A Proposal of Metaheuristics Based in the Cooperation between Operators in Combinatorial Optimization Problems

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Abstract

In the context of optimization problems, metaheuristics are tools that stand out by its excellent results and generality. A lot of metaheuristics are formed by a population of agents that operates in a search space. A frame of metaheuristics inspired in cooperation between unrelated individuals is proposed and three different methods of cooperation are suggested.

The implementation of the cooperation between agents is made using Soft Computing techniques. A fuzzy rules system has been designed concretely to perform the cooperation. Details about the implementation of three methods of cooperation and the computation of the fuzzy rules are offered for the models considered. A framework of experimentation over the combinations of methods and models is proposed.

1. Introduction

Using heuristic techniques are more and more frequent to approach difficult problems in combinatorial optimization. [1, 4] Many of these techniques are inspired by the behaviors of biological agents and we find very frequently in the literature that the biological language makes its appearance in metaheuristics in a figurative sense (with greater or smaller success).

In many metaheuristics we found a population of solutions and/or operators which are moving in a space and, by means of diverse procedures of competition and/or cooperation, they approach the solution of the problem that they are pretending to solve.

The aim of this work is to present a generic frame able to represent several models of metaheuristics and methods of cooperation between the operators who they conform. This frame tries to show an approach based on Soft Computing to tackle the design and implementation of the methods of cooperation between operators. Soft Computing is assumed as the theoretical base of the area of the Approximate Models. Within Soft Computing we can find [9] on the one hand the Approximate Reasoning that includes the Probabilistic Models and the Fuzzy Logic, and on the other hand the Functional Approach and Optimization Methods that include the Neuronal Networks and Metaheuristics. Examples of the inclusion of techniques of Soft Computing in the field of the metaheuristics can be found in [8, 5].

In our paper we use in a systematic and continued way a set of fuzzy rules to design and implement the cooperative behavior of the operators.

The paper is divided in 5 sections plus the references:

- This introduction is the first section.
- In second place, we review in a general way different methods of cooperation between no relatives individuals in the animal world.
- In section three we present our proposal based on three models of design of the metaheuristics and we analyze different implementations of every method of cooperation presented in the former section.

  These details are based in Soft Computing techniques, concretely in Fuzzy Logic. We apply these methods to the cooperation between operators for the solution of an optimization problem.
- In the fourth section we present a scheme of experimentation from our proposal as well as a discussion about the predictable advantages and disadvantages of each one of these models.

1-4244-0054-6/06/$20.00

- The paper ends with the conclusions and references.

2. Cooperation between individuals without kinship in the animal world

The role that plays the cooperation between individuals not related by kinship (cooperation that cannot be understood directly by means of a strictly Darwinist explanation) is being studied more frequently and profoundly in diverse scopes of the knowledge. Studies in Behavioral Ecology, Ethology, Anthropology, Economy, Sociology, etc., converge together with the classic tools of Game Theory and the algorithmic and computer science' technologies to analyze every time with more subtlety and depth the consequences of cooperation in different contexts. [3]

From the set of these studies and models, it can be extracted excellent information for our subject of study: the optimization problems and specially the hard problems to solve of combinatorial optimization. In fact, in the last years it has been experienced a non-systematic approach to the concepts of sciences of the life in different useful heuristic methods in combinatorial optimization; Genetic Algorithms, Ant Colonies, Clusters of Particles, etc. [4]

In all these models of population-based heuristics, it is understood that the population stand for a set of solutions that by means of some process (Darwinist, dynamic, etc.) is refined and approaching the optimal that is looked for. Another approach not so common is to consider as population a set of agents (operators, algorithms) who act on the search space or a subset of it, and they cooperate in the search of the optimal one. Our previous works have explored the cooperation by means of centralized coordination and the role that plays the memory in this type of coordination [6].

Nevertheless, this way is not commonest nor necessarily it has to be most effective to cooperate between individuals. We want hence to explore other non centralized models of cooperation between individuals.

