GLOBALLY OPTIMAL UNEVEN ERASURE-PROTECTED MULTI-GROUP PACKETIZATION OF SCALABLE CODES

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ABSTRACT

We study the problem of rate-distortion optimal packetization with uneven erasure protection (UEP) of scalable source sequence, into multiple groups of packets. The grouping of packets is needed when the length of the channel code, hence the number of packets, has to be modest for low decoding complexity. The problem was previously addressed in the literature but only locally optimal solution was proposed. We develop an algorithm for globally optimal solution and show that it has the same complexity as optimal UEP packetization into a single group of packets, i.e., quadratic in transmission budget.

1. INTRODUCTION

Modern packet switched communications systems such as ATM and the Internet have to overcome the problems of packet loss. In the case of streaming a scalable source sequence of compressed digital media, packetization with uneven erasure protection (UEP) has become a strategy of choice to alleviate the impact of packet loss.

The essential feature of scalable compression algorithms like SPIHT [6] or EBCOT [9] is that the source can be reconstructed to some degree from any prefix of the code stream. One UEP packetization scheme which effectively combines this feature with the erasure correction capability of Reed-Solomon (RS) codes, uses a collection of (RS) block codes of the same length but decreasing strengths to protect subsequent segments of the source code, and forms the packets across the channel codewords. Any set of received packets can be used to reconstruct the source to some fidelity, and the fidelity increases in the number of received packets. Optimal packetization techniques, in the sense of maximizing the expected fidelity at the receiver side subject to a given transmission budget, have been extensively studied [1, 2, 3, 4, 5, 7, 8].

A drawback of RS-coded UEP is high decoding complexity when the length of the RS code, hence the number N of packets, is too large. A remedy is to impose a limit on the number of packets. Thus, if the number of packets N is bounded, then in order to use the whole transmission budget for packetization the packet size L has to be increased. But this may not be desirable especially in a high loss network. In order to use a large transmission budget while keeping both N and L small, a possibility is to packetize the scalable source stream into multiple groups, each group containing only N packets. For this the scalable stream is partitioned into several sub-streams and each sub-stream is packetized separately, obtaining thus several groups of packets.

This idea of packetizing the scalable stream into multiple groups of packets to reduce the decoding complexity was proposed by Thie and Taubman [10]. They addressed the problem of optimizing such a scheme in the rate-fidelity sense. Their formulation of the problem allows for fractional bit allocation for the redundancy assignment, while in practice only integer bit allocation is possible. Also, the algorithm proposed only finds a locally optimal solution to the problem.

In this paper we are interested in finding an exact solution to this problem. In our formulation integer bit allocation is imposed for the redundancy assignment. Thus, the problem becomes one of combinatorial optimization. We present an algorithm that finds the globally optimal solution. Interestingly, even if the problem appears to be more complex than optimal UEP packetization into a single group of packets, the computational time requirement for the globally optimal solution is the same, i.e., quadratic in the transmission budget [1].

2. UNI-GROUP UEP PACKETIZATION

We refer to the packetization of a scalable sequence into a single group a packets as uni-group UEP packetization.

Let N be the number of packets, and L the number of symbols in each packet (a symbol is a block of a fixed number of bits). Actually only a prefix of the scalable source sequence is packetized. This prefix of the source code stream is partitioned into L consecutive segments, and each of these segments is protected by RS code. Let m_i be the length (in symbols) of the i-th source segment, then the channel code assigned to protect it is the (N, m_i) RS code. The stream of these m_i source symbols followed by the $f_i = N - m_i$ redundancy symbols constitutes the i-th slice of the joint source-channel code. The packets are formed across the slices, i.e., the n-th packet contains the n-th symbol of each slice. The effect of the (N, m_i) RS code applied to the i-th source segment is that, if at most f_i of N packets are lost, then all the m_i source symbols of the i-th slice can be correctly recovered. However, since the scalable source sequence is only sequentially refinable, the i-th source segment can be decoded only if the previous $i-1$ segments are available. This leads to the necessity that the number of redundancy symbols assigned to a slice be monotonically non-increasing in the slice index: $f_1 \geq f_2 \geq \cdots \geq f_L$, or equivalently, the number of source symbols allocated to each slice be monotonically non-decreasing in the slice index:

