JOINT SOURCE-CHANNEL DECODING OF MULTIPLE DESCRIPTION QUANTIZED AND VARIABLE LENGTH CODED MARKOV SEQUENCES

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ABSTRACT

This paper proposes a framework for joint source-channel decoding of Markov sequences that are encoded by an entropy coded multiple description quantizer (MDQ), and transmitted via a lossy network. This framework is particularly suited for lossy networks of inexpensive energy-deprived mobile source encoders. Our approach is one of maximum *a posteriori* probability (MAP) sequence estimation that exploits both the source memory and the correlation between different MDQ descriptions. The MAP problem is modeled and solved as one of the longest path in a weighted directed acyclic graph.

For MDQ-compressed Markov sequences impaired by both bit errors and erasure errors, the proposed joint source-channel MAP decoder can achieve 5dB higher SNR than the conventional hard-decision decoder. Furthermore, the new MDQ decoding technique unifies the treatments of different subsets of the K descriptions available at the decoder, circumventing the thorny issue of requiring up to $2^K - 1$ MDQ side decoders.

1. INTRODUCTION

Suppose that a multimedia signal to be encoded and communicated via noisy channel(s) is a Markov sequence $\chi^{\mathcal{M}} = \chi_1$, χ_2, $\cdots$, $\chi_{\mathcal{M}}$. Limited by battery capacity and computing power (e.g., on mobile devices), the encoder cannot afford optimal compression (e.g., context-based arithmetic coding) nor channel coding. It simply quantizes (scalar or vector) $\chi^{\mathcal{M}}$ into $K \geq 2$ descriptions, and then send these descriptions through a lossy network, either in fixed length code (no entropy coding) or in a simple variable length (VLC) code (e.g., Huffman code). To keep multiple description coding simple, multiple description scalar quantizer (MDSQ) or multiple description lattice vector quantizer (MDLVQ) should be used.

This simple encoder leaves three forms of statistical redundancy: 1) the memory of the Markov sequence that is unexploited due to scalar coding or simple suboptimal block code (e.g., lattice VQ), 2) residual source redundancy for no or suboptimal entropy coding, and 3) the correlation that is intentionally introduced between the K descriptions by MDQ. It is up to the decoder to exploit these available redundancies to correct the channel errors.

In [1], we proposed a joint source-channel MDQ decoding technique for fixed-rate code. This paper generalizes the work of [1] to variable length code (VLC). VLC achieves a rate closer to the entropy, but it is very sensitive to channel noise. Any loss of synchronization of source symbols makes correct joint decoding of multiple descriptions impossible for a hard-decision decoder. The proposed algorithm can simultaneously utilize the inter-description and intra-description redundancies, thus successfully evades the difficulty in merging multiple desynchronized descriptions. Joint source-channel decoding of MDSQ and VLC coded Markov sequences was also reported in [2], but the memory of the Markov source and the correlation between the descriptions of MDSQ were exploited in tandem.

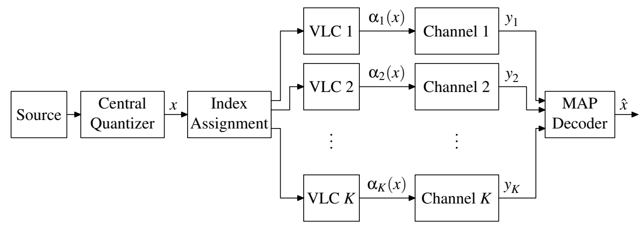

Fig. 1. Block diagram of a MDSQ based communication system with a MAP decoder.

We first pose the problem as one of MAP sequence estimation, and then solve it by a graph theoretical algorithm. For MDSQ-coded Gaussian Markov sequences the algorithm complexity can be reduced. Moreover, the MDQ decoding algorithm eliminates the need for $2^K - 1$ side decoders, which poses a great difficulty for a hard-decision MDQ decoder.

The paper is structured as follows. Section 2 formulates a general framework for joint source-channel MAP decoding of MDQ-coded Markov sequences. Section 3 presents a longest path algorithm for solving the MAP MDQ decoding problem. In Section 4, a more efficient solution is developed for Gaussian Markov sequences. Simulation results are reported in Section 5. Section 6 concludes.