From the specialized bibliography on cooperation in natural environment we can extract three great methods of cooperation between no relative individuals. We want to emphasize the interest in that the individuals are not relatives because in this type of cooperation no transmission of information from parents to children takes part, so the model it does not put the emphasis in the reproductive processes.

These models, therefore, are the adequated ones if we want to apply them to contexts where the population is set of agents who operate on the space of solutions and where improvements in the search are not translated necessarily in mechanisms of representation of solutions that pass of one generation to the following one. It does not mean that these models have not populations, nor they avoid some way of reproduction. We emphasize that the model does not try to analyze the transmission of characters of a generation to another one nor its genes dynamics. In the problems that we deal generally with the improvement of a solution is usually more difficult according we near (in terms of the objective function) to the optimal one. So the computational difficulty that it supposes to pass from an error of 30 percent to an error of 25 percent is much smaller than the needed to pass from 10 percent to 5 percent and very much smaller than the corresponding one to go from 5 percent to the optimal. Therefore, the cooperation mechanisms can allow the survival of search strategies (agents) that in certain moments are not efficient, but that at other moments do. In addition these mechanisms are dynamic and self tuned, and they can change the population of agents (its parameters of adjustment) in agreement with they are progressing the search.

The three methods of cooperation between unrelated individuals that we consider are [2, 3]:

1. *By product Mutualism*: The individuals who find a situation favorable but that they do not have by himself, capacity to take advantage of it, shares it among their equal ones. The individuals need sufficient cognitive capacities to evaluate the potentiality of a situation.

2. *Reciprocity*: The individuals with more capacity make favors with hope of being compensated ahead in an unfavorable situation. A system of memory or recognition of the last helpers is needed for this method of cooperation. The individuals with an altruist past with respect to the donor are helped with more frequency than those than they were selfish. A popular version of this idea is the proverb: "Today for me, tomorrow for you".

3. *Selection of groups*: In this method, the selection does not operate on the unit of standard study of the Darwinist model (the individual), but on the group like evolutionary unit. We want not discuss here about how correct is the model from the biological point of view [10], but in the formal operation of the model, the individuals of a group cooperate among them because exists a strong intergroup competition. In this model at least are necessary two groups of individuals. The individuals must also have cognitive abilities of recognition of own and strange so that the cooperation occurs.

These three methods of cooperation highly depend on the space over the individuals are moving in. So, we present below three different situations of this space in the context of a optimization problem. We call every situation as Model 1 to Model 3. For each model, we present at once the construction of the fuzzy rules that govern the three different methods.

3. Metaheuristics Based in the Cooperation between Operators

Let us consider an optimization problem. Without loss of generality we suppose that we looked for a maximum. The space of permissible solutions is S and f the objective function. Let us suppose that each agent A_i is an algorithm or operator that depends on a set of parameters.

We use in this paper the terms operator, agent, search strategy or search algorithm as synonymous. For the aim of this paper we consider an operator as a black box with input a solution x_i and output the changed solution x_{i+1}.

Let us suppose that whenever agent A_i changes a solution x_i, this agent modifies its resources in an amount that has the same sign that $f(x_{i+1})-f(x_i)$: it gains resources if it is positive and loses them otherwise. Thus an agent in addition to its parameters will have an amount of resources variable throughout the development of the run. We suppose that below a certain threshold of resources the agent is extinguished and over another threshold the agent can to reproduce.

We are going to present three different models of organization of the search either giving priority to different aspects of the problem or to the operators.

3.1 Model 1. The standard model

The agents move through the set of solutions S following different trajectories as it is usual in metaheuristics. There are many ways to model the cooperation methods. We suggest the following ones.

3.1.1 By product Mutualism

In this case we want modeling a situation as the following one: an agent with a sufficient amount of resources is able to recognize a certain situation like favorable and therefore, it carries out a tending behavior to share this situation with the nearest agents (or the neediest).