$$m_1 \leq m_2 \leq \cdots \leq m_L, \qquad (1)$$

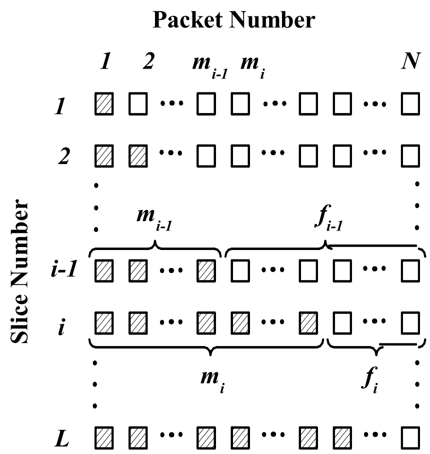

Fig. 1. Uni-group UEP packetization scheme. The slices are positioned horizontally and the packets vertically. The shaded squares represent the source symbols and the white squares represent redundancy symbols.

Let $\mathbf{m} = (m_1, m_2, \cdots, m_L)$ be the vector whose components are the number of source symbols allocated to the slices. We call $\mathbf{m}$ the L-slice source allocation vector. Figure 1 illustrates the uni-group UEP packetization scheme.

Let $\phi(r)$ be the rate-fidelity function of the scalable source sequence, which is a monotonically non-decreasing function in rate $r \in [0, R_{max}]$, where r denotes the number of symbols in a prefix of the source sequence, and R_{max} is the total number of source symbols. Let $p_N(n)$, for $0 \le n \le N$, denote the probability of loosing n packets out of N. The efficiency of the packetization scheme is measured by the expected fidelity of the reconstructed sequence at the decoder side, denoted by $\Phi(\mathbf{m})$. This quantity can be expressed as [2, 3]

$$\Phi(\mathbf{m}) = P_N(N)\phi(0) + \sum_{i=1}^{L} P_N(f_i)(\phi(r_i) - \phi(r_{i-1})) =$$
$$P_N(N)\phi(0) + \sum_{i=1}^{L} P_N(N - m_i)(\phi(r_i) - \phi(r_{i-1})),$$

where $P_N(k) = \sum_{n=0}^{k} p_N(n)$, $k = 0, 1, \cdots, N$, and $r_i = \sum_{k=1}^{i} m_k$, $1 \le i \le L$, $r_0 = 0$.

The objective of optimal uni-group UEP packetization under the rate-fidelity criterion is to find the L-slice source allocation vector $\mathbf{m} = (m_1, m_2, \cdots, m_L)$ that maximizes $\Phi(\mathbf{m})$, for given N, L, $p_N(n)$, and $\phi(r)$. Various algorithms have been proposed in the literature to find exact or approximate solutions [1, 2, 3, 4, 5, 7, 8]. Among the algorithms which provide globally optimal solution to the most general setting of the problem, the most efficient one has running time $O(N^2 L^2)$, i.e., quadratic in the transmission budget [1].

3. MULTI-GROUP UEP PACKETIZATION

In multi-group UEP packetization the source sequence is partitioned into K sub-streams and each sub-stream is packetized separately producing a group of packets. Each group of packets has N packets, each of size L.

This packetization strategy was proposed in [10] for the case when the channel codewords are constrained in length in order to limit the decoding complexity.

