Neurorobotics

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Neurorobotics

- Biorobotics with emphasis on neural sources of model inspiration
- **Biologically Inspired Robots <> (BIO/Neurorobots)**
- Biologically inspired robots use biology as metaphors to solve practical problems in robotics (Brooks/Arkin)
- Bi/Neurorobots are simulation of biological systems
Simulation & the Scientific Method

Barbara Webb 2001
Robots have been strongly inspired by biological systems. In a sense, all robots are “biologically inspired”
GE ELEKTRO & Sparko1937

• ELEKTRO
  – 7’ Tall 265 LBS
  – Could “walk” by “voice command”
  – Mouth coordinated with 78 RPM record
  – Eyes could distinguish Red vs Green
• Sparko
  – Walk
  – Move toward bright light
• Used to demonstrate automated controls
• Example of animatronic technology
Unimation 1956-

- Unimation Founded by Engelberger
- George Devol writes patents
- Installs robots at GE
- Unimation is acquired by GE
- Billion Dollar industry Spawned
GE Walking Robot

- Ralph Mosher GE 1960’s
- Too complex to control without onboard computer
Significantly, biorobots were an early ‘simulation’ tool
Grey Walter’s Turtle 1950’s

• Key feature was “decision” making capability
  – Seek Light
  – Recharge

• Interested in mechanistic explanations of psychological phenomena
Grey Walter’s Turtle Elsie
Circa 1950
Telluride Historical Note

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Classical robotics

• Robotics requires a *rational basis for design*
  – Analytic geometry gives us great power in finding solutions
    • The Classic Elements of robotics are Kinematics, Inverse Kinematics, Statics, Dynamics and Trajectory
  – Bayesian probability has assumed a prominent role in the last 10 years
Example

The position of the body can be described by joint angles: configuration space.

Cartesian coordinates: world space
Example

\[ y_2 = l_2 \cos(\theta'_2) \]

\[ x_2 = l_2 \sin(\theta'_2) \]

\[ y_1 = l_1 \cos(\theta_1) \]

\[ x_1 = l_1 \sin(\theta_1) \]
Example

Kinematics:

\[
\begin{align*}
x &= l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\
y &= l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)
\end{align*}
\]

\[
\begin{align*}
x_2 &= l_2 \sin(\theta_1 + \theta_2) \\
y_2 &= l_2 \cos(\theta_1 + \theta_2) \\
x_1 &= l_1 \sin(\theta_1) \\
y_1 &= l_1 \cos(\theta_1)
\end{align*}
\]
Inverse Kinematics

• Harder problem
  – Kinematics: Automatic solution
  – Inverse Kinematics: Use insight
Example

- Inverse Kinematics

\[ P = \sqrt{x^2 + y^2} \]

\[ \theta_2 = \arccos \left( \frac{l_1^2 + l_2^2 - p^2}{2l_1l_2} \right) \]

\[ \alpha = \arccos \left( \frac{l_1^2 + p^2 - l_2^2}{2l_1p} \right) \]

\[ \alpha' = a \tan 2(y, x) \]

\[ \theta_1 = \alpha' - \alpha \]
Differential motion

\[ d\vec{x} = f(d\vec{\theta}) \]

Vice Versa:

\[ d\vec{\theta} = g(d\vec{x}) \]
Differential motion

• Q: What is the relationship between small changes in configuration space and small changes in world coordinates?

\[
\ddot{x}_c + \Delta \ddot{x} = F(\ddot{\theta}_c) + J(\ddot{\theta}) \Delta \dot{\theta} + \ldots
\]

• A: The Jacobian:

\[
d\ddot{x} = J(\ddot{\theta})d\dot{\theta}
\]

• Example:

\[
\begin{bmatrix}
    dx \\
    dy
\end{bmatrix} =
\begin{bmatrix}
    -l_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\
    l_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2)
\end{bmatrix}
\begin{bmatrix}
    d\theta_1 \\
    d\theta_2
\end{bmatrix}
\]
Differential motion

• Q: What is the relationship between small changes in world coordinates and small changes in configuration space?

