Topology-Transparent Duty Cycling for Wireless Sensor Networks

Yu Chen[1], Eric Fleury[2] and Violet R. Syrotiuk[3]

[1,2] ARES/INRIA
INSA de Lyon, France
[1]Yu.Chen@inrialpes.fr, [2]Eric.Fleury@inria.fr

[3] Computer Science& Engineering
Arizona State University
syrotiuk@asu.edu

Abstract

Our goal is to save energy in wireless sensor networks *(WSNs) by periodic duty-cycling of sensor nodes. We schedule sensor nodes between active (transmit or receive) and sleep modes while bounding packet latency in the presence of collisions. In order to support a dynamic WSN topology, we focus on topology-transparent approaches to scheduling; these are independent of detailed topology information. Much work has been done on topology-transparent scheduling in which all nodes are active. In this work, we examine the connection between topology-transparent duty-cycling and such* non-sleeping *schedules. This suggests a way to construct topology-transparent duty-cycling schedules. We analyse the performance of topology-transparent schedules with a focus on throughput in the worst case. A construction of topology-transparent duty-cycling schedules based on a topology-transparent non-sleeping schedule is proposed. The constructed schedule achieves the maximum average throughput in the worst case if the given non-sleeping schedule satisfies certain properties.*

1 Introduction

Wireless sensor networking has been a growing research area for the last years. It has a wide range of potential applications, such as environment monitoring, smart spaces, medical systems and robotic exploration. In sensor networks, sensor nodes are normally battery-operated and energy efficiency has become one of the most important constraints on sensor networks [1, 12]. Studies have identified that idle listening is a significant consumer of power [17, 14, 24, 21, 26, 15]. Previous works (e.g., [24, 25, 4, 15]) show that, in networks where the traffic load is light most of the time, energy efficiency can be achieved by periodic duty cycling of sensor nodes, that is, scheduling sensor nodes between active and sleep mode.

An efficient synchronization mechanism is required by most duty cycling schemes (e.g., [24, 15]). Compared to a TDMA scheme, duty cycling schemes require a much looser synchronization. This low precision requirement enables us to save energy by reducing message exchanges for synchronization. Efficient synchronization protocols have been proposed for sensor networks (e.g., [8, 11, 20, 23, 10]); in particular, works have been done on synchronization for duty cycling (e.g., [23, 10]). In this work, we assume an efficient synchronization scheme is available and we describe system behavior in terms of time slots.

Although light traffic networks are the scenarios for which duty cycling schemes are primarily considered, it does not mean collision is no longer a concern. Consider a network in which each node is scheduled to be awake in one of k slots. Since a node has to wait until the receiver wakes up before it can forward the packet, transmissions from neighbors, which were distributed in k slots, now happen in one slot, making a collision very likely. In this work, we aim to save energy by scheduling periodic nodes' duty cycles while preserving communication connectivity and bounding packet latency in the presence of collisions.

In order to handle dynamic topology, we focus on *topology-transparent* approaches that are independent of topology changes. Given a set of networks $\mathcal{N}$, we say a schedule is topology-transparent with respect to $\mathcal{N}$ if, for any network in $\mathcal{N}$, it allows each node to transmit without collisions to each neighbor node infinitely often; the range of $\mathcal{N}$ represents the level of transparency. Such an approach tolerates topology changes since for every possible topology, given the same topology-transparent schedule, connectivity of each link is guaranteed. As existing works on topology-transparent schedules [2, 13, 3, 6], our work focuses on topology-transparency in networks where the number of nodes and node degrees are no more than given

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bounds. Research has been done on topology-transparent schedules where no node is scheduled to sleep [2, 13, 3]; we refer to such schedules as *non-sleeping schedules*.

In this work, we consider duty cycling of sensor nodes which aims to save energy by switching nodes between active and sleep mode. Letting α_T and α_R be parameters that capture applications' requirement on energy efficiency, we focus on schedules in which the number of nodes that are allowed to transmit (receive resp.) per slot is no more than α_T (α_R resp.); we call such a schedule an (α_T, α_R)-schedule. Such schedules have been considered in [6], where the requirements on topology-transparent (α_T, α_R)-schedules are described by a general combinatorial model and a graph construction algorithm is examined with the focus on a special type of schedules in which the number of nodes transmitting and receiving per slot are equal. Different from [6], our work focuses on general schedules and has three main contributions.

First, we examine in section 4 the connection between topology-transparent (α_T, α_R)-schedules and non-sleeping schedules. Our analysis indicates that, given bounds on the number of nodes and node degrees, a necessary condition for the existence of a topology-transparent (α_T, α_R)-schedule, for any α_T and α_R, is the existence of a topology-transparent non-sleeping schedule; furthermore, any construction of a topology-transparent (α_T, α_R)-schedule involves the construction of a topology-transparent non-sleeping schedule.

Secondly, we present our analyses in section 5 on schedules' performance; as most works on topology-transparent schedules (e.g., [13, 3]), we consider throughput in the worst case, that is, each node has the maximum degree and each neighbor has a packet to transmit. We show that the average throughput in the worst case only depends on the *number* of nodes that are allowed to transmit and receive per slot. An upper bound on the average throughput, together with the optimal numbers of transmitters and receivers per slot to achieve this upper bound, is given for both general schedules and (α_T, α_R)-schedules for given α_T and α_R. Our analyses indicate that a necessary condition for the existence of a topology-transparent (α_T, α_R)-schedule to achieve this upper bound is the existence of a topology-transparent non-sleeping schedule that has certain properties.

Thirdly, we investigate in section 6 whether a topology-transparent non-sleeping schedule (that has the appropriate properties) can be converted to a topology-transparent (α_T, α_R)-schedule (that has good performance). This is motivated by our observation in sections 4 and 5 that a necessary condition of the existence of a topology-transparent (α_T, α_R)-schedule (that has good performance) is the existence of a non-sleeping topology-transparent schedule (that satisfies certain properties), and any construction of a topology-transparent (α_T, α_R)-schedule involves the construction of a topology-transparent non-sleeping schedule. Our answer to this question is positive and we present a construction of an (α_T, α_R)-schedule based on a non-sleeping schedule. Such a conversion is feasible since much work has been done on the construction of topology-transparent non-sleeping schedules (e.g. [2, 13, 22, 3]), and it has been pointed out in [22, 3] that topology-transparent non-sleeping schedules can be constructed by cover-free families, which have been well investigated by numerous researchers (e.g., [9, 7, 16, 19, 18, 5]).

Our construction is very straightforward and it shows good properties. We prove that given *any* topology-transparent non-sleeping schedule, a topology-transparent (α_T, α_R)-schedule can be constructed by our approach for any α_T and α_R. The performance of the constructed schedule depends on the number of transmitters per slot in the non-sleeping schedule — given n and D, the constructed schedule is optimal in terms of average worst-case throughput if the number of transmitters per slot in the non-sleeping schedule is at least $\min\{\alpha_T, \lceil \frac{n-D}{D} \rceil\}$, otherwise the larger is the minimum number of transmitters per slot, the better average throughput can be achieved. We also give a lower bound on the minimum throughput.

2 Related work

Research has been done on topology-transparent non-sleeping schedules [2, 13, 22, 3]. These works aim to guarantee communication between any pair of adjacent nodes in the presence of collisions. It is pointed out in [22, 3] that topology-transparent non-sleeping schedules can be constructed by a cover-free family, and the constructions in [2, 13] are indeed to construct a cover-free family using an orthogonal array. Cover-free families were first introduced in [9] and have been considered in different subjects such as information theory, combinatorics and group testing by numerous researchers (e.g., [7, 16, 19, 18, 5]). All these research results provide a strong base for the construction of a topology-transparent duty cycling based on a topology-transparent non-sleeping schedule. The details of how to construct non-sleeping schedules and cover-free families are out of the scope of this paper; see [3, 5] for more information.