Let $(a, b]$ denote the sub-stream obtained by removing the first a symbols and the suffix which starts on the position $b + 1$ from the whole code sequence. Using this notation the whole code stream can be denoted by $(0, R_{max}]$. Let the K sub-streams be $(u_0, u_1], (u_1, u_2], \cdots (u_{K-1}, u_K]$ where $0 = u_0 < u_1 < \cdots < u_K \le R_{max}$. Each $(u_{k-1}, u_k]$ is packetized as described in the previous section, forming the k^{th} group of packets. Let $\mathbf{m}_k = (m_{k,1}, m_{k,2}, \cdots, m_{k,L})$ denote the L-slice source allocation vector corresponding to the k^{th} group of packets, where $m_{k,i}$ denotes the number of source symbols on the i^{th} slice of the k^{th} group of packets. The number of redundancy symbols is thus $f_{k,i} = N - m_{k,i}$. The constraint (1) becomes:

$$1 \le m_{k,1} \le m_{k,2} \le \cdots \le m_{k,L} \le N \text{ for all } k, 1 \le k \le K. \tag{2}$$

Moreover because the whole sub-stream $(u_{k-1}, u_k]$ is packetized in the k^{th} group the following equality holds:

$$m_{k,1} + m_{k,2} + \cdots + m_{k,L} = u_k - u_{k-1}. \tag{3}$$

Let $s_i(\mathbf{m}_k) = m_{k,1} + m_{k,2} + \cdots + m_{k,i}$ denote the total number of source symbols on the first i slices of the k^{th} group. By convention, $s_0(\mathbf{m}_k) = 0$. Relation (3) can be written as $s_L(\mathbf{m}_k) = u_k - u_{k-1}$.

Note that the multi-group packetization is uniquely determined by the K vectors of L-slice source allocation: $\mathbf{m}_1, \cdots, \mathbf{m}_K$. Indeed, if these vectors are known then the source sub-streams packetized in each group can be determined according to equation (3). More exactly, we have $u_0 = 0$ and

$$u_k = \sum_{j=1}^{k} s_L(\mathbf{m}_j), \tag{4}$$

for each k. Therefore the expected fidelity of the source reconstruction at the receiver can be treated as a function of the source allocation vectors and we denote it by $\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K)$. Let us find now its expression. The packets of the k^{th} group available at the receiver are useful to restore a prefix of the k^{th} source substream. Because the source sequence is scalable this can be further used for source decoding only if the whole prefix of the source stream packetized in the first $k-1$ groups (i.e., $(0, u_k]$) can be recovered at the receiver. Let us denote by $\Delta\Phi(a, \mathbf{m}_k)$ the expected increment in fidelity due to decoding of the received packets of the k^{th} group, under the assumption that the prefix $(0, a]$ has been restored at the receiver, where $u_k = a$. Then,

$$\Delta\Phi(a, \mathbf{m}_k) = \sum_{i=1}^{L} P_N(f_{k,i})[\phi(a + s_i(\mathbf{m}_k)) - \phi(a + s_{i-1}(\mathbf{m}_k))].$$

On the other side, the prefix $(0, u_k]$ can be entirely recovered if and only if the number of lost packets from each group j is at most $f_{j,L}$ for $1 \le j \le k-1$. The probability of this event is $\Pi_{j=1}^{k-1} P_N(f_{j,L})$, under the assumption that packet losses in different groups are independent. It follows that the expected increment in fidelity due to decoding the received packets from the k^{th} group is

$$\Pi_{j=1}^{k-1} P_N(f_{j,L}) \Delta\Phi(a, \mathbf{m}_k). \tag{5}$$

The overall expected fidelity at the receiver, denoted by $\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K)$, can be obtained by adding up the expected increments in fidelity due to decoding the received packets from each group:

$$\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K) = P_N(N)\phi(0) + \Delta\Phi(0, \mathbf{m}_1) + \sum_{k=2}^{K} \Pi_{j=1}^{k-1} P_N(f_{j,L}) \Delta\Phi(u_{k-1}, \mathbf{m}_k). \quad (6)$$

The objective of optimal multi-group UEP packetization under the rate-fidelity criterion is to find the L-slice source allocation vectors $\mathbf{m}_1, \mathbf{m}_2, \cdots, \mathbf{m}_K$, that maximize $\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K)$, for given N, L, K, $p_N(n)$, and $\phi(r)$.

