the speed to increase the accuracy or vice versa according to the requirement.

V. Conclusion

In this paper, a direct extraction algorithm for partial reluctance matrix is proposed. In our method, we take the high frequency effect into account by meshing the conductor segments into filaments non-uniformly. According to the excellent locality of partial reluctance matrix, we use an efficient window selection strategy to select a window for each conductor. In each window, the partial reluctance is directly extracted without the expensive matrix inversion usually employed in previous methods. Numerical results demonstrate that our new algorithm has several tens to hundreds speedup ratio to FastHenry while preserving high accuracy.

The future work may be the combination of our algorithm to the reluctance based circuit simulator, and the improvement of equation solution for each window.

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TABLE I
Error Distribution of Loop Inductance for the Examples (Compared with FastHenry)

Error of loop inductance(%)	<3%	3-6%	6-9%	9-12%	12-15%	>15%
example 1	95.5%	4.2%	0.3%	0.0%	0	0
example 2	46.4%	26.4%	12.3%	6.8%	2.5%	5.1%
example 3	72.7%	20.7%	5.7%	0.8%	0.1%	0.0%

TABLE II
Computational Time for the Examples (Unit in Second)

	FastHenry	Our algorithm	Speed-up ratio
example 1	855.2	9.12	**94**
example 2	5650.2	9.83	**575**
example 3	5796.3	122.51	**47**