Topology-transparent duty cycling which aims to achieve energy efficiency by scheduling nodes to sleep was first considered in [6], in which a general combinatorial model is described. In [6], a graph construction algorithm is examined with the focus on a special type of schedules in which the number of nodes transmitting and receiving per slot are equal. Note such schedules are optimal when the cost to transmit and receive are the same order of magnitude; furthermore, the number of interferences is bounded in the constructed schedules. In this work, we focus on general cases and a different construction is considered.

Throughput in the worst case has been investigated in [13, 3] for topology-transparent non-sleeping schedules that are constructed in certain ways. The focus in [13] is on schedules constructed using *polynomial functions of degree k mod p*; note polynomial function is only one of the standard constructions of cover-free family based on orthogonal arrays. The focus in [3] are on schedules based on cover-free families constructed from *orthogonal arrays* and *Steiner systems*. Different from these works, we consider throughput for *general* schedules; by saying general schedules, we mean there is no constraint on how schedules are constructed, schedules can be topology-transparent or not, and schedules can be non-sleeping or not.

3 System model

We assume time is structured into discrete units called *slots*. A schedule of node activities is represented by a pair $\langle T, R\rangle$, where T and R are two disjoint arrays with the same length, say L, such that $\forall i \in [0, L-1]$, $T[i] \subseteq V$ and $R[i] \subseteq V$; $T[i]$ and $R[i]$ are the sets of nodes that are eligible to transmit and receive respectively in slots $i + Ll$, $l = 0, 1, \ldots$, while other nodes turn off their radio and stay in sleep mode. We say T is the *transmission schedule* and R is the *reception schedule*. We call the L continuous slots $lL, lL+1, \ldots, lL+L-1$, $\forall l \geq 0$, a *frame*, and L is the *frame length*. Given a schedule $\langle T, R\rangle$ and a node x, we denote the set of slots in which x is allowed to transmit and receive by $tran_{\langle T,R\rangle}(x) = \{i \in [0, L-1] | x \in T[i]\}$ and $recv_{\langle T,R\rangle}(x) = \{i \in [0, L-1] | x \in R[i]\}$ respectively. The subscript $\langle T, R\rangle$ is omitted when it is clear from context.

In this work, our goal is to design a schedule of nodes' activities where nodes are put into sleep mode to meet applications' requirement on energy saving. We use two parameters α_T and α_R to describe applications' requirement on energy efficiency: the maximum number of nodes that are allowed to transmit and receive per slot are no more than α_T and α_R respectively. Given α_T and α_R, we aim to design a schedule $\langle T, R\rangle$ that has the desirable property, that is, $\forall i \in [0, L-1]$, $|T[i]| \leq \alpha_T$ and $|R[i]| \leq \alpha_R$, where $L = |T| = |R|$; we call such a schedule *an* (α_T, α_R)-*schedule*. We say a schedule $\langle T, R\rangle$ is *a non-sleeping schedule* if all nodes are active in each slot, that is, $\forall i \in [0, L-1]$, $T[i] \cup R[i] = V$. Since such a schedule can be completely decided by T, we abbreviate the representation $\langle T, R\rangle$ to $\langle T\rangle$.

Our focus is on *topology-transparent* scheduling. Given a set $\mathcal{N}$ of networks, a schedule is topology-transparent for networks in $\mathcal{N}$ if it ensures that, for any network in $\mathcal{N}$, each node is guaranteed to successfully transmit a packet to each adjacent node in at least one slot of each frame. In practice, packet transmissions might fail due to many reasons. We restrict our attention to failures caused by *collisions* — by saying node x is guaranteed to successfully transmit to node y in some slot, we mean in this slot y is eligible to receive and x is the only node in y's neighborhood that is allowed to transmit. Given two adjacent nodes x and y, if there exist slots in which node x is guaranteed to successfully transmit to node y, we say the *connectivity* from node x to y is guaranteed. As most works on topology-transparency, given two parameters n and D, $2 \leq D \leq n$, we consider a class of networks, denote by $\mathcal{N}_n^D$, that consist of at most n nodes, denoted by V_n, and in which node degrees are no more than D.

4 Requirements on topology-transparency

In this section we consider the requirements on topology-transparent schedules. Informally, a schedule is topology-transparent if the *connectivity* between any pair of adjacent nodes is guaranteed in any network where the number of nodes and node degrees are bounded by given values. The requirement on topology-transparent duty cycling has been proposed in [6]. Here we give an equivalent requirement which presents a connection between topology-transparent (α_T, α_R)-schedules and non-sleeping schedules. This connection implies that any construction of a topology-transparent (α_T, α_R)-schedule involves the construction of a topology-transparent non-sleeping schedule. It also suggests a way to construct a topology-transparent (α_T, α_R)-schedule based on a topology-transparent non-sleeping schedule.

Since our construction is based on a topology-transparent non-sleeping schedule, we present in Requirement 1 the requirement proposed in [3] on topology-transparent non-sleeping schedules. Given a schedule $\langle T, R\rangle$, a node x and a set Y of nodes, we define a denotation to represent the set of slots in which node x is guaranteed to successfully transmit to a node $y \in Y$ whose neighborhood is $Y - \{y\} \cup \{x\}$. Intuitively, in each of these slots, x is the only node that is allowed to transmit among the neighbors of y.

$$freeSlots_{\langle T,R\rangle}(x, Y) \equiv tran_{\langle T,R\rangle}(x) - \bigcup_{y \in Y} tran_{\langle T,R\rangle}(y)$$

Note given any two nodes x, y and any set $S \subseteq V_n - \{x, y\}$ of $D-1$ nodes, there exists network in $\mathcal{N}_n^D$ such that x and y are adjacent and y's neighborhood is $\{x\} \cup S$. Thus a topology-transparent schedule should guarantee $\forall x, \forall y$, $\forall\ S \subseteq V_n - \{x, y\}$ such that $|S| = D-1$, $freeSlots(x, S \cup \{y\}) \neq \emptyset$, which is equivalent to $\forall x$, $\forall$ $Y \subseteq V_n - \{x\}$ such that $|Y| = D$, $freeSlots(x, Y) \neq \emptyset$. On the other hand, if this condition is true, for any network in $\mathcal{N}_n^D$, given any two adjacent nodes x and y, denoting y's neighborhood by $S \cup \{x\}$, we have $freeSlots(x, S \cup \{y\}) \subseteq freeSlots(x, Y) \neq \emptyset$, where Y is some set of D nodes such that $S \cup \{y\} \subseteq Y$, thus the connectivity from x to y is guaranteed. This requirement is formally addressed below.

Requirement 1 *[3] A non-sleeping schedule $\langle T \rangle$ ensures a successful transmission in each frame between any pair of adjacent nodes in any network in $\mathcal{N}_n^D$ provided that*

- $\forall x \in V_n$, $\forall$ *set* $Y \subseteq V_n - \{x\}$ *of* D *nodes,*

$$freeSlots_{\langle T \rangle}(x, Y) \neq \emptyset$$

We give in Requirement 2 the requirements proposed in [6] on topology-transparent schedules where nodes might switch to sleep mode. Intuitively, $\sigma(x, y)$ denotes the set of slots in which node x can successfully transmit to node y. It is required at least one slot in $\sigma(x, y)$ is collision-free for any two nodes x and y under any possible neighborhood of y.

