Piece-wise Approximations of *RLCK* Circuit Responses using Moment Matching

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ABSTRACT

Capturing *RLCK* circuit responses accurately with existing model order reduction (MOR) techniques is very expensive. Direct metrics for fast analysis of *RC* circuits exist but there is no such technique for *RLCK* circuits. This paper introduces a new family of MOR techniques based on piece-wise functions to capture *RLCK* circuit responses accurately using only four or five moments. The time-domain response is approximated using a piece-wise function whose pieces are simple polynomials. The proposed method is fast and guaranteed stable and it avoids the calculation of poles and residues associated with existing model order reduction techniques. Results for many different industrial netlists indicate that delay and transition time can be captured within 5% error using only four moments. To the authors' knowledge, there is no existing method that can extract as much information about *RLCK* circuits with only four or five moments.

Categories and Subject Descriptors

B.7.2 Integrated Circuits (Design Aids), *Simulation*

General Terms

Algorithms, Performance

Keywords

Interconnect timing analysis, moments, *RC*, *RLC, RLCK* circuits

1. INTRODUCTION

Interconnect timing analysis is one of the fundamental components of a static timing analysis tool. Until recently, most interconnect timing analyzers were tuned for handling *RC* circuits. The ease and simplicity of computing circuit moments from large extracted netlists made model order reduction (MOR) techniques ([1]-[3]) based on moment matching very useful for interconnect timing analysis. Only a few moments can produce good results for *RC* circuits. However, tens of moments are required for accurate estimation of *RLCK* circuit responses and existing techniques can suffer instability, inaccuracies, and/or long runtimes. Inductive effects have become much more significant recently ([4],[5]) and interconnect timing analyzers must be modified to efficiently handle complex *RLCK* networks present in high speed digital circuits.

One feature common to all existing model order reduction techniques is their approximation of a time-domain circuit response $y(t)$ using a sum of exponential (SOE) terms as given by

$$y(t) \approx \tilde{y}_n(t) = \sum_{i=1}^{n} k_i e^{p_i t} \qquad (1)$$

where k_i and p_i are generally complex numbers. The values p_i in the above equation are closely related to the dominant eigenvalues or *poles* of the original circuit and the values k_i are the *residues*.

There are many problems associated with the sum of exponential representation. First, the approximation of the exact circuit response can be unstable if any p_i has a positive real part. Second, it can be very expensive to find accurate approximations for *RLCK* circuits. Third, it is hard to extract timing information such as delay and risetime from the SOE form.

The form shown in equation (1) cannot be readily used for timing analysis. In any case, static timing analyzers do not need all the details contained in the sum of exponential representation. Most static timing analyzers require the computation of timing parameters such as delay for computing path delay and transition time for computing next gate delay using pre-characterized cell library lookups. Extracting these parameters from the form shown in (1) is usually expensive and requires iterative methods. Hence, the SOE form is unnecessarily complex for static timing purposes.

Direct metrics for delay and transition time calculation were introduced to avoid problems associated with model order reduction techniques and to reduce runtime. Elmore and Wyatt delay models approximate the 50% delay as $-m_1$ and $-0.693m_1$, respectively, where m_1 is the first order moment of the response in frequency domain ([6],[7]). Recently, more accurate delay and transition time metrics have been developed based on higher order moments ([8]-[15]). These metrics also assume that the circuit response follows some time-domain function.

Most of the direct metrics are based on only two or three moments and they give accurate delay and transition time for *RC* circuits. However, similar direct metrics do not exist for *RLCK* circuits whose responses are very complex and require more moments. Another problem with the direct methods is that the closed form expressions for delay and transition time are not scalable with the number of moments. There is no general method to extend these methods to make use of more information contained in higher order moments.

In this paper, a new family of model order reduction techniques based on piece-wise functions is introduced to approximate *RLCK* circuit responses using moment matching. As with previous methods, it is assumed that the response follows some pre-specified time-domain function, which is piece-wise here. An example of such a function is the piece-wise linear (PWL) function (see Figure 1). Obvious extensions to the PWL function are the piece-wise quadratic (PWQ) and piece-wise cubic (PWC) functions. In fact the methodology presented in this paper can be applied to any piece-wise polynomial function of time efficiently.