2. PROBLEM FORMULATION

Fig. 1 schematically depicts the proposed joint source-channel MDQ decoding system. The input to the system is a finite Markov sequence $\chi^{\mathcal{M}} = \chi_1, \chi_2, \cdots, \chi_{\mathcal{M}}$. A central quantizer $q : \mathbb{R} \rightarrow \mathcal{C}$ maps a source symbol (MDSQ) or a block of source symbols (MDVQ) to a codeword in central codebook $\mathcal{C} = \{1, 2, \cdots, L\}$, where L is the number of codecells of the central quantizer. Let the VLC codebooks of the K side quantizers be $\mathcal{C}_k = \{c_{k,1}, c_{k,2}, \cdots, c_{k,L_k}\}$, where L_k is the number of codecells of side quantizer k, $L \le \prod_{k=1}^{K} L_k$ and $L_k \le L$, $k = 1, 2, \cdots, K$. The K-description MDQ is specified by an index assignment function $\alpha_k : \mathcal{C} \to \mathcal{C}_k$ [3]. The redundancy carried by the descriptions is reflected by a rate $1 - \log_2 L / \sum_{k=1}^{K} \log_2 L_k$.

Let $\boldsymbol{x} = x_1 x_2 \cdots x_M \in \mathcal{C}^M$ be the output sequence of $\chi^{\mathcal{M}}$ produced by the central quantizer, $M = \mathcal{M}$ for MDSQ, or $M = \iota\mathcal{M}$ for MDVQ with ι being the VQ dimension. The K descriptions of MDQ, $\alpha_k(\boldsymbol{x}) \in \mathcal{C}_k^M$, $k = 1, 2, \cdots, K$, are transmitted via K noisy channels. We assume that the K noisy channels are memoryless, mutually independent, and do not introduce phase errors such as insertion or deletion of code symbols or bits. Consequently, a received description may have inversion or/and erasure errors, but it has the same number of bits as the one generated by MDQ. Denote the received code streams by $\boldsymbol{y}_k$, with length $N_k = |\alpha_k(\boldsymbol{x})| = \sum_{m=1}^{M} |\alpha_k(x_m)|$, where $|\cdot|$ is the number of bits in a bitstream.

Since VLC is used, the parsing of $\boldsymbol{y}_k$ is not unique. Any given $\boldsymbol{x}$ with $|\alpha_k(\boldsymbol{x})| = N_k$ uniquely determines a parsing of $\boldsymbol{y}_k$, which is called the parsing of $\boldsymbol{y}_k$ with respect to $\boldsymbol{x}$. It parses the bit stream $\boldsymbol{y}_k$ into a sequence of codewords delimited by $(b_{k,0}, b_{k,1}, \cdots, b_{k,M})$. We write the m^{th} codeword parsed out of $\boldsymbol{y}_k$ as $y_k(b_{k,m-1}, b_{k,m}]$, where $b_{k,0} = 0$, $b_{k,m} - b_{k,m-1} = |\alpha_k(x_m)|, 1 \le m \le M$ and $b_{k,M} = N_k$.

Having the source and channel statistics and knowing the design of MDQ, the MDQ decoder can perform joint source-channel decoding of $\boldsymbol{y}_k$, $k = 1, 2, \cdots, K$, to best reconstruct $\boldsymbol{x}$. In a departure from the current practice of designing multiple side decoders (up to $2^K - 1$ of them!), we develop a single unified MDQ decoder that operates the same way regardless what subset of the K descriptions is available to the decoder. Our MDQ decoder takes the approach of MAP sequence estimation, and it reconstructs, given the observed sequences $\boldsymbol{y}_k$, $k = 1, 2, \cdots, K$, the input sequence $\boldsymbol{x}$ such that the *a posteriori* probability $P(\boldsymbol{x}|\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K)$ is maximized. Namely,

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x} \in \mathcal{C}^*} \log P(\boldsymbol{x}|\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K). \quad (1)$$