Requirement 2 *[6] A schedule $\langle T, R \rangle$ ensures a successful transmission in each frame between any pair of adjacent nodes in any network in $\mathcal{N}_n^D$ provided that*

- $\forall x, y \in V$, $x \neq y$, *and* $\forall$ *set of* $d \leq D - 1$ *nodes* $\{y_1, \ldots, y_d\} \subseteq V_n - \{x, y\}$,

$$\bigcup_{i=1}^{d} \sigma_{\langle T,R \rangle}(y_i, y) \not\supseteq \sigma_{\langle T,R \rangle}(x, y)$$

where, letting $L = |T| = |R|$, *we define*

$$\begin{aligned} \sigma_{\langle T,R \rangle}(a, b) &\equiv \{j \in [0, L-1] | a \in T[j] \wedge b \in R[j]\} \\ &= tran_{\langle T,R \rangle}(a) \cap recv_{\langle T,R \rangle}(b) \end{aligned}$$

Since much work has been done on topology-transparent non-sleeping schedule, here we examine the connection between topology-transparent non-sleeping schedules and duty cycling. We propose a requirement equivalent to Requirement 2. Intuitively, given any node x and any set Y of D nodes, for any $y_k \in Y$, condition (1) requires the existence of slots in which x is the only node in y_k's neighborhood that is allowed to transmit and condition (2) requires y_k is eligible to receive in at least one of these slots. Note condition (1) is implied by condition (2); we write it separately to emphasize that condition (1) states that the non-sleeping schedule $\langle T \rangle$ should be topology-transparent.

Requirement 3 *A schedule $\langle T, R \rangle$ ensures a successful transmission in each frame between any pair of adjacent nodes in any network in $\mathcal{N}_n^D$ provided that*

- $\forall$ $x \in V_n$ *and* $\forall$ *set of* D *nodes* $Y = \{y_0, \ldots, y_{D-1}\} \subseteq V_n - \{x\}$,

$$freeSlots_{\langle T,R \rangle}(x, Y) \neq \emptyset \quad (1)$$

$$\forall k \in [0, D-1], recv_{\langle T,R \rangle}(y_k) \cap freeSlots_{\langle T,R \rangle}(x, Y) \neq \emptyset \quad (2)$$

Theorem 1 *Requirement 2 and Requirement 3 are equivalent.*

Proof. We first prove if a schedule satisfies Requirement 2, then it satisfies Requirement 3; we only need to prove condition (2) of Requirement 3, which implies condition (1). Assume in contradiction that condition (2) is not true, that is, $\exists$ node x and D nodes $y_0, \ldots, y_{D-1}$, and $k \in [0, D-1]$, such that $recv(y_k) \cap freeSlots(x, \{y_0, \ldots, y_{D-1}\}) = \emptyset$. By the definition of $freeSlots$, we have $recv(y_k) \cap \big(tran(x) - \cup_{i=0}^{D-1} tran(y_i)\big) = \emptyset$, which implies $recv(y_k) \cap tran(x) \subseteq \cup_{i=0}^{D-1} tran(y_i)$, that is, $\sigma(x, y_k) \subseteq \cup_{i=0}^{D-1} tran(y_i)$. Now for any $t \in \sigma(x, y_k)$, we prove $\exists k' \in [0, D-1], k' \neq k$, such that $t \in \sigma(y_{k'}, y_k)$ as follows: since $\sigma(x, y_k) \cap tran(y_k) = tran(x) \cap recv(y_k) \cap tran(y_k) = \emptyset$ and $\sigma(x, y_k) \subseteq \cup_{i=0}^{D-1} tran(y_i)$, we have $\exists k' \in [0, D-1], k' \neq k$, such that $t \in tran(y_{k'})$; since $t \in recv(y_k)$, we have $t \in \sigma(y_{k'}, y_k)$. Thus we prove $\cup_{i \neq k, i=0}^{D-1} \sigma(y_i, y_k) \supseteq \sigma(x, y_k)$, which contradicts to Requirement 2.

Now we prove if a schedule satisfies Requirement 3, then it satisfies Requirement 2. Consider $\forall x, y \in V$, $x \neq y$, and $d \leq D - 1$ nodes $y_1, \ldots, y_d \in V_n - \{x, y\}$. By condition (2) of Requirement 3, we have $recv(y) \cap freeSlots(x, \{y_1, \ldots, y_d\}) \neq \emptyset$, which implies $\sigma(x, y) \cap (tran(x) - \cup_{i=1}^{d} tran(y_i)) \neq \emptyset$. Letting t be a slot in $\sigma(x, y) \cap (tran(x) - \cup_{i=1}^{d} tran(y_i))$, we have $t \notin \cup_{i=1}^{d} tran(y_i)$. Thus $\forall i \in [1, d]$, $t \notin \sigma(y_i, y)$ and we prove $\cup_{i=1}^{d} \sigma(y_i, y) \not\supseteq \sigma(x, y)$. ■

Given any topology-transparent schedule $\langle T, R \rangle$, Requirement 3 states that the non-sleeping schedule $\langle T \rangle$ is also topology-transparent, which implies that any construction of a topology-transparent (α_T, α_R)-schedule involves the construction of a topology-transparent non-sleeping schedule. Furthermore, condition (2) of Requirement 3 indicates that given a non-sleeping schedule, as far as connectivity is concerned, it is not necessary to keep all the non-transmitting nodes active; instead, nodes can be scheduled to sleep provided that they are active in at least one of the free slots. Based on this observation, we consider constructing a topology-transparent schedule in two steps: we first construct a topology-transparent non-sleeping schedule, and then reduce the numbers of nodes transmitting and receiving per slot without violating the topology-transparency requirement. In the sequel, we discuss in section 5 the throughput achievable in networks with the number of nodes and node degrees less than given bounds; then we propose our construction in section 6, with the focus on the second step since much work has been done on constructing topology-transparent non-sleeping schedules,

5 Upper bounds on throughput in $\mathcal{N}_n^D$

In this section, we discuss the throughput of *general schedules* and *general* (α_T, α_R)*-schedules* for networks in $\mathcal{N}_n^D$. Our focus is on throughput in the worst case, that is, each node has D neighbors and each neighbor has a packet to transmit in each of the slots in which it is allowed to transmit. We first give the definitions of *the minimum throughput* in the worst case and *the average throughput* in the worst case, then we present our analyses on the average throughput. We give upper bounds on the average throughput that can be achieved by general schedules and general (α_T, α_R)-schedules in the worst case, as well as the condition for a schedule to achieve the upper bound.

Given any pair of adjacent nodes x, y, we consider the number of guaranteed successful transmissions from x to y. Letting S be the set of y's neighbors other than x, we define $\mathcal{T}_{\langle T,R\rangle}(x, y, S)$ as the set of slots in which transmissions from x to y are guaranteed to be successful under schedule $\langle T, R\rangle$:

$$\mathcal{T}_{\langle T,R\rangle}(x, y, S) \equiv recv_{\langle T,R\rangle}(y) \cap freeSlots_{\langle T,R\rangle}(x, \{y\} \cup S)$$

Note $\mathcal{T}_{\langle T,R\rangle}(x, y, S) \supseteq \mathcal{T}_{\langle T,R\rangle}(x, y, S')$ if $S \subseteq S'$. Since we focus on throughput in the worst case, in our definition of the minimum throughput, we only need to consider S such that $|S| = D - 1$.