The condition' first part (an agent with sufficient amount of resources) is easily modeled. Labels over the resources values of the agent could be easily built. The second part is more delicate: How to know a priori, how much favorable is a situation?

Of course a local optimum is the less favorable, because this is what we are running away. A point of the space search in whose neighborhood we found appreciable differences of level but not local optima can be candidate to be a 'very favorable situation'.

A way to measure the potentiality of a situation is the value:

$$\Delta(x_i) = \frac{max_{x \in N_k(x_i)}(f(x) - f(x_i))}{k'} \qquad (1)$$

where $N_k(x)$ is the neighborhood of radio k of the solution $x \in S$ and we suppose that $k > 1$ and $k' \leq k < 1$ is the value of the radius of the neighborhood where the maximum takes place.

On the value $\Delta(x_i)$ we can build a linguistic variable *favorable*. If x_i is the local optimum in a radius k, we will have $\Delta(x_i) < 0$; if $\Delta(x_i)$ is very high, we will have a great potentiality point, so there is a high gradient in some direction around x_i.

If we maintain a record of values we can easily obtain how favorable is a situation to an agent. We consider the vector composed by the last h values of $\Delta(x_i)$ and we call it H_i. We build the following labels 'unfavorable' and 'favorable' fuzzy sets, namely μ_{UnFav} and μ_{Fav}, respectively. They are defined by means of the following membership functions:

$$\mu_{UnFav}(x) = \begin{cases} 1 & \text{if } x < \alpha \\ (x-\beta)/(\alpha-\beta) & \text{if } \alpha \leq x \leq \beta \\ 0 & \text{if } x > \beta \end{cases} \qquad (2)$$

$$\mu_{Fav}(x) = \begin{cases} 0 & \text{if } x < \alpha' \\ (x-\alpha')/(\beta'-\alpha') & \text{if } \alpha' \leq x \leq \beta' \\ 1 & \text{if } x > \beta' \end{cases} \qquad (3)$$

where x is the percentile rank of a value in the samples stored in H_i, and the parameters $\alpha, \beta, \alpha', \beta'$ stand for admission levels in this membership functions. For example, if $\alpha = 10$ and $\beta = 30$, we consider unfavorable, with degree 1, any value of $\Delta(x_i)$ below the 10^{th} percentile in the vector H_i. Any value over the thirty percentile will get degree 0 and linearly decreasing in the middle values.

Thus the cooperation in this context can be transformed in the fuzzy rules:

1. If an agent has *MANY RESOURCES* and it is in a *FAVORABLE* situation then *COMMUNICATE* its position

2. If an agent has *FEW RESOURCES* and it is in *UNFAVORABLE* situation and there is *SOME POSITION* communicated then *APPROACH* to the nearest communicated position

Let's say that we define the set $\mathcal{A}$ of agents with elements A_i. We consider here that each agent lies in $x_i \in S$ (its current solution) and we establish a metric $\delta : \mathcal{A} \times \mathcal{A} \rightarrow [0,1]$, such that $\delta(A_i, A_j) = d(x_i, x_j)$, where $d : S \times S \rightarrow [0,1]$ is a distance in the space of solutions.

We understand *APPROACH* as: Agent A_i approaches to agent A_j when we replaced the current solution of A_j by a solution of the segment that link A_i with A_j.

This approach could be implemented as a fuzzy set that defines the distance decrement between both agents in the space S. A definition of this fuzzy set (μ_{APP}) could be the following:

$$\mu_{APP}(x) = \begin{cases} 0 & \text{if } x < 0 \\ (x/s) & \text{if } 0 \leq x \leq s \\ 1 & \text{if } x > s \end{cases} \quad (4)$$

where $s \leq \delta(A_i, A_j)$ stands for the maximum admissible approach of A_j to A_i.