According to the Bayes' theorem,

$$\begin{aligned} & P(\boldsymbol{x}|\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K) \\ &= \frac{P(\boldsymbol{x})P(\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K|\boldsymbol{x})}{P(\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K)} \\ &\overset{(a)}{\propto} P(\boldsymbol{x})P(\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_K|\boldsymbol{x}) \\ &\overset{(b)}{=} P(\boldsymbol{x}) \prod_{k=1}^{K} P(\boldsymbol{y}_k|\alpha_k(\boldsymbol{x})) \\ &\overset{(c)}{=} \prod_{m=1}^{l(\boldsymbol{x})} \left\{ P(x_m|x_{m-1}) \prod_{k=1}^{K} P_k(y_k(b_{k,m-1}, b_{k,m}]|\alpha_k(x_m)) \right\}, \end{aligned} \quad (2)$$

where we let $P(x_1|x_0) = P(x_1)$ as convention. In the above derivation, step (a) is due to the fact that $\boldsymbol{y}_1$ through $\boldsymbol{y}_K$ are fixed in the objective function; step (b) is from the mutual independency of the K channels; and step (c) is under the assumption that $\boldsymbol{x}$, the output of the central quantizer, is first-order Markovian and the channels are memoryless. This assumption certainly holds, if the original source sequence $\chi^{\mathcal{M}}$ before MDQ is first-order Markovian, and it remains a good approximation for a high-order Markov sequence $\chi^{\mathcal{M}}$ as well, if $\chi^{\mathcal{M}}$ is vector quantized.

3. JOINT SOURCE-CHANNEL MDQ DECODING

Now we devise a graph theoretical algorithm for joint source-channel MDQ decoding. Combining (1) and (2), we have

$$\begin{aligned} \hat{\boldsymbol{x}} = \arg \max_{\substack{\boldsymbol{x} \in \mathcal{C}^* \\ \boldsymbol{\alpha}(\boldsymbol{x}) = \boldsymbol{N}}} \sum_{m=1}^{l(\boldsymbol{x})} \Bigg\{ & \log P(x_m|x_{m-1}) \\ & + \sum_{k=1}^{K} \log P_k(y_k(b_{k,m-1}, b_{k,m}]|\alpha_k(x_m)) \Bigg\}. \end{aligned} \quad (3)$$

where $\boldsymbol{N} = (N_1, \cdots, N_K)$, $\boldsymbol{\alpha}(\boldsymbol{x}) = (|\alpha_1(\boldsymbol{x})|, \cdots, |\alpha_K(\boldsymbol{x})|)$. The additivity of (3) breaks the MAP estimation problem into the following subproblems:

$$\begin{aligned} w(\boldsymbol{n}, a) = \max_{\substack{\boldsymbol{x} \in \mathcal{C}^* \\ \boldsymbol{\alpha}(\boldsymbol{x}) = \boldsymbol{n} \\ x_{l(\boldsymbol{x})} = a}} \sum_{m=1}^{l(\boldsymbol{x})} \Bigg[& \log P(x_m|x_{m-1}) \\ & + \sum_{k=1}^{K} \log P_k(y_k(b_{k,m-1}, b_{k,m}]|\alpha_k(x_m)) \Bigg], \\ & 1 \le n_k \le N_k, \ k = 1, 2, \cdots, K, \ a \in \mathcal{C}, \end{aligned} \quad (4)$$

where $\boldsymbol{n} = (n_1, \cdots, n_K)$. Then, the solution of the optimization problem (1) is given by

$$\hat{\boldsymbol{x}} = \arg\max_{c \in \mathcal{C}} w(\boldsymbol{N}, c), \quad (5)$$

The subproblems $w(\cdot, \cdot)$ can be expressed recursively as

$$\begin{aligned} w(\boldsymbol{n}, a) = & \max_{b \in \mathcal{C}} \left\{ w(\boldsymbol{n} - \boldsymbol{\alpha}(a), b) + \log P(a|b) \right\} \\ & + \sum_{k=1}^{K} \log P_k(y_k(n_k - |\alpha_k(a)|, n_k]|\alpha_k(a)). \end{aligned} \quad (6)$$

The above recursion reduces the MAP estimation problem to one of finding the longest path in a weighted directed acyclic graph (WDAG) [4], which is given in Fig. 2. The underlying graph G has $L \prod_{k=1}^{K} N_k + 1$ vertices, which forms a hyper-trellis of dimension $K + 1$, with the K dimensions represent the K received bitstreams $\boldsymbol{y}_1, \cdots, \boldsymbol{y}_K$, and the remaining dimension corresponds to L codecells of the central quantizer. There is also one starting node s, corresponding to the beginning of $\boldsymbol{x}$.