Definition 1 *Given a schedule $\langle T, R\rangle$, its minimum worst-case throughput $Thr^{min}_{\langle T,R\rangle}$ in $\mathcal{N}_n^D$ is defined below, where $L = |T| = |R|$:*

$$Thr^{min}_{\langle T,R\rangle} \equiv \min_{\forall x,y \in V_n, S \subseteq V_n - \{x,y\}, |S|=D-1} \left\{ \frac{|\mathcal{T}_{\langle T,R\rangle}(x, y, S)|}{L} \right\}$$

It is easy to see the average value of $|\mathcal{T}_{\langle T,R\rangle}(x, y, S')|$ over all the S' such that $|S'| \le D-1$ is no less than the average value of $|\mathcal{T}_{\langle T,R\rangle}(x, y, S)|$ over all S such that $|S| = D-1$. So the number of guaranteed successful transmissions in each frame from x to y averaged over all the possible y's neighborhoods in $\mathcal{N}_n^D$ is at least:

$$tx_{\langle T,R\rangle}(x, y) \equiv \frac{\sum_{S \subseteq V_n - \{x,y\}, |S|=D-1} |\mathcal{T}_{\langle T,R\rangle}(x, y, S)|}{\binom{n-2}{D-1}}$$

We define the average worst-case throughput $Thr^{ave}_{\langle T,R\rangle}$ of a schedule $\langle T, R\rangle$ in $\mathcal{N}_n^D$ as the ratio of the average number of $tx_{\langle T,R\rangle}(x, y)$ over all pairs of nodes x and y to the frame length.

Definition 2 *Given a schedule $\langle T, R\rangle$, its average worst-case throughput $Thr^{ave}_{\langle T,R\rangle}$ in networks in $\mathcal{N}_n^D$ is defined below, where $L = |T| = |R|$:*

$$Thr^{ave}_{\langle T,R\rangle} \equiv \frac{\sum_{\forall x,y \in V_n} tx_{\langle T,R\rangle}(x, y)}{n(n-1)L} = \frac{F_{\langle T,R\rangle}}{n(n-1)\binom{n-2}{D-1} L}$$

where

$$F_{\langle T,R\rangle} \equiv \sum_{x,y \in V_n} \sum_{S \subseteq V_n - \{x,y\}, |S|=D-1} |\mathcal{T}_{\langle T,R\rangle}(x, y, S)|$$

Since our focus is on throughput in the worst case, in the sequel we abbreviate “throughput in the worst case” to “throughput” for presentation simplicity. Note $Thr^{min}_{\langle T,R\rangle}$ and $Thr^{ave}_{\langle T,R\rangle}$ are not defined particularly for topology-transparent schedules. A schedule $\langle T, R\rangle$ is topology-transparent if and only if $Thr^{min}_{\langle T,R\rangle}>0$. If a schedule $\langle T, R\rangle$ is not topology-transparent, we have $Thr^{min}_{\langle T,R\rangle}=0$, that is, there exists two nodes x, y and y's neighborhood S such that no slot exists in which the transmissions from x to y is guaranteed to be successful; the average throughput $Thr^{ave}_{\langle T,R\rangle}$ can still be computed according to Definition 2.

Requirements 2 and 3 show whether a schedule is topology-transparent depends on how nodes are scheduled to transmit and receive. The theorem below states that the average throughput only depends on the *number* of transmitters and receivers per slot; furthermore, higher average throughput can be achieved by allowing more nodes to receive. So if there is no constraint on the numbers of receivers and transmitters, the maximum throughput is achieved by a non-sleeping schedule. Note it is not necessary true if only specific topologes are considered, as shown by an example presented later in section 5.2.

Theorem 2 *Given a schedule $\langle T, R\rangle$, we denote $L = |T| = |R|$. The average throughput $Thr^{ave}_{\langle T,R\rangle}$ of $\langle T, R\rangle$ in networks $\mathcal{N}_n^D$ is*

$$Thr^{ave}_{\langle T,R\rangle} = \frac{\sum_{i=0}^{L-1} |T[i]| \cdot |R[i]| \binom{n-|T[i]|-1}{D-1}}{n(n-1)\binom{n-2}{D-1} L}$$

Proof. We define $\mathcal{C}_{\langle T,R\rangle}(i)$ as the set of tuples $\langle x, y, S\rangle$ such that $i \in \mathcal{T}_{\langle T,R\rangle}(x, y, S)$, where x, y are two different nodes and S is any set of $D-1$ nodes other than x and y. Formally,

$$\mathcal{C}_{\langle T,R\rangle}(i) \equiv \{\langle x, y, S\rangle \quad | i \in \mathcal{T}_{\langle T,R\rangle}(x, y, S), \text{ where } x, y \in V_n, x \ne y, S \subseteq V_n - \{x, y\}, |S| = D-1\}$$

We have $F_{\langle T,R\rangle} = \sum_{i\in[0,L-1]} |\mathcal{C}_{\langle T,R\rangle}(i)|$. Since given i, for x,y and S such that $i \in \mathcal{T}_{\langle T,R\rangle}(x, y, S)$, x can be any node in $T[i]$, y can be any node in $R[i]$, and S can be any $D-1$ nodes of those other than nodes in $T[i]$ and y, we have $\left|\mathcal{C}_{\langle T,R\rangle}(i)\right| = |T[i]| \cdot |R[i]| \binom{n-|T[i]|-1}{D-1}$ and the theorem is proved. ■

In the rest of this section, we examine the maximum average throughput in $\mathcal{N}_n^D$ that can be achieved by general schedules and by general (α_T, α_R)-schedules for given α_T

and α_R. Before that, we present two properties of function $g_{n,D}(x) = \frac{x\binom{n-x}{D}}{n\binom{n-1}{D}}$ for integer x. These properties will be used in our proofs and x will represent the number of nodes that are allowed to transmit in a slot. Intuitively, $g_{n,D}(x)$ represents the throughput of a non-sleeping schedule in which the number of transmitters per slot is fixed at x.

(1) $\forall x \in [0, n-1]$, $g_{n,D}(x) \leq \frac{nD^D}{(n-D)(D+1)^{D+1}}$.

(2) $\exists x_0 \in \left\{\left\lfloor\frac{n-D}{D+1}\right\rfloor, \left\lceil\frac{n-D}{D+1}\right\rceil\right\}$, such that $g_{n,D}(x_0) \geq g_{n,D}(x)$, $\forall\, x \in [0, n-1]$.

Property (1) is true since $g_{n,D}(x) = x\,\frac{(n-x)}{n}\frac{(n-1-x)}{n-1}\cdots\frac{(n-(D-1)-x)}{n-(D-1)}\,\frac{1}{n-D} = x\left(1-\frac{x}{n}\right)\left(1-\frac{x}{n-1}\right)\cdots\left(1-\frac{x}{n-(D-1)}\right)\left(\frac{1}{n-D}\right) \leq x\left(1-\frac{x}{n}\right)^D\left(\frac{1}{n-D}\right) \leq \frac{nD^D}{(n-D)(D+1)^{D+1}}$. Property (2) can be proved by showing the following properties for $x \leq n-(D-1)$: (a) $g_{n,D}(x)$ is monotonically increasing up to some point and then monotonically decreasing, and (b) $g_{n,D}(x) \geq g_{n,D}(x+1)$ if and only $x \geq \frac{n-D}{D+1}$ since $\frac{g_{n,D}(x)}{g_{n,D}(x+1)} = \frac{x(n-x)}{(x+1)(n-D-x)}$.

5.1 An upper bound on average throughput

In this section, we consider the average throughput that can be achieved in networks in $\mathcal{N}_n^D$ by general schedules, where there is no constraint on the numbers of transmitters and receivers per slot. The theorem below presents an upper bound on the average throughput, as well as the condition for a schedule to achieve this upper bound: the number of transmitters in each slot is about $\frac{n-D}{D+1}$ and the number of receivers in each slot is about $n - \frac{n-D}{D+1}$.