For the fuzzy rule 1 we need to set up a value λ in order to fire the rule consequent. Let us call the membership function of label 'Many resources' as μ_{MR}, then the activation of rule 1 can be reduced to the condition:

$$min(\mu_{MR}(r_i), \mu_{Fav}(x_i)) > \lambda, \quad (5)$$

where r_i stands for the resources of the agent A_i. When this condition holds, the consequent could be represented by a binary variable $Comm(A_i) = 1$

For the fuzzy rule 2, let us suppose that the agent is A_j. We must obtain a value to approach A_j to A_i. This value is obtained as in a classical fuzzy implication: first we compute if there are any $A_i \neq A_j$ with $Comm(A_i) = 1$. The rule only works if the answer is true. We then take the nearest A_i in this condition and we compute the rule antecedent:

$$min(\mu_{FR}(r_j), \mu_{UnFav}(x_j)), \quad (6)$$

where μ_{FR} stands for the membership function 'Few resources' label. So we use the label consequent inverse function to compute the value of the approach of A_j to A_i

$$\mu_{APP}^{-1}(min(\mu_{FR}(r_j), \mu_{UnFav}(x_j))). \quad (7)$$

Concretely, if before applying the second rule $\delta(A_i, A_j) = K$, then after its application holds that:

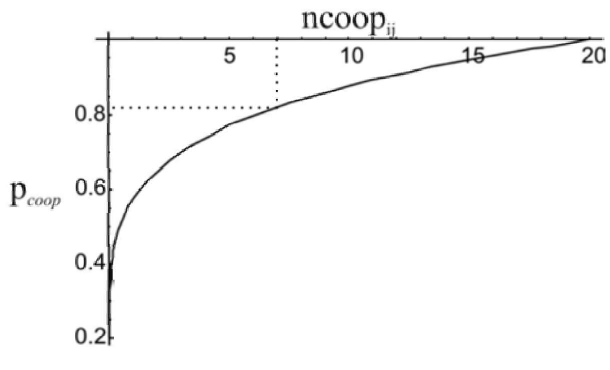

Figure 1. Graphic of p_{coop}

$$\delta(A_i, A_j) = K - \mu_{APP}^{-1}(min(\mu_{FR}(r_j))), \mu_{UnFav}(x_j))). \quad (8)$$

3.1.2 Reciprocity

In this case an agent with many resources distributes (with probability or possibility p_{coop}) a fraction of them with the near agents.

The value p_{coop} will depend on: a) a value base q_i of agent A_i that indicates the agent's tendency to cooperate and b) the agent cooperations record, namely C_i.

C_i will be a vector of c components that keeps the agents subindex who shared resources with agent A_i in the last c times.

Let us suppose that we call $ncoop_{ij}$ the number of times which the agent A_j appears in the record of cooperations of A_i. A possible function that give us p_{coop} must be increasing in the variable $ncoop_{ij}$ and with a minimum in 0 and a maximum (normally equal to 1) in c. The minimum stands for the probability/possibility that the A_i cooperates with an agent that is not in the record. The maximum stands for the value for the situation with the entire vector C_i full of j. For example the following function holds these conditions for the parameters $c = 20$ y $q_i = 0.2$, a graphic (Figure 1) for the possible values of $ncoop_{ij}$ are shown:

$$P_{coop} = q_i \times \sqrt{1 + (\frac{1}{q_i^2} - 1) \times (\frac{ncoop_{ij}}{c})^{0.4}} \quad (9)$$

For example, in the figure 1, the value of p_{coop} for $ncoop_{ij} = 7$ is slightly over 0.8.

We can see p_{coop} as a membership function of a label '*to share with*'. So, the reciprocity cooperation method could be modeled with fuzzy rules as:

3. If an agent A_i has MANY RESOURCES and an agent A_j has FEW RESOURCES and A_i 'SHARE WITH' A_j then transfer resources from A_i to A_j

The computation of this rule is done in a very similar way to the previous ones.

3.1.3 Selection of groups

Let us suppose in this case that the agents are grouped in G groups and they are able to know if another agent belongs or not to the same group. We can model the selection of groups as in the former taking into account that the agents only cooperate with agents from the same group. Nevertheless, they can rob resources to another near agent of a different group with certain probability/possibility p_r.