Let us denote

$$\Delta\Phi(a, \mathbf{m}_k, \cdots, \mathbf{m}_K) = \Delta\Phi(a, \mathbf{m}_k) + \sum_{i=k+1}^{K} \Pi_{j=k}^{i-1} P_N(f_{j,L}) \Delta\Phi(u_{i-1}, \mathbf{m}_i), \quad (7)$$

for any a and any L-tuples $\mathbf{m}_k, \cdots, \mathbf{m}_K$. Likewise is defined $\Delta\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_{k-1})$. It follows that

$$\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K) = P_N(N)\phi(0) + \Delta\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_{k-1}) + \Pi_{j=1}^{k-1} P_N(f_{j,L}) \Delta\Phi(u_{k-1}, \mathbf{m}_k, \cdots, \mathbf{m}_K). \quad (8)$$

The above equality implies that in order for $\Phi(0, \mathbf{m}_1, \cdots, \mathbf{m}_K)$ to be maximal over all possible L-tuples $\mathbf{m}_1, \cdots, \mathbf{m}_K$, $\Delta\Phi(u_{k-1}, \mathbf{m}_k, \cdots, \mathbf{m}_K)$ has to be also maximal over all L-tuples $\mathbf{m}_k, \cdots, \mathbf{m}_K$, given fixed u_{k-1}. This observation leads to the idea of solving the optimization problem by recursively maximizing $\Delta\Phi(a, \mathbf{m}_k, \cdots \mathbf{m}_K)$ for all a and k. Let us denote

$$\Delta\hat{\Phi}(a, k : K) = \max_{\mathbf{m}_k, \cdots, \mathbf{m}_K} \Delta\Phi(a, \mathbf{m}_k, \cdots, \mathbf{m}_K), \quad (9)$$

where the maximum is taken over all L-tuples $\mathbf{m}_k, \cdots, \mathbf{m}_K$ which satisfy the condition (2). By convention, $\Delta\hat{\Phi}(a, K+1 : K) = 0$ for all a. Clearly, solving the optimization problem is equivalent to computing $\Delta\hat{\Phi}(0, 1 : K)$. The following propositions shows that this can be done recursively.

Proposition 1. For all $k, 1 \le k \le K$, and $a, 0 \le a \le R_{max}$, we have

$$\Delta\hat{\Phi}(a, k : K) = \max_{\mathbf{m}_k} [\Delta\Phi(a, \mathbf{m}_k) + P_N(f_{k,L}) \Delta\hat{\Phi}(a + s_L(\mathbf{m}_k), k+1 : K)], \quad (10)$$

where the maximum is taken over all L-tuples $\mathbf{m}_k$ which satisfy the condition (2).

Proof. Relation (7) implies that

$$\Delta\Phi(a, \mathbf{m}_k, \cdots, \mathbf{m}_K) = \Delta\Phi(a, \mathbf{m}_k) + P_N(f_{k,L}) \Delta\Phi(a + s_L(\mathbf{m}_k), \mathbf{m}_{k+1}, \cdots, \mathbf{m}_K). \quad (11)$$

It further follows that

$$\max_{\mathbf{m}_k, \cdots, \mathbf{m}_K} \Delta\Phi(a, \mathbf{m}_k, \cdots, \mathbf{m}_K) = \max_{\mathbf{m}_k} \{\Delta\Phi(a, \mathbf{m}_k) + P_N(f_{k,L}) \cdot \max_{\mathbf{m}_{k+1}, \cdots, \mathbf{m}_K} \Delta\Phi(a + s_L(\mathbf{m}_k), \mathbf{m}_{k+1}, \cdots, \mathbf{m}_K)\}, (12)$$

which implies the claim. □

Solving recursion (10) by exhaustive search over all L-tuples $\mathbf{m}_k$ is intractable. We show next how (10) can be solved in finer recursive steps. For this we need to introduce some more notations. For any integers $a, 0 \le a \le R_{max}, j, 1 \le j \le N$, and $1 \le m_1 \le m_2 \le \cdots \le m_j \le N$, denote

$$W(a, m_1, m_2, \cdots, m_j) = \sum_{i=1}^{j} P_N(N - m_i)[\phi(a + m_1 + \cdots + m_i) - \phi(a + m_1 + \cdots + m_{i-1})]. \quad (13)$$