Theorem 3 ***(An Upper Bound on Average Throughput of General Schedules in $\mathcal{N}_n^D$)*** *Given any schedule $\langle T, R\rangle$, letting $Thr^{ave}_{\langle T,R\rangle}$ be its average throughput in networks in $\mathcal{N}_n^D$, we have*

$$Thr^{ave}_{\langle T,R\rangle} \leq \frac{\alpha_T^\star\binom{n-\alpha_T^\star}{D}}{n\binom{n-1}{D}} \leq \frac{nD^D}{(n-D)(D+1)^{(D+1)}}$$

where

$$\alpha_T^\star = \begin{cases} \left\lfloor\frac{n-D}{D+1}\right\rfloor, \textit{if } \left\lfloor\frac{n-D}{D+1}\right\rfloor\binom{n-\left\lfloor\frac{n-D}{D+1}\right\rfloor}{D} \geq \left\lceil\frac{n-D}{D+1}\right\rceil\binom{n-\left\lceil\frac{n-D}{D+1}\right\rceil}{D} \\ \left\lceil\frac{n-D}{D+1}\right\rceil, \textit{otherwise} \end{cases}$$

Furthermore, defining $Thr^\star \equiv \frac{\alpha_T^\star\binom{n-\alpha_T^\star}{D}}{n\binom{n-1}{D}}$, we have $Thr^{ave}_{\langle T,R\rangle} = Thr^\star$ if and only if $\langle T, R\rangle$ is a non-sleeping schedule such that $\forall i \in [0, L-1]$, $|T[i]| = \alpha_T^\star$ and $|R[i]| = n - \alpha_T^\star$, where $L = |T| = |R|$.

Proof. Given any schedule $\langle T, R\rangle$, we have $|R[i]| \leq n - |T[i]|, \forall i \in [0, L-1]$. By Theorem 2, we have

$$\begin{aligned} Thr^{ave}_{\langle T,R\rangle} &= \frac{\sum_{i=0}^{L-1}|T[i]||R[i]|\binom{n-|T[i]|-1}{D-1}}{n(n-1)\binom{n-2}{D-1}L} \\ &\leq \frac{\sum_{i=0}^{L-1}|T[i]|(n-|T[i]|)\binom{n-|T[i]|-1}{D-1}}{n(n-1)\binom{n-2}{D-1}L} \\ &= \frac{\sum_{i=0}^{L-1}|T[i]|\binom{n-|T[i]|}{D}}{n\binom{n-1}{D}L} = \frac{1}{L}\sum_{i=0}^{L-1}g_{n,D}(|T[i]|) \end{aligned}$$

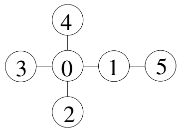
Note $|T[i]| \in [1, n-1]$. By property of function $g_{n,D}(x)$, we have $Thr^{ave}_{\langle T,R\rangle} \leq \frac{nD^D}{(n-D)(D+1)^{D+1}}$. In order to maximize $Thr^{ave}_{\langle T,R\rangle}$, $g_{n,D}(|T[i]|)$ should be maximum for all $i \in [0, L-1]$. By property of $g_{n,D}(x)$, the maximum $g_{n,D}(|T[i]|)$ is achieved when $|T[i]| \in \left\{\left\lfloor\frac{n-D}{D+1}\right\rfloor, \left\lceil\frac{n-D}{D+1}\right\rceil\right\}$. ■

Theorem 3 indicates only schedules in which all the nodes are active can achieve the proposed upper bound. Since the number of transmitters and receivers might be constrained due to energy efficiency, we consider in the next section the throughput of schedules with bounded numbers of transmitters and receivers.

5.2 An upper bound on average throughput of (α_T, α_R)-schedules

When only *connectivity* between any pair of adjacent nodes is considered, Requirement 3 shows it is not necessary to keep all the nodes active. However, Theorem 2 indicates that, the performance is affected by putting nodes into sleep mode if we focus on the average throughput between all pairs of nodes for *all the networks in $\mathcal{N}_n^D$*. It is worth pointing out that, if only a specific topology is considered, it is possible to save energy by scheduling nodes to sleep while preserving the same throughput. An example is given in Figure 1, where nodes are denoted by the numbers in circles and arrays T and R are given. We consider two schedules $\langle T\rangle$ and $\langle T, R\rangle$: schedule $\langle T\rangle$ is a non-sleeping schedule and in schedule $\langle T, R\rangle$ some nodes are scheduled to sleep. It is easy to see these two schedules have the same throughput.

Now we consider the average throughput $Thr^{ave}_{\langle T,R\rangle}$ of an (α_T, α_R)-schedule $\langle T, R\rangle$ in $\mathcal{N}_n^D$. By Theorem 2, we have $Thr^{ave}_{\langle T,R\rangle} = \frac{\sum_{i=0}^{L-1}|T[i]|\cdot|R[i]|\binom{n-|T[i]|-1}{D-1}}{n(n-1)\binom{n-2}{D-1}L} \leq \frac{\alpha_R\sum_{i=0}^{L-1}|T[i]|\binom{n-1-|T[i]|}{D-1}}{n(n-1)\binom{n-2}{D-1}L} = \frac{\alpha_R}{nL}\sum_{i=0}^{L-1}g_{n-1,D-1}(|T[i]|)$. By properties of $g_{n-1,D-1}(x)$, we have $Thr^{ave}_{\langle T,R\rangle} \leq \frac{\alpha_R}{n}$



slots	0	1	2	3	4
T	{0}	{1}	{2,5}	{3}	{4}
R	{1,2,3,4}	{0,5}	{0,1}	{0}	{0}

Figure 1. An example of networks in which throughput can be preserved when nodes are scheduled to sleep

$\frac{(n-1)(D-1)^{D-1}}{(n-D)D^D}$, and the maximum of $g_{n-1,D-1}(|T[i]|)$ is achieved when $|T[i]| \in \left\{\left\lfloor\frac{n-D}{D}\right\rfloor, \left\lceil\frac{n-D}{D}\right\rceil\right\}$. Since $|T[i]|$ is constrained by the requirement that $|T[i]| \le \alpha_T$, we have the following theorem on the average throughput of an (α_T, α_R)-schedule in $\mathcal{N}_n^D$ networks. This theorem implies, in order to achieve the best average throughput, the number of receivers in each slot should be as large as possible, while the number of transmitters should be as close to $\frac{n-D}{D}$ as possible.

Theorem 4 ***(An Upper Bound on Average Throughput of*** (α_T, α_R)***-schedules in*** $\mathcal{N}_n^D$***).*** *Given any* (α_T, α_R)*-schedule* $\langle T, R\rangle$*, letting* $Thr^{ave}_{\langle T,R\rangle}$ *be its average throughput in networks in* $\mathcal{N}_n^D$*, we have*

$$Thr^{ave}_{\langle T,R\rangle} \le \frac{\alpha_R \alpha_T^\star \binom{n-\alpha_T^\star-1}{D-1}}{n(n-1)\binom{n-2}{D-1}} \le \frac{\alpha_R(n-1)(D-1)^{D-1}}{n(n-D)D^D}$$

where $\alpha_T^\star = \min\{\alpha_T, \alpha\}$*, and*

$$\alpha \equiv \begin{cases} \left\lfloor\frac{n-D}{D}\right\rfloor \text{ if } \left\lfloor\frac{n-D}{D}\right\rfloor \binom{n-\lfloor\frac{n-D}{D}\rfloor-1}{D-1} \ge \left\lceil\frac{n-D}{D}\right\rceil \binom{n-\lceil\frac{n-D}{D}\rceil-1}{D-1} \\ \left\lceil\frac{n-D}{D}\right\rceil \text{ otherwise} \end{cases}$$