We use a $(K + 1)$-dimensional vector $(\boldsymbol{n}, x)$, $1 \le n_k \le N_k$, $x \in \mathcal{C}$ to label a node in G. From node $(\boldsymbol{n} - \boldsymbol{\alpha}(a), b)$ to node $(\boldsymbol{n}, a)$, $a, b \in \mathcal{C}$, there is a directed edge, with weight

$$\log P(a|b) + \sum_{k=1}^{K} \log P_k(y_k(n_k - |\alpha_k(a)|, n_k]|\alpha_k(a)). \quad (7)$$

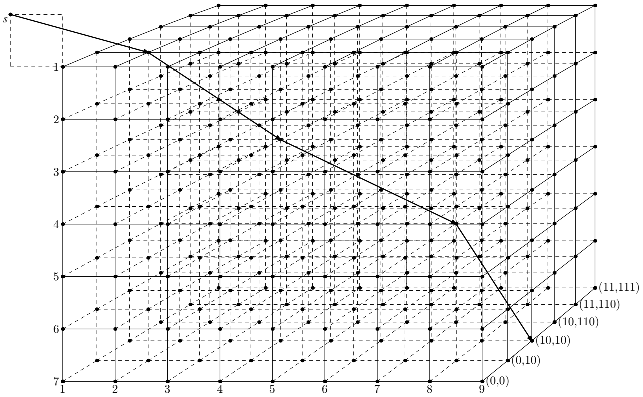

Fig. 2. Graph G constructed for the joint source-channel MDQ decoding ($K = 2,\ L = 6,\ \mathcal{C}_1 = \{0, 10, 11\},\ \mathcal{C}_2 = \{0, 10, 110, 111\},\ N_1 = 7$ and $N_2 = 9$).

From the starting node s to each node $(\boldsymbol{\alpha}(a), a)$, there is an edge whose weight is

$$\log P(a) + \sum_{k=1}^{K} \log P_k(y_k(0, |\alpha_k(a)|]|\alpha_k(a)). \tag{8}$$

In graph G, the solution of the subproblem $w(\boldsymbol{n}, a)$ is the weight of the longest path from the starting node s to node $(\boldsymbol{n}, a)$, which can be calculated recursively using dynamic programming. The MAP decoding problem is then converted into finding the longest path in graph G from the starting node s to nodes $(\boldsymbol{N}, c), c \in \mathcal{C}$.

To analyze the algorithm complexity we note that the dynamic programming algorithm proceeds from the starting node s to the nodes $(\boldsymbol{N}, c)$, through all $L \prod_{k=1}^{K} N_k$ nodes in G. The quantities $\log P_k(y_k(n_k - |\alpha_k(a)|, n_k]|\alpha_k(a))$, $\log P(a)$ and $\log P(a|b)$ can be precomputed and stored in lookup tables so that they will be available to the dynamic programming algorithm in $O(1)$ time. Hence (7) and (8) can be computed in $O(K)$ time. Therefore the value of $w(\boldsymbol{n}, a)$ can be evaluated in $O(L + K)$ time, according to (6). Thus the total time complexity of this algorithm is $O(L(L+K) \prod_{k=1}^{K} N_k)$. To reconstruct the input sequence, the selection in (6) should be recorded at each node, which results in a space complexity of $O(L \prod_{k=1}^{K} N_k)$.

4. COMPLEXITY REDUCTION

First let us convert our recursion formula into a matrix search problem [4]. Define an $L \times L$ matrix $A_{\boldsymbol{n}}$ such that

$$\begin{aligned} A_{\boldsymbol{n}}(a, b) =& w(\boldsymbol{n}, b) + \log P(a|b) \\ &+ \sum_{k=1}^{K} \log P_k(y_k(n_k, n_k + |\alpha_k(a)|]|\alpha_k(a)). \end{aligned} \tag{9}$$

Then (6) is equivalent to finding the row maxima of $A_{\boldsymbol{n}}$.