Let us further denote

$$\hat{W}_{k,j}(a, n) = \max_{n \le m_{k,j} \le \cdots \le m_{k,L} \le N} \{W(a, m_{k,j}, \cdots, m_{k,L}) + P_N(N - m_{k,L}) \Delta\hat{\Phi}(a + m_{k,j} + \cdots + m_{k,L}, k+1 : K)\} (14)$$

for any $0 \le a \le R_{max}$, $1 \le n \le N$, $1 \le k \le K$, and $1 \le j \le N$.

Proposition 2.

i) For any a and $k, 1 \le k \le K$

$$\Delta\hat{\Phi}(a, k : K) = \hat{W}_{k,1}(a, 1). \quad (15)$$

ii) For any $a, k, 1 \le k \le K$ and $n, 1 \le n \le N-1$

$$\hat{W}_{k,L}(a, n) = \max\{\hat{W}_{k,L}(a, n+1), (\phi(a+n) - \phi(a) + \hat{W}_{k+1,1}(a+n, 1)) P_N(N-n)\}. \quad (16)$$

iii) For $1 \le l \le L-1$

$$\hat{W}_{k,l}(a, n) = \max\{\hat{W}_{k,l}(a, n+1), (\phi(a+n) - \phi(a)) P_N(N-n) + \hat{W}_{k,l+1}(a+n, n)\}. \quad (17)$$

Proof. i) According to definition (14) we have that

$$\hat{W}_{k,1}(a, 1) = \max_{1 \le m_{k,1} \le \cdots \le m_{k,L} \le N} \{W(a, m_{k,1}, \cdots, m_{k,L}) + P_N(N - m_{k,L}) \Delta\hat{\Phi}(a + m_{k,1} + \cdots + m_{k,L}, k+1 : K)\}.$$

But $W(a, m_{k,1}, \cdots, m_{k,L})$ is actually $\Delta\Phi(a, \mathbf{m}_k)$. It follows that,

$$\hat{W}_{k,1}(a, 1) = \max_{\mathbf{m}_k} [\Delta\Phi(a, \mathbf{m}_k) + P_N(f_{k,L}) \Delta\hat{\Phi}(a + s_L(\mathbf{m}_k), k+1 : K)]. \quad (18)$$

Applying further Proposition 1, the conclusion follows.

ii) Applying definition (14) we obtain

$$\hat{W}_{k,L}(a, n) = \max_{n \le m_{k,L} \le N} \{W(a, m_{k,L}) + P_N(N - m_{k,L}) \Delta\hat{\Phi}(a + m_{k,L}, k+1 : K)\}. \quad (19)$$

The above maximum is taken over the set of integers $m_{k,L}$ in the range $[n, N]$. Such an integer is either equal to n or situated between $n+1$ and N. Consequently,

$$\hat{W}_{k,L}(a, n) = \max\{[W(a, n) + P_N(N-n) \Delta\hat{\Phi}(a+n, k+1 : K)], \max_{n+1 \le m_{k,L} \le N} [W(a, m_{k,L}) + P_N(N - m_{k,L}) \Delta\hat{\Phi}(a + m_{k,L}, k+1 : K)]\}. \quad (20)$$

Since $\Delta\hat{\Phi}(a+n,k+1:K) = \hat{W}_{k+1,1}(a+n,1)$ according to the conclusion of point i), using further the definitions of $W(a,n)$ and $\hat{W}_{k,L}(a,n+1)$ it follows that

$$\hat{W}_{k,L}(a,n) = \max\{(\phi(a+n)-\phi(a)+ \hat{W}_{k+1,1}(a+n,1))P_N(N-n), \hat{W}_{k,L}(a,n+1)\}. \quad (21)$$

iii) Definition (14) implies

$$\hat{W}_{k,l}(a,n) = \max_{n\le m_{k,l}\le\cdots\le m_{k,L}\le N}\{W(a,m_{k,l},\cdots,m_{k,L}) + P_N(N-m_{k,L})\Delta\hat{\Phi}(a+m_{k,l}+\cdots+m_{k,L},k+1:K)\}.$$