Furthermore, defining $Thr^\star_{\alpha_R,\alpha_T} \equiv \frac{\alpha_R\cdot\alpha_T^\star\binom{n-\alpha_T^\star-1}{D-1}}{n(n-1)\binom{n-2}{D-1}}$*, we have* $Thr^{ave}_{\langle T,R\rangle} = Thr^\star_{\alpha_R,\alpha_T}$ *if and only if* $\forall i \in [0, L-1]$*,* $|R[i]| = \alpha_R$ *and* $|T[i]| = \alpha_T^\star$*.*

Theorem 4 presents an upper bound on the throughput that is achievable by an (α_T, α_R)-schedule in $\mathcal{N}_n^D$ networks. An interesting question is whether there exists a *topology-transparent* (α_T, α_R)-schedule that achieves this upper bound. Requirement 3 indicates, given any topology-transparent (α_T, α_R)-schedule $\langle T, R\rangle$, $\langle T\rangle$ is a topology-transparent non-sleeping schedule, and Theorem 4 states, in order to achieve the maximum average throughput, it is required $|T[i]| = \alpha_T^\star, \forall i \in [0, |T|-1]$. Thus a necessary condition for the existence of a *topology-transparent* (α_T, α_R)-schedule that achieves this upper bound is the existence of a non-sleeping topology-transparent schedule such that the number of transmitters in each slot is at least $\alpha_T^\star$. Our construction indicates such a condition is also a sufficient condition.

6 Our construction of topology-transparent (α_T, α_R)-schedules

In this section, we present our construction of topology-transparent (α_T, α_R)-schedules. Given a topology-transparent non-sleeping schedule, our goal is to reduce the number of nodes that are allowed to transmit and receive in each slot, while preserving the connectivity between any pair of adjacent nodes.

In our construction, we first compute the optimal number $\alpha_T^\star$ of transmitters per slot, then we construct an $(\alpha_T^\star, \alpha_R)$-schedule by calling function **Construct**$(\alpha_T^\star, \alpha_R, \langle T\rangle)$. In this function, for each slot i, we divide $T[i]$ into subsets $T[i] = T_0 \cup \ldots \cup T_{k_T-1}$ and $V - T[i]$ into subsets $V_n - T[i] = R_0 \cup \ldots \cup R_{k_R-1}$. Note subsets in $\{T_i | i \in [0, k_T-1]\}$ (subsets in $\{R_i | i \in [0, k_R-1]\}$ resp.) are not necessarily disjoint. Instead, in order to achieve maximal throughput, we require each $|T_i|$ to be as close to $\alpha_T^\star$ as possible, and each $|R_i|$ to be as close to α_R as possible. In the constructed schedule $\langle\bar{T}, \bar{R}\rangle$, we add $k_T k_R$ entries to guarantee that each subset of $T[i]$ transmits to each subset of $V_n - T[i]$; in particular, we require $\forall i_t \in [0, k_T-1]$ and $\forall i_r \in [0, k_R-1]$, $\exists k$ such that $\bar{T}[k] = T_{i_t}$ and $\bar{R}[k] \subseteq R_{i_r}$. If $|R_{i_r}| < \alpha_R$, we add nodes in $V_n - T_{i_t}$ to $R[k]$ to get $|\bar{R}[k]| = \alpha_R$, which is feasible since $T_{i_t} \le \alpha_T$ and $\alpha_T + \alpha_R \le n$. Note the way to divide $T[i]$ (line 3) and $R[i]$ (line 4) and the way to add nodes in $V_n - T_{i_t}$ to R_{i_r} (line 8) are not unique, and the specific way they are computed will not affect the correctness, frame length and average worst-case throughput of the constructed schedule, as we show in Theorem 6, Theorem 7 and Theorem 8 respectively. The code is given in Figure 2.

7 Correctness and performance analyses

In this section, we present correctness proofs and performance analyses of our construction. Given any topology-transparent non-sleeping schedule $\langle T\rangle$, we first show the correctness of the schedule constructed by the algorithm in Figure 2. Then we discuss the *frame length* and *throughput* of the constructed schedule. We show that the constructed schedule is optimal in terms of average throughput if the topology-transparent non-sleeping schedule $\langle T\rangle$ satisfies $\forall i \in [0, L-1]$, $|T[i]| \ge \min\left\{\alpha_T, \left\lceil\frac{n-D}{D}\right\rceil\right\}$.

We first consider the correctness of our construction. In the following lemma, we show that the constructed schedule is topology-transparent.

Input: integers n, D, α_T, α_R, and schedule $\langle T\rangle$, such that $n \geq D \geq 2$, $\alpha_T + \alpha_R \leq n$, and $\langle T\rangle$ is topology-transparent for networks in $\mathcal{N}_n^D$.
Output: An (α_T, α_R)-schedule $\langle \bar{T}, \bar{R}\rangle$ that is topology-transparent in networks in $\mathcal{N}_n^D$.
Main Program:

- Let $\alpha_T^\star$ be the optimal number of transmitters per slot computed as in Theorem 4.
- $\langle \bar{T}, \bar{R}\rangle$=**Construct** $(\alpha_T^\star, \alpha_R, \langle T\rangle)$;
- **return** $\langle \bar{T}, \bar{R}\rangle$;

Function **Construct** $(\alpha_T^\star, \alpha_R, \langle T\rangle)$
Input: integers $\alpha_T^\star$, α_R and a non-sleeping schedule $\langle T\rangle$
Output: $\langle \bar{T}, \bar{R}\rangle$

1 k=0;
2 **for** $i = 0; i < |T|; i++$
3 Divide $T[i]$ into $k_T = \left\lceil \frac{|T[i]|}{\alpha_T^\star} \right\rceil$ subsets $T[i] = T_0 \cup \ldots \cup T_{k_T-1}$, such that $|T_i| = \min\{\alpha_T^\star, |T[i]|\}, \forall i \in [0, k_T - 1]$;
4 Divide $R[i] = V - T[i]$ into $k_R = \left\lceil \frac{|R[i]|}{\alpha_R} \right\rceil$ subsets $R[i] = R_0 \cup \ldots \cup R_{k_R-1}$, such that $|R_i| = \min\{\alpha_R, |R[i]|\}, \forall i \in [0, k_R - 1]$;
5 **for** $i_t = 0; i_t < k_T; i_t++$
6 **for** $i_r = 0; i_r < k_R; i_r++$
7 $\bar{T}[k] = T_{i_t}, \bar{R}[k] = R_{i_r}, k++$;
8 **if** $|R_{i_r}| < \alpha_R$ **then** Add $\alpha_R - |R_{i_r}|$ nodes in $V_n - \bar{T}[k]$ to $\bar{R}[k]$;
9 **endfor**
10 **endfor**
11 **endfor**

Figure 2. The code of constructing a topology-transparent (α_T, α_R)-schedule based on a topology-transparent non-sleeping schedule

Lemma 5 *If $\langle T\rangle$ is a topology-transparent non-sleeping schedule for networks in $\mathcal{N}_n^D$, then the schedule $\langle \bar{T}, \bar{R}\rangle$ constructed by function **Construct**$(\langle T\rangle, \alpha_T^\star, \alpha_R)$ in Figure 2 is a schedule that is topology-transparent for networks in $\mathcal{N}_n^D$.*

Proof. We consider any node $x \in V_n$ and any set $Y \subseteq V_n - \{x\}$ of D nodes $y_0, \ldots, y_{D-1}$. Since $\langle T\rangle$ is a topology-transparent non-sleeping schedule, there exists $t \in [0, |T| - 1]$, such that $t \in freeSlots_{\langle T\rangle}(x, Y)$. That is, $x \in T[t]$ and $\forall i \in [0, D-1]$, $y_i \notin T[t]$.