A matrix $A = A(a, b)$ is said to be *totally monotone* with respect to row maxima if

$$A(a, b) \le A(a, b') \Rightarrow A(a', b) \le A(a', b'),\ a < a', b < b'. \tag{10}$$

If an $n \times n$ matrix A is totally monotone, then the row maxima of A can be found in $O(n)$ time [5]. A sufficient condition for (10) is

$$A(a, b') + A(a', b) \le A(a, b) + A(a', b'),\ a < a', b < b', \tag{11}$$

To apply the fast algorithm to the joint source-channel MDSQ decoding problem, we check if matrix $A_{\boldsymbol{n}}$ satisfies the total monotonicity. Substituting $A_{\boldsymbol{n}}$ in (9) for A in (11), we have

$$\begin{aligned} \log P(a|b') + \log P(a'|b) \le \log P(a|b) + \\ \log P(a'|b'),\ a < a', b < b', \end{aligned} \tag{12}$$

which is a sufficient condition for $A_{\boldsymbol{n}}$ to satisfy the total monotonicity and therefore, for the fast algorithm to be applicable. This condition, which depends only on the source statistics not the channels, is exactly the same as the one derived in [4]. It was shown by [4] that (12) holds if the source is Gaussian Markov, which includes a large family of signals studied in practice and theory.

Finally, we conclude that the time complexity of MAP decoding of MDSQ can be reduced to $O((L + K) \prod_{k=1}^{K} N_k)$ for Gaussian Markov sequences.

5. SIMULATION RESULTS

(a) L=15

1							
2	3						
	4	5					
		6	7				
			8	9			
				10	11		
					12	13	
						14	15

(b) L=21

1	2						
3	4	6					
	5	7	8				
		9	10	11			
				12	14		
				13	15	16	
					17	18	20
						19	21

Fig. 3. The index assignments for two two-description scalar quantizers as proposed by [6].

We implemented the proposed MAP-based MDQ decoding algorithm and tested it on three first-order, zero-mean, unit-variance Gaussian Markov sequences with the correlation coefficient ρ being 0.1, 0.5 and 0.9 respectively. Two different two-description scalar quantizers (2DSQ) were used in our experiments, which are uniform and have the index assignment matrices shown in Fig. 3. One of them has $L = 15$ central codecells, and the other $L = 21$ codecells. For both 2DSQ's, the two side quantizers each has $L_1 = L_2 = 8$ codecells. The 2DSQ with two diagonals in its index assignment matrix has a stronger correlation between the two descriptions than the 2DSQ of three diagonals, i.e., the former has higher degree of redundancy than the latter.

For each description k, $k = 1, 2$, Huffman codes are generated according to the distribution of the side quantization codecell. The encoded bitstreams $\alpha_k(\boldsymbol{x})$ are then transmitted over two error-and-erasure channels with erasure probability p_ϕ and inversion probability p_c varying. The new MDQ decoding algorithm is compared with 1) MAP decoder for single description scalar quantization, and 2) conventional hard-