We can think about three subclasses within this method: the intragroup cooperation occurs as in 3.1.1 or like in 3.1.2 or allowing both cases simultaneously. So the agents on the same group can cooperate with a by product mutualism or reciprocity or both, while there is a competition among the different groups. Details of calculus for this method are very similar to previous ones.

3.2 Model 2. Subset of solutions matrix

Let us suppose a certain subset M of the search space S where the agents lie. We can set $|M| = m^2$, thus M is representable in a matrix of solutions with m rows and m columns. These agents change the subset M, so it converges to another subset of the space S in which better solutions may be found.

This model has been used in [7] as a framework to simulate different cooperation schemes between operators. The methods of cooperation in this model 2 could be implemented in a similar way to the previous one taking into account that now, the situation of the agent does not agree with the representation of the solution in the matrix M. That is, we maintain the same structure that in model 1 but the distances between the agents in anyone of the three methods are not calculated in terms of the space of solutions S but in the matrix M terms. The calculation of distances between the agents become therefore through a metric $\delta : \mathcal{A} \times \mathcal{A}' \rightarrow [0, 1]$, based on a conventional metric $d' : M \times M' \rightarrow [0, 1]$. For example if agent A_i it is located in cell m_{kl} and agent A_j is in cell $m_{k'l'}$ a possible distance $\delta(A_i, A_j) = d'(m_{kl}, m_{k'l'})$. The rules calculus in this model can be made as in the standard model by substituting the δ definition. For this reason we omit the calculus of the three cooperation methods in this model.

3.3 Model 3. Subset matrix of agents

Let us suppose that the set of agents $|\mathcal{A}| = m^2$, thus $\mathcal{A}$ is representable in a matrix of solutions with m rows and m columns.

When a solution falls in a cell a_{ij}, the agent A_{ij} is applied to it. In this model therefore the agents are fixed and the solutions move. In this model the agents cooperate only with its neighbors's cells. When the resources fall below a threshold, the agent cell become empty and then is occupied by the neighboring agent that can reproduce itself with more resources available.

In this model the number of agents is fixed. The implementations that we suggest for the cooperation methods in this model are very simple for the first and second ones, since in these cases it is enough giving a vicinity radius to consider. So we have one constant list of neighbors for each cell in all the execution of the metaheuristic, since these do not move.

For the third cooperation method, the only important difference to consider is the geometric arrangement of the groups in the matrix. In effect, one arrangement very scattered would give rise to a very competing profile between groups since the agents of different groups would be well mixed.

A very compact arrangement would cause a flow of cooperation in the inner agents of the nucleus (that do not compete) with those of the outside (that does it) on the other hand it would be necessary to stand rules of replacement for agents who are extinguished (Figure 2).

Two possibilities must be considered: we could allow the extinction of a group or maintain the number of groups and its elements constant. Models 2 and 3 both display an advantage respect to the standard model, since they are easily representable. The tools of graphical representation of the performance of metaheuristics can help us to understand the processes that underlie to search the problem optimum.

3.4 Scheme of experimentation

In order to check the interest and performance of these models and methods, it is necessary to develop a set of experiments. The combination of the three methods of cooperation between operators and the three models of organization of the metaheuristics gives us a rich frame of experimentation. With this frame we can obtain one scheme of 21 different combinations, since the cooperation methods can coexist in a same model. Concretely we have 3 models by 3 methods (9 combinations) and 3 models by 3 pairs of methods (9 combinations) and 3 models with the 3 methods at time (3