We consider y_k, $\forall k \in [0, D-1]$. Let T_{i_t} be the subset of $T[t]$ such that $x \in T_{i_t}$ in line 3 of Figure 2, and R_{i_t} be the subset of $R[t] = V - T[t]$ such that $y_k \in R_{i_t}$ in line 4 of Figure 2. Let t' be an index such that $\bar{T}[t'] = T_{i_t}$ and $\bar{R}[t'] \supseteq R_{i_r}$. Given $\forall i = [0, D-1]$, since $y_i \notin T[t]$ and $\bar{T}[t'] = T_{i_t} \subseteq T[t]$, we have $y_i \notin \bar{T}[t']$, that is, $t' \notin tran_{\langle \bar{T}, \bar{R}\rangle}(y_i)$. Since $x \in T_{i_t} = \bar{T}[t']$, we have $t' \in tran_{\langle \bar{T}, \bar{R}\rangle}(x)$. So $t' \in tran_{\langle \bar{T}, \bar{R}\rangle}(x) - \bigcup_{i=0}^{D-1} tran_{\langle \bar{T}, \bar{R}\rangle}(y_i) = freeSlots_{\langle \bar{T}, \bar{R}\rangle}(x, Y)$. Since $y_k \in R_{i_r} \subseteq \bar{R}[t']$, we have $t' \in recv_{\langle \bar{T}, \bar{R}\rangle}(y_k)$. So $t' \in recv_{\langle \bar{T}, \bar{R}\rangle}(y_k) \cap freeSlots_{\langle \bar{T}, \bar{R}\rangle}(x, Y)$, that is, $recv_{\langle \bar{T}, \bar{R}\rangle}(y_k) \cap freeSlots_{\langle \bar{T}, \bar{R}\rangle}(x, Y) \neq \emptyset$. Thus by Requirement 3, we prove $\langle \bar{T}, \bar{R}\rangle$ is a topology-transparent schedule for $\mathcal{N}_n^D$. ■

It is easy to see the number of transmitters in each slot is no more than $\alpha_T^\star \leq \alpha_T$ and the number of receivers in each slot is no more than α_R. Thus we prove the correctness of our construction.

Theorem 6 ***(Correctness).*** *The schedule constructed by the algorithm in Figure 2 is an (α_T, α_R)-schedule that is topology-transparent for networks in $\mathcal{N}_n^D$.*

Our construction also indicates that, given any α_T', α_R', $\alpha_T' + \alpha_R' \leq n$, if a topology-transparent non-sleeping schedule $\langle T\rangle$ such that $|T[i]| \geq \alpha_T'$, $\forall i \in [0, |T| - 1]$ is available, then a topology-transparent schedule such that the number of transmitters in each slot is *exactly* α_T' and the number of receivers in each slot is *exactly* α_R' can be constructed by calling function **Construct**$(\alpha_T', \alpha_R', \langle T\rangle)$.

The *frame length* of the constructed schedule can be obtained directly from the code.

Theorem 7 ***(Frame Length).*** *The frame length of the schedule constructed by the algorithm in Figure 2 is $\sum_{i=0}^{L-1}\left(\left\lceil \frac{|T[i]|}{\alpha_T^\star}\right\rceil \left\lceil \frac{n-|T[i]|}{\alpha_R}\right\rceil\right) \leq \left\lceil \frac{M_{ax}}{\alpha_T^\star}\right\rceil \left\lceil \frac{n-M_{in}}{\alpha_R}\right\rceil L$, where $L = |T|$, $M_{ax} = \max\{|T[i]|, \forall i \in [0, L-1]\}$ and $M_{in} = \min\{|T[i]|, \forall i \in [0, L-1]\}$.*

In order to compare the average throughput of the constructed (α_T, α_R)-schedule in $\mathcal{N}_n^D$ networks to the maximum average throughput, we define function

$$r(x) \equiv \left(\frac{x}{\alpha_T^\star}\right) \prod_{i=1}^{D-1} \left(\frac{n-i-x}{n-i-\alpha_T^\star}\right)$$

where $\alpha_T^\star$ is computed as in Theorem 4. Note $\alpha_T^\star \approx \min\{\alpha_T, \frac{n-D}{D}\}$ only relies on n, D and α_T. Given an (α_T, α_T)-schedule $\langle T, R\rangle$ such that the number of receivers per slot is α_R, the optimality of $\langle T, R\rangle$ in terms of average throughput can be represented as follows:

$$\frac{Thr_{\langle T,R\rangle}^{ave}}{Thr_{\alpha_R,\alpha_T}^\star} = \frac{1}{L}\sum_{i=0}^{L-1} r(|T[i]|)$$

where $Thr_{\alpha_R,\alpha_T}^\star$ is the maximum average throughput that is achievable by an (α_T, α_R)-schedule (see Theorem 4). The average throughput of our construction can be computed by Theorem 2. We present below a lower bound on the ratio of our average throughput to the maximum average throughput, which indicates the constructed schedule is optimal in terms of average throughput if the number of transmitters per slot in the non-sleeping schedule is at least $\min\left\{\alpha_T, \left\lceil\frac{n-D}{D}\right\rceil\right\}$, otherwise the larger is the minimum number of transmitters per slot, the better average throughput can be achieved.

Theorem 8 ***(Average Throughput and Optimality).*** *Consider the construction in Figure 2. We have the following lower bound on the ratio of the average throughput $Thr^{ave}_{\langle\bar{T},\bar{R}\rangle}$ of $\langle\bar{T},\bar{R}\rangle$ to the maximum average throughput $Thr^{\star}_{\alpha_T,\alpha_R}$ that is achievable by (α_T,α_R)-schedules in $\mathcal{N}_n^D$ (see Theorem 4).*

$$\frac{Thr^{ave}_{\langle\bar{T},\bar{R}\rangle}}{Thr^{\star}_{\alpha_T,\alpha_R}} \geq \frac{r(M_{in})|A_1|+c|A_2|}{|A_1|+c|A_2|}$$

where

- $M_{in} = \min\{|T[i]|, i \in [0, L-1]\}$, *where* $L = |T|$,
- $A_1 = \{i \mid |T[i]| < \alpha_T^{\star}\}$ *and* $A_2 = \{i \mid |T[i]| \geq \alpha_T^{\star}\}$,
- $c = \frac{\left\lceil\frac{n}{\alpha_m}\right\rceil - 1}{\left\lceil\frac{n-M_{in}}{\alpha_R}\right\rceil}$, *where* $\alpha_m = \max\{\alpha_T^{\star}, \alpha_R\}$.

In particular, $Thr_{\langle\bar{T},\bar{R}\rangle} = Thr^{\star}_{\alpha_T,\alpha_R}$ *if* $M_{in} \geq \alpha_T^{\star}$.