• A: The Jacobian Inverse:

$$J^{-1}(\vec{\theta})d\vec{x} = d\vec{\theta}$$

• Example cont:

$$\begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix} = \begin{pmatrix} -l_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{pmatrix}^{-1} \begin{pmatrix} dx \\ dy \end{pmatrix}$$
Singularity

- Singular point exist, eg $\theta_2 = 0$ no inverse

\[ \det |J(\vec{\theta})| = 0 \]

\[ 0 = L_1 L_2 \left( \cos(\theta_1 + \theta_2) \sin(\theta_1) - \sin(\theta_1 + \theta_2) \cos(\theta_1) \right) \]
Statics

• Effect of torques and forces acting on a static structure
• Useful Preliminary Concepts
  – Generalized Force
    • Force
      – Newton
    • Torque
      – Newton-Meter
  – Work
    • Force x distance
    • Torque x angle
    • Joule=Newton*Meter
  – Power
    • Force x Velocity
    • Torque x angular velocity
    • Watt/sec
Statics

• What is the relationship between end-tip force and joint torques?
  – Work done at tip=∑joint work!
  – Implies $J^T \mathbf{F} = \mathbf{\tau}$

\[
\Delta \tilde{W}^{tip} = \Delta \bar{X}^T \mathbf{\bar{F}} \\
\Delta \tilde{W}_i^{j\text{joint}} = \Delta \theta_i \cdot \mathbf{\tau}_i \\
\Delta \bar{X}^T \mathbf{\bar{F}} = \Delta \bar{\theta}^T \mathbf{\bar{\tau}}
\]

\[
J d\theta = dx \rightarrow J \Delta \bar{\theta} \approx \Delta \bar{X}
\]

\[
( J \Delta \bar{\theta} )^T \mathbf{\bar{F}} = \Delta \bar{\theta}^T \mathbf{\bar{\tau}}
\]

\[
\Delta \bar{\theta}^T J^T \mathbf{\bar{F}} = \Delta \bar{\theta}^T \mathbf{\bar{\tau}}
\]

\[
\Rightarrow \quad J^T \mathbf{\bar{F}} = \mathbf{\bar{\tau}}
\]
Redundancy

Forward Kinematics unique

\[ f: Q \rightarrow X \]

Inverse Kinematics not unique

\[ f^{-1}: X \rightarrow Q \]

Think of Posture and Balance Problems
Dynamics

- Real systems have mass and evolve through time
- Lagrangian summarizes the dynamics of a system
- Automatic Soln
- 3 dof practical limit of ability to do dynamics by hand

\[ L = KE^{TOTAL} - PE^{TOTAL} \]

\[ KE = \frac{1}{2} \ddot{\mathbf{x}}^T M \ddot{\mathbf{x}} \]

\[ KE = \frac{1}{2} \frac{d}{dt} \mathbf{\dot{\theta}}^T J^* (\mathbf{\dot{\theta}})^T M J^* (\mathbf{\dot{\theta}}) \frac{d\mathbf{\dot{\theta}}^T}{dt} \]

\[ PE = g \sum_i \text{height}_i \cdot m_i \]

\[ PE = \left[ L^*_1 \sin(\theta_1) \cdot m_1 + \left( L^*_1 \sin(\theta_1) + L^*_2 \sin(\theta_1 + \theta_2) \right) \cdot m_2 \ldots \right] \cdot g \]

\[ \mathbf{\tau} = \frac{d}{dt} \left( \frac{\partial L}{\partial \mathbf{\dot{q}}} \right) - \left( \frac{\partial L}{\partial q} \right) \]

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Trajectory Generation

• Basic idea: generate a spline from knot-points
• Spline satisfies endpoint, vel, and acc constraints
• Goal of robot is to track trajectory
Notions in Legged Locomotion

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Models of Locomotion

- Inverted Pendulum Model
- Spring Model: Leg is modeled as a prismatic joint with spring in line

inverted pendulum = walking
spring-mass model = running
Models of Locomotion: SLIP model

- Spring loaded Inverted Pendulum (SLIP)
Speed Control in Hopping

- Neutral point: landing point were no net acceleration occurs.
- Longer stride, same take off power: deceleration
- Shorter stride, same take off power: acceleration
- Greater takeoff power: Longer stride, flight phase: greater net speed.