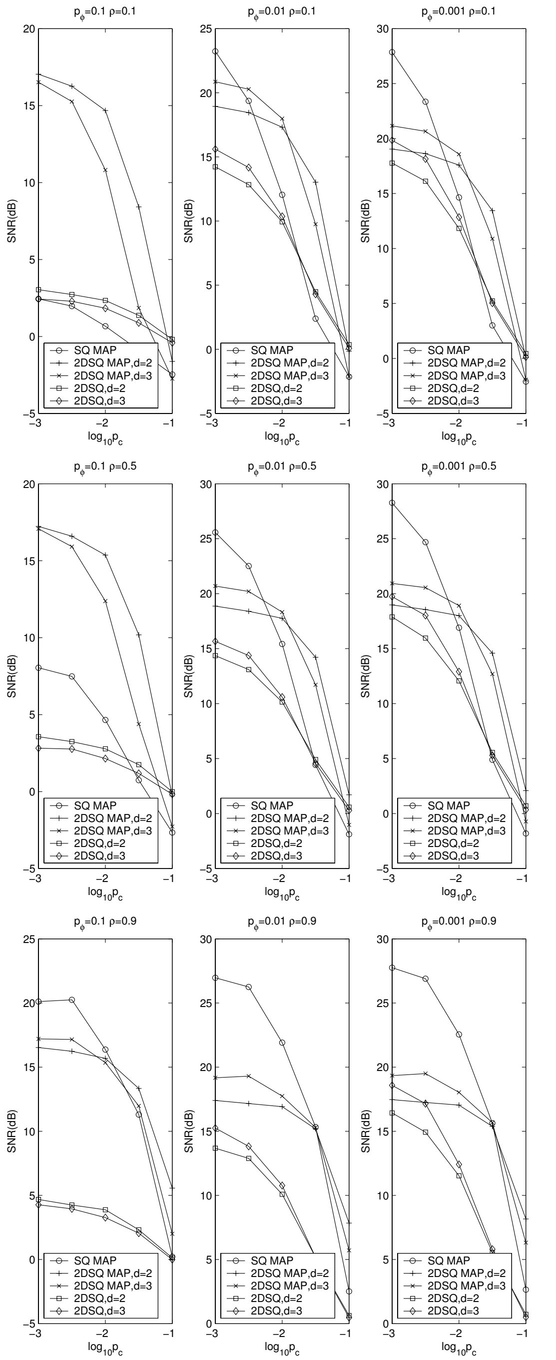

Fig. 4. SNR performances of different MDQ decoders ($\rho = 0.1, 0.5$ and 0.9), where d is the number of diagonals in the 2DSQ index assignment matrix.

decision MDQ decoder. The system performance measure is the signal-to-noise ratio (SNR).

The simulation results are plotted in Fig. 4. Over all values of ρ, p_c and p_ϕ, the joint source-channel MAP MDQ decoder outperforms the conventional hard-decision MDQ decoder, regardless the level of correlation between the two side descriptions. Not surprisingly, the performance gap between the two approaches increases as the amount of memory in the Markov source (ρ) increases. This is because the hard-decision MDQ decoder cannot benefit from the residual source redundancy in $\boldsymbol{x}$. The gap also increases as the erasure error probability p_ϕ increases, indicating that the MAP MDQ decoder can make a better use of inter-description correlation. Also, as expected, the MAP SQ decoder achieves higher SNR than the MAP MDQ decoder when the channel quality is very good, but the former loses to the latter as the channel condition deteriorates. This is when the redundancy of MDQ starts to pay off. More interestingly, we notice that joint source-channel MAP decoding of MDQ is advantageous even when the source memory is weak (see the curves for $\rho = 0.1$).

6. CONCLUSIONS

We proposed a framework for optimal (in MAP sense) joint source-channel decoding of Markov sequences compressed by entropy coded MDQ. This framework allows both inter-description and intra-description correlations to be exploited for correcting bit errors as well as erasure errors. It is suitable for lossy communications involving low-power inexpensive encoders.

The new MDQ decoding technique unifies the treatments of different subsets of descriptions available at the decoder, overcoming the difficulty of having a large number of side decoders that hinders the design of a good hard-decision MDQ decoder. Moreover, our joint source-channel decoder considers simultaneously the processes of decoding and merging of multiple descriptions, thus evades the difficulty in merging two desynchronized descriptions, which hard decision MDQ decoders have to face.

7. REFERENCES

[1] X. Wu, X. Wang, and J. Wang, "Joint source-channel decoding of multiple decription quantized Markov sequences," in *Data Compression Conference*, 2006, accepted.

[2] T. Guionnet, C. Guillemot, and E. Fabre, "Soft decoding of multiple descriptions," in *IEEE International Conference on Multimedia, ICME*, vol. 2, Lausanne, Switzerland, August 26-29 2002, pp. 601–604.

[3] S. D. Servetto, V. A. Vaishampayan, and N. J. A. Sloane, "Multiple description lattice vector quantization," in *Data Compression Conference*, 1999, pp. 13–22.

[4] X. Wu, S. Dumitrescu, and Z. Wang, "Monotonicity-based fast algorithms for MAP estimation of Markov sequences over noisy channels," *IEEE Trans. on Information Theory*, vol. 50, no. 7, pp. 1539–1544, July 2004.

[5] A. Aggarwal, M. Klave, S. Moran, P. Shor, and R. Wilber, "Geometric applications of a matrix-searching algorithm," *Algorithmica*, vol. 2, pp. 195–208, 1987.

[6] V. A. Vaishampayan, "Design of multiple description scalar quantizers," *IEEE Trans. on Information Theory*, vol. 39, no. 3, pp. 821–834, May 1993.