Proof. Given $\forall k \in [0, L-1]$, let I_k be the set of indices of the $\left\lceil\frac{|T[k]|}{\alpha_T^{\star}}\right\rceil \left\lceil\frac{n-|T[k]|}{\alpha_R}\right\rceil$ entries in $\langle\bar{T},\bar{R}\rangle$ that are computed in lines 5-10 of function **Construct**() in the k'th iteration of the for-loop in lines 2-11 in Figure 2. Note I_k is disjoint for different i and $[0, \bar{L}-1] = \bigcup_{i=0}^{L-1} I_k$, where $\bar{L}=|\bar{T}|=|\bar{R}|$. Letting $\bar{A}_1 = \cup_{k\in A_1} I_k$ and $\bar{A}_2 = \cup_{k\in A_2} I_k$, we have $[0, \bar{L}-1] = \bar{A}_1 \cup \bar{A}_2$. Denoting $x_k = |T[k]|$, we have (1) $\forall i \in [0, \bar{L}-1]$, $\left|\bar{R}[i]\right| = \alpha_R$, (2) $\forall k \in A_1$ and $i \in I_k$, $\left|\bar{T}[i]\right| = x_k$, and (3) $\forall k \in A_2$ and $i \in I_k$, $\left|\bar{T}[i]\right| = \alpha_T^{\star}$. Thus $\forall k \in A_1$, $\forall i \in I_k$, $r(\left|\bar{T}[i]\right|) = r(x_k)$, and $\forall k \in A_2$, $\forall i \in I_k$, $r(\left|\bar{T}[i]\right|) = 1$. So we have

$$\begin{aligned}
\frac{Thr^{ave}_{\langle\bar{T},\bar{R}\rangle}}{Thr^{\star}_{\alpha_T,\alpha_R}} &= \frac{1}{\bar{L}}\sum_{i=0}^{\bar{L}-1} r(\left|\bar{T}[i]\right|) \\
&= \frac{\sum_{k\in A_1}\sum_{i\in I_k} r(\left|\bar{T}[i]\right|) + \sum_{k\in A_2}\sum_{i\in I_k} r(\left|\bar{T}[i]\right|)}{\left|\bar{A}_1\right|+\left|\bar{A}_2\right|} \\
&= \frac{\sum_{k\in A_1} r(x_k)|I_k| + \left|\bar{A}_2\right|}{\left|\bar{A}_1\right|+\left|\bar{A}_2\right|} \geq \frac{r(M_{in})\left|\bar{A}_1\right|+\left|\bar{A}_2\right|}{\left|\bar{A}_1\right|+\left|\bar{A}_2\right|}
\end{aligned}$$

The theorem can be proved if $\frac{|\bar{A}_2|}{|\bar{A}_1|} \geq c\ \frac{|A_2|}{|A_1|}$, which can be shown because (1) we have $\left|\bar{A}_1\right| \leq \left\lceil\frac{n-M_{in}}{\alpha_R}\right\rceil |A_1|$, since $\forall k \in A_1$, $|I_k| = \left\lceil\frac{n-x_k}{\alpha_R}\right\rceil \leq \left\lceil\frac{n-M_{in}}{\alpha_R}\right\rceil$; and (2) we have $\left|\bar{A}_2\right| \geq \left(\left\lceil\frac{n}{\alpha_m}\right\rceil - 1\right) |A_2|$, since $\forall k \in A_2$, $|I_k| = \left\lceil\frac{x_k}{\alpha_T^{\star}}\right\rceil \left\lceil\frac{n-x_k}{\alpha_R}\right\rceil \geq \left\lceil\frac{x_k}{\alpha_m}\right\rceil \left\lceil\frac{n-x_k}{\alpha_m}\right\rceil \geq \left\lceil\frac{n}{\alpha_m}\right\rceil - 1$. ∎

We have shown in Section 5 that a necessary condition for the existence of a *topology-transparent* (α_T,α_R)-schedule that achieves the maximum average throughput $Thr^{\star}_{\alpha_R,\alpha_T}$ is the existence of a non-sleeping topology-transparent schedule such that the number of transmitters in each slot is at least $\alpha_T^{\star}$. Theorem 8 indicates such a condition is also a sufficient condition.

Now we discuss the *minimum throughput*. We give a lower bound in Theorem 9 on the minimum throughput of the (α_T,α_R)-schedule constructed by the algorithm in Figure 2. In our proof, we consider the number of slots in which the transmissions from x to y are guaranteed successful for any pair of adjacent nodes x, y and any neighborhood of y. We prove that the number of such slots in each frame of the constructed schedule is no less than that of the original schedule. The degradation of the minimum throughput is due to a smaller number of active nodes.

Theorem 9 ***(Minimum Throughput).*** *Consider the construction in Figure 2. Denoting $L = |T|$ and $\bar{L} = |\bar{T}| = |\bar{R}|$, we have the following relation on the minimum throughput $Thr^{min}_{\langle T\rangle}$ of $\langle T\rangle$ and the minimum throughput $Thr^{min}_{\langle\bar{T},\bar{R}\rangle}$ of $\langle\bar{T},\bar{R}\rangle$ in networks in $\mathcal{N}_n^D$:*

$$Thr^{min}_{\langle\bar{T},\bar{R}\rangle} \geq \frac{L}{\bar{L}} Thr^{min}_{\langle T\rangle} \geq \frac{Thr^{min}_{\langle T\rangle}}{\left\lceil\frac{M_{ax}}{\alpha_T^{\star}}\right\rceil \left\lceil\frac{n-M_{in}}{\alpha_R}\right\rceil}$$

where $M_{ax} = \max\{|T[i]|, \forall i \in [0, L-1]\}$ *and* $M_{in} = \min\{|T[i]|, \forall i \in [0, L-1]\}$.

Proof. By Theorem 7 and Definition 9, the theorem can be proved by showing that $\forall x, y$, $\forall S \subseteq V_n - \{x, y\}$ such that $|S| = D-1$, $|\mathcal{T}_{\bar{T},\bar{R}}(x,y,S)| \geq |\mathcal{T}_{T,R}(x,y,S)|$. Given $\forall i \in \mathcal{T}_{T,R}(x,y,S)$, we have $x \in T[i]$, $y \in V_n - T[i]$ and $S \subseteq V_n - T[i]$. Let I_i be the set of indices of the $k_{T_i} \cdot k_{R_i}$ entries in $\bar{T}$ and $\bar{R}$ that are computed in lines 5-10 of the ith iteration of the for-loop in lines 2-11 of Figure 2. Let T_{i_t} be the subset of $T[i]$ such that $x \in T_{i_t}$ (line 3) and R_{i_r} be the subset of $V_n - T[i]$ such that $y \in R_{i_r}$ (line 4). Let $t' \in I_i$ such that $\bar{T}[t'] = T_{i_t}$ and $\bar{R}[t'] \supseteq R_{i_r}$. We have $S \subseteq V_n - T[i] \subseteq V_n - T_{i_t}$. Thus $t' \in \mathcal{T}_{\bar{T},\bar{R}}(x,y,S)$. Since I_i is disjoint for different i, we prove $|\mathcal{T}_{\bar{T},\bar{R}}(x,y,S)| \geq |\mathcal{T}_{T,R}(x,y,S)|$. ∎

In some scenarios, applications require balanced energy consumption in the sense that (1) the same number of nodes are active in each slot, and (2) the percentage of slots in which node x is active is the same for all $x \in V_n$. It is easy to verify that if energy consumption is balanced in the non-sleeping schedule $\langle T\rangle$, then these two properties can be preserved in the constructed schedule by modifying the division of $T[i]$ and $R[i]$ (lines 3-4) to satisfy that the number of subsets T_i (R_i resp.) such that node $x \in T_i$ ($x \in R_i$ resp.) is the same for all $x \in T[i]$ ($x \in R[i]$ resp.); it is easy to see such divisions exist.

8 Conclusion

We consider topology-transparent duty cycling in wireless sensor networks. The connection between topology-transparent duty cycling and non-sleeping schedules is investigated and analyses on throughput are presented. We propose a construction of topology-transparent duty cycling schedules based on a topology-transparent non-sleeping schedule. The constructed schedule is optimal in terms of average throughput if the non-sleeping schedule satisfies certain conditions.

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