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Hodgins Raibert 1991
CoP

- COP: Center of the resulting foot forces
- ZMP: ZMP is the point where the vertical reaction force intersects the ground (Hemani and Golliday 1977).
- COP=ZMP (Goswami, 1999)
In conventional robots: CoP never touching edge of convex hull of support
CoP Movement/Humans

Wong et al 2008
Limitations of the geometric approach

• Geometrics methods
  – Useful when simulating mechanical systems
  – Useful as a tool for understanding
  – Useful when analyzing biomechanical aspects
  – Useful as a source for certain biorobots

• Analytic Geometry is probably not the technology of the brain
  – Neurorobots requires a different source: *Neurons*
Neural building blocks

• ANN have been shown to be universal approximators
• Therefore functions can be created using classic ANN
  – Kinematics
  – IK
  – Dynamics
  – Inverse dynamics
  – Statics
  – Trajectory tracking
• Subsumes part of the functionality of geometrical methods
Neurons

• Cells are the building blocks multi-cellular organisms.

• Special cells exist which are electrically excitatable
  – these include muscle cells
  – and neural cells (neurons)
Cell construction

- The cell membrane is the bag which contains the cellular machinery.
- We do not care about DNA, mitochondria, cell nuclei etc. (computationally)

We care that:
- Cells are bags of electrolytes.
- The cell membrane has a very high resistance.
- ION channels penetrate the bilipid membrane and are highly selective to ion species.
Capacitor

• The membrane/electrolyte system forms a capacitor
• The electrical model of an ideal capacitor is:

\[ C \frac{dV}{dt} = i \]

• where \( C \) is the capacitance, \( i \) is a current flow, \( v \) is the voltage potential measured relative to the inside
Ion Flow

• Driven by **diffusion** and **voltage**
• Ion pumps maintain concentrations
• Ion channels allow current flow
Current Flow

Net flow of particles from higher to lower concentration
If the particles are charged, a current is generated
Balancing Current Flow

Zero net flow at proper potential
Nernst potential

\[ E = \frac{RT}{zF} \ln \left( \frac{[\text{outside}]}{[\text{inside}]} \right) \]

- where \( E \) is the potential, \( z \) the particle charge \( [\cdot] \) indicate concentration of an ion species
For real cells..

\[ E_K = [-70...- 90 \text{mv}] \]
\[ E_{Na} = [50 \text{mv}] \]
\[ E_{Ca^{2+}} = [150 \text{mv}] \]
Models of I

\[ C \frac{dV}{dt} = i \]
Hodgkin Huxley

\[ i_m = g_L(V - E_L) + g_K n^4 (V - E_K) + g_{Na} m^4 h (V - E_{Na}) \]

• \( n,m,h \) evolve according to complex equations
• Difficult to integrate-> Not practical in real-time simulations
Integrate and Fire

\[ c_m \frac{dV}{dt} = -g_l (V - E_L) + \frac{I}{A} \]

if \( V > V_{thres} \) then \( V = V_{reset} \)

• Insufficient for motor models
Integrate and Fire With Adaptation

\[ c_m \frac{dV}{dt} = -g_l(V - E_L) + r_m g_{sra}(V - E_k) + I \]

\[ \tau_{sra} \frac{dg_{sra}}{dt} = -g_{sra} \]

if \((V > V_{thres}) \rightarrow V = V_{reset}, g_{sra} \rightarrow g_{sra} + \Delta g_{sra}\)
Leaky Integrator

\[ \tau \frac{dv}{dt} = -v_i + \sum w_{i,j} x_j \]

\[ x_j = f(v) \]

- \( x \) represents the average short-term firing rate
- \( f(v) \) is a sigmoidal function
- \( \tau \) is \( C \) and \( \frac{1}{g_L} \)
Matsuoka Oscillator

\[ \tau_i u_i = -u_i - \beta f(v_i) + \sum_{j \neq i} w_{ij} f(u_j) + u_0, \]
\[ \dot{v}_i = -v_i + f(u_i), \]
\[ (f(u) = \max(0, u)), \quad (i = 1, 2) \]