The maximum is taken over all $(L-l+1)$ tuples $(m_{k,l},\cdots,m_{k,L})$ such that $n\le m_{k,l}\le\cdots\le m_{k,L}\le N$. For any such tuple either $m_{k,l}=n\le m_{k,l+1}\le\cdots\le m_{k,L}\le N$ or $n+1\le m_{k,l}\le m_{k,l+1}\le\cdots\le m_{k,L}\le N$ hold. The maximum over the tuples in the second category is actually $\hat{W}_{k,l}(a,n+1)$. Consequently,

$$\hat{W}_{k,l}(a,n) = \max\{\hat{W}_{k,l}(a,n+1), \max_{n\le m_{k,l+1}\le\cdots\le m_{k,L}\le N}\{W(a,n,m_{k,l+1},\cdots,m_{k,L}) + P_N(N-m_{k,L})\Delta\hat{\Phi}(a+n+ m_{k,l+1}+\cdots+m_{k,L},k+1:K)\}\}. \quad (22)$$

From (13) it follows that

$$W(a,n,m_{k,l+1},\cdots,m_{k,L}) = (\phi(a+n)- \phi(a))P_N(N-n) + W(a+n,m_{k,l+1},\cdots,m_{k,L}), \quad (23)$$

which implies that

$$\max_{n\le m_{k,l+1}\le\cdots\le m_{k,L}\le N}\{W(a,n,m_{k,l+1},\cdots,m_{k,L}) + P_N(N-m_{k,L})\Delta\hat{\Phi}(a+n+m_{k,l+1}+\cdots +m_{k,L},k+1:K)\} = (\phi(a+n)-\phi(a))P_N(N-n) + \max_{n\le m_{k,l+1}\le\cdots\le m_{k,L}\le N}\{W(a+n,m_{k,l+1},\cdots,m_{k,L}) + P_N(N-m_{k,L})\Delta\hat{\Phi}(a+n+m_{k,l+1}+\cdots +m_{k,L},k+1:K)\} = (\phi(a+n)-\phi(a))P_N(N-n) + \hat{W}_{k,l+1}(a,n).$$

By replacing in (22) relation (17) follows. □

According to Proposition 2 i), solving the problem of optimal multi-group UEP packetization is equivalent to computing $\hat{W}_{1,1}(0,1)$. This can be done by recursively computing the quantities $\hat{W}_{k,l}(a,n)$ for all integers a,n,k,l, $0\le a\le R_{max}$, $1\le n\le N$, $1\le k\le K$, $1\le l\le L$, by using the recursions established by Proposition 2 ii), and iii). The computations are organized in decreasing order of k,l and n. More exactly, the following nested loop briefly describes the algorithm.

```
for     k := K down to 1
        for l := L down to 1
                for a := 0 to R_max
                        for n := N down to 1
                                compute Ŵ_{k,l}(a,n)
                        end for
                end for
        end for
end for
```

The value $\hat{W}_{k,l}(a,n)$ is computed by applying recursion (16) or (17), which requires constant time. Thus the time necessary to complete all the four nested loops is $O(KLNR_{max}) = O(K^2L^2N^2)$, i.e., quadratic in the transmission budget.

4. CONCLUSION

The problem of of rate-distortion optimal packetization with uneven erasure protection (UEP) of scalable source sequence, into multiple groups of packets, is addressed. The packetization into multiple groups was previously proposed in order to decrease the decoding complexity, but only a locally optimal solution to the problem was given. We present an algorithm for globally optimal solution and show that it has the same time complexity as optimal UEP packetization into a single group of packets, i.e., quadratic in transmission budget.

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