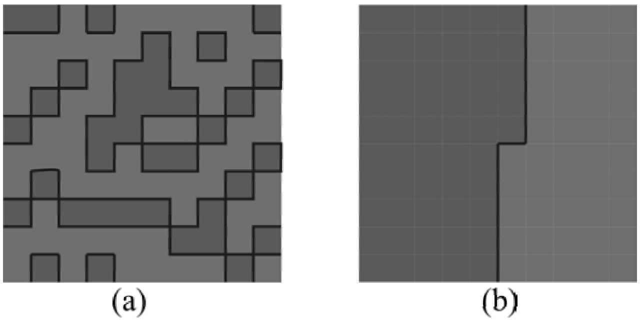

Figure 2. Different arrangement of groups in the grid: (a)High level of conflict, (b) Low level of conflict

combinations) summing up 21. In effect, nothing prevents us using two or three methods simultaneously in a same run, because these methods are implemented as a fuzzy rules base, there is not obstacles to consider them all simultaneously or separately. Indeed, for some of these combinations (method three) we can define different subclasses as we said on section 3, well by changing the groups disposition or well allowing one or two kind of cooperation intragroup. In order to conclude with the experimentation scheme we will have to also count with the definition of the following elements:

- Elements independent of the problem
  - Rules of reproduction of agents (information transference between ancestors and descendants).
  - Selection of groups, criteria of selection (balanced or unequal).
  - Rules of movement of agents and/or solutions in models 2 and 3.
- Elements dependent of the problem
  - Selection of the type of agents or operators.
  - Selection of initials populations (agents and solutions).

Of course, the choice of the problem is a subject of fundamental importance in this experimentation. All these terms will be boarded in later works.

3.5 Conclusions

Three methods of cooperation among agents based in the cooperation among unrelated individuals in the animal world has been shown. The methods approached are: By product mutualism, Reciprocity and Selection of groups. These methods has been applied in the context of combinatorial optimization problems using fuzzy rules. Three different ways of organize the search have been described: the first is the standard model, i.e. each agent produces a trajectory in the search space, in the second model the agents move in a matrix M of solutions, in the third model the agents are fixed in a matrix A and the solutions move in. All the combinations of methods and models have been described and analyzed. Definitions of membership functions of the labels involved in the fuzzy rules have been proposed. A framework of experimentation over these combinations has been proposed.

Acknowledgments

This work was supported in part by project TIN2005-08404-C04-01 from the Spanish Ministry of Science and Education.

References

[1] C. Blum and A. Roli. Metaheuristics in combinatorial optimization: Overview and conceptual comparison. *ACM Computing Surveys*, 3(35):268–308, 2003.

[2] L. Dugatkin. Animal cooperation among unrelated individuals. *Naturwissenschaften*, (89):533–541, 2002.

[3] L. Dugatkin and H. Reeves, editors. *Game Theory and Animal Behavior*. 2000.

[4] R. Martí, editor. *Handbook of Metaheuristics*. Kuwler Academic, 2003.

[5] D. Pelta, N. Krasnogor, C. Bousono-Calzon, J.Hirst, E. Burke, and J. Verdegay. Applying a fuzzy sets-based heuristic for the protein structure prediction problem. *Fuzzy Sets and Systems*, 1(151):103–123, 2005.

[6] D. Pelta, A. Sancho-Royo, C. Cruz, and J. L. Verdegay. Using memory and fuzzy rules in a cooperative multithread strategy for optimization. *Information Sciences*, (In Press):Available online in Sciencedirect.com, Aug. 2005.

[7] A. Sancho-Royo, D. Pelta, J. Verdegay, and J. González. Evaluation of cooperative strategies in optimization problems. In *Nature-inspired Smart Information Systems (NiSIS'05)*, october 2005.

[8] J. Verdegay, editor. *Fuzzy Sets based Heuristics for Optimization*. Studies in Fuzziness and Soft Computing. Physica-Verlag, 2003.

[9] J. Verdegay. A review on methodologies that integrate the soft computing (in spanish). In *Actas del I Simposio sobre Lgica Fuzzy y Soft Computing (LFSC'2005)*, pages 151–156, 2005.

[10] M. Wade. A critical review of the models of group selection. *Q.Rev. Biol.*, (50):1–33, 1978.