- Popular in CPG modes controlling robots

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Izhikevich Model

\[
\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I
\]

\[
\frac{du}{dt} = a(bv - u)
\]

if \(v > 30mv\): \(v \leftarrow c, u \leftarrow u + d\)
Izhikevich Model

(A) tonic spiking
(B) phasic spiking
(C) tonic bursting
(D) phasic bursting

(E) mixed mode
(F) spike frequency adaptation
(G) Class 1 excitable
(H) Class 2 excitable

(I) spike latency
(J) subthreshold oscillations
(K) resonator
(L) integrator

(M) rebound spike
(N) rebound burst
(O) threshold variability
(P) bistability

(Q) depolarizing after-potential
(R) accommodation
(S) inhibition-induced spiking
(T) inhibition-induced bursting

Note: The diagrams illustrate the behavior of the Izhikevich model.
## Izhikevich Model

| Models                              | biophysically meaningful | phase-spiking | tonic-spiking | bursting | mixed-mode | class 1 excitability | class 2 excitability | spike latency | AP threshold | spike frequency adaptation | integrate-and-fire | integrate-and-fire with adaptation | integrate-and-fire-or-burst | resonate-and-fire | quadratic integrate-and-fire | accommodation | inhibition | conductance | chaotic | # of FLOPS |
|-------------------------------------|--------------------------|---------------|---------------|----------|------------|----------------------|----------------------|----------------|--------------|----------------------------|-----------------------|-------------------------------------------------|--------------------|-----------------|--------------------------|---------|-----------|
| Izhikevich (2003)                   | -                        | +             | +             | +        | +          | +                    | +                    | +             | +            | +                          | +                     | +                                              | +                  | +              | +                        | +       | 13        |
| FitzHugh-Nagumo                    | -                        | +             | -             | -        | -          | +                    | +                    | +             | +            | +                          | +                     | +                                              | -                  | -              | -                        | +       | 72        |
| Hindmarsh-Rose                     | -                        | +             | +             | +        | +          | +                    | +                    | +             | +            | +                          | +                     | +                                              | +                  | +              | +                        | +       | 120       |
| Morris-Lecar                       | +                        | +             | -             | +        | +          | +                    | +                    | +             | +            | +                          | +                     | +                                              | +                  | +              | +                        | +       | 600       |
| Wilson                              | -                        | +             | +             | +        | +          | +                    | +                    | +             | +            | +                          | +                     | +                                              | +                  | +              | +                        | +       | 180       |
| Hodgkin-Huxley                     | +                        | +             | +             | +        | +          | +                    | +                    | +             | +            | +                          | +                     | +                                              | +                  | +              | +                        | +       | 1200      |
Solving via Euler Integration

\[
\frac{dx}{dt} = f(x)
\]

\[
dx = f(x) \cdot dt
\]

\[
\Delta x \approx f(x) \cdot \Delta t
\]

\[
x_{n+1} = x_n + f(x) \cdot \Delta t
\]

\[
x_0 = x(0)
\]
Better Integrators Exist

- Runge-Kutta >> Euler for many problems despite greater number of evaluations per time step
- Time steps can be much greater
- Time can be ignored in many problems...

\[
\begin{align*}
\frac{dx}{dt} &= f(t,x) \\
x_{n+1} &= x_n + \frac{h}{6}(a + 2b + 2c + d) \\
a &= f(t_n, x_n) \\
b &= f(t_n + \frac{h}{2}, x_n + \frac{h}{2}a) \\
c &= f(t_n + \frac{h}{2}, x_n + \frac{h}{2}b) \\
d &= f(t_n + h, x_n + hc)
\end{align*}
\]
Building Oscillators: Brown Half-Centered Oscillator

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Lamprey example

Ekeberg 1993
Fig. 2. Photograph of the robot.
Fig. 4. Pacemaker coordination. Legs controlled by each pacemaker are indicated by subscripts.
Fig. 10. Generation of the tripod gait. The two stable configurations of the central network during the tripod gait are shown. Neurons that are active are filled, inhibited neurons are white.
Case Neuron model

\[
C_i \frac{dV_i}{dt} = -\frac{V_i}{R_i} + \sum w_{ij} f_j(V_j) + INT_i + EXT_i
\]
high speed gait
Medium Speed
Low Speed

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Lesion Experiments
Forward swing L1 Sensor
Taga, 1995

Fig. 1. Model for human locomotion
Taga dynamic model

Fig. 2. The model of the human body, composed of the HAT (head, arms, and trunk), pelvis, thigh, shank, and foot. There are seven joints, two each at the hips, knees, and ankles, and one at the trunk.
Matsuoka Oscillator

\[
\begin{align*}
\tau_i \dot{u}_i &= -u_i - \beta f(v_i) + \sum_{j \neq i} w_{ij} f(u_j) + u_0, \\
\dot{v}_i &= -v_i + f(u_i), \\
(f(u) &= \max(0, u)), \quad (i = 1, 2)
\end{align*}
\]
Global State

Fig. 3. Global pattern of movement within a gait cycle represented by a cyclic sequence of the global states. A gait cycle is defined as the time interval between successive instances of initial foot-to-floor contact with the same foot. $s_{gh}$ is determined by the global angle $\phi$ and the position of the foot contacting the ground. The single-support phase is divided into two periods: the first half, when the global angle is less than $\pi/2$, and the second half, when the global angle exceeds $\pi/2$. 

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Global Angle

• Between CoG and COP

\[ \phi = \cos^{-1} \left[ \frac{(x_{cp} - x_{cg})}{(x_{cp} - x_{cg})^2 + (y_{cg} - y_{cp})^2} \right]^{\frac{1}{2}} \]  

(5)
Oscillation

A Neural Rhythm Generator

1 2

5 3 4

6 7 8

Neural oscillators:

trunk neural oscillator

hip neural oscillator

knee neural oscillator

ankle neural oscillator

Neural oscillators:

left

right

B Muscles

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

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Correlation of muscle and neural activations
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Fig. 6. A Stick figure of walking movement. Given a set of initial conditions, the walking movement converged to a steady state. The stick figure was traced every 0.2 s. B Stick figure of walking movement in the steady state within a gait cycle. The stick figure was traced every 0.01 s.
Fig. 7. Activities of the neural oscillators. The time courses of the state variables of each neuron are shown.

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Neural Activities

\[ u_i \]

1, 2 \quad \text{trunk oscillator}
3, 4 \quad \text{hip oscillator}
7, 8 \quad \text{knee oscillator}
11, 12 \quad \text{ankle oscillator}

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Muscle Torques

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Oscillators network

Fig. 3. CPG network for (a) Patrush and (b) Tekken. The subscripts \( i, j = 1, 2, 3, 4 \) correspond to LF, LH, RF, RH. L, R, F and H denote the left, right, fore or hind legs, respectively.
Equations

\[
\begin{align*}
\tau \dot{u}_{e,\mu} &= -u_{e,\mu} + w_{fe} y_{f,\mu} - \beta v_{e,\mu} \\
&\quad + u_0 + \text{Feed}_{e,\mu} + \sum_{j=1}^{n} w_{ij} y_{e,\mu} \\
\gamma_{e,\mu} &= \max (u_{e,\mu}, 0) \\
\tau \dot{v}_{e,\mu} &= -v_{e,\mu} + y_{e,\mu}.
\end{align*}
\]

\[Feed_{e,\mu} = k_{\mu} (\theta - \theta_0), \quad Feed_{f,\mu} = -Feed_{e,\mu}\]

- Matsuoka oscillator
- e: Extensor
- f: flexor
- Feedback: joint angles and sensors
Tekken using spring/damper system

- implemented electronically
- varies depending on state
Fig. 18. Photographs of walking on irregular terrain: (a) walking over a step 4 cm in height; (b) walking up and down a slope of 10° in a forward direction; (c) walking over slopes of 3 and 5° in a sideways direction; (d) walking over a series of obstacles 2 cm in height.
Building Biologically Realistic Legs

• Human body has 244 Kinematics DOFs and ~660 muscles
• more muscles than are needed(!)
• Biarticulate muscles are ubiquitous.
Leg Muscle
Redrawn from Prilutsky and Zatsiorsky 2002

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Klein et al 2008
Transfer of power via Gastrocnemius

Klein et al 2008

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Figure 4. Ankle work done by SO, GA and both SO and GA together during return from squat.

*Klein et al 2008*
Figure 6. Max power at ankle versus delay between GA activation and SO activation.

Klein et al 2008