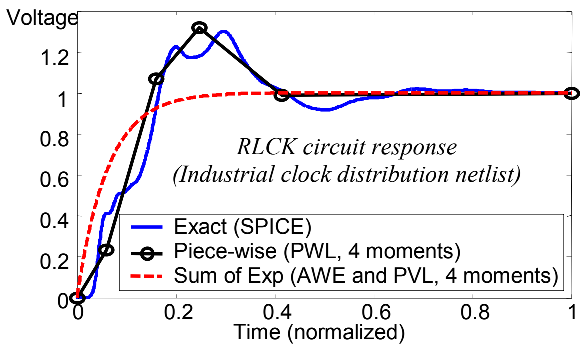

Figure 1. Piece-wise waveform (PWL) is better than SOE using just 4 moments.

Representing circuit responses using piece-wise functions has several advantages over the SOE form for computing delays and transition times. Finding the piece-wise approximation is much easier than finding the SOE approximation. Additionally, much fewer moments are needed to get the required information about signals (see Figure 1) than required for a single-piece SOE representation. As seen in Figure 1, the *RLCK* circuit response is approximated well using the PWL function using just four moments. In comparison, the SOE form returns very inaccurate result with four moments. Although the method is easily extendable to use the information contained in higher order moments, it will be shown in the results section that using only four to five moments produces very good results for complex *RLCK* responses. The method is stable since poles are not involved at all. As long as the piece-wise function is kept simple (PWL, PWQ, etc.), it is very easy to extract timing parameters such as delay and transition time from the piece-wise description. The runtime is comparable to some of the direct methods which only handle *RC* circuits.

The rest of the paper is organized as follows. Background information is presented in section 2 and the new theory is presented in section 3. Results are provided in section 4 for *RC* and *RLCK* circuits. Finally, the paper is concluded in section 5.

2. BACKGROUND

Many model order reduction techniques assume that the time-domain response can be represented by the SOE form given by (1), which resembles the exact response for n^{th} order interconnect systems. A reduced order model is produced when n is less than the order of the original system. The poles and residues of the reduced order system are computed by moment matching.

Moment matching has been famous since the introduction of AWE because of the ease of computing circuit moments ([16]). Matching the circuit response moments m_i for i=0,…,(2n-1) with the moments of (1) yields the system of equations

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & \cdots & p_n^{-1} \\ p_1^{-2} & p_2^{-2} & \cdots & p_n^{-2} \\ \vdots & \vdots & \ddots & \vdots \\ p_1^{-2n} & p_2^{-2n} & \cdots & p_n^{-2n} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = - \begin{bmatrix} m_0 \\ m_1 \\ \vdots \\ m_{2n-1} \end{bmatrix}. \qquad (2)$$

The above system is a set of 2n nonlinear equations in 2n unknowns and it can be solved using the method in [1].

The SOE form works well for *RC* circuits. For *RLCK* circuits however, higher orders are needed and AWE typically fails at higher orders because of numerical instability issues. Methods such as PRIMA and PVL were introduced to capture these responses but they are more complex and they typically require many moments to accurately capture *RLCK* responses. Figure 2 shows an exact *RLCK* circuit response obtained using SPICE and its approximation using the SOE form (equation (1)) for many different approximation orders n. The circuit was extracted from an industrial layout and the response shown is for a step input. The figure shows that a second order approximation using four moments is not sufficient at all to capture the complex response. The SOE form requires as many as 20 moments to get close to the exact response and 40 or more moments for exactness.

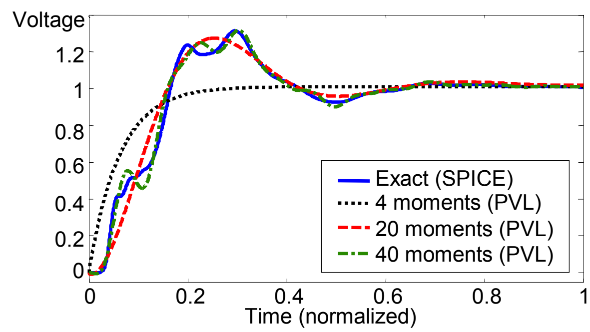

Figure 2. The SOE (SOE) representation requires too many moments for *RLCK* circuit response approximation, requiring long runtimes.

3. THEORY

In this section, the theory for calculating a piece-wise approximation from circuit moments will be presented. Section 3.1 presents the details for calculating the moments of a general piece-wise function. The details about calculating piece-wise linear, piece-wise quadratic, and piece-wise hybrid approximations from the circuit moments are presented in sections 3.2, 3.3, and 3.4, respectively. The method is general enough to be extended easily for other types of piece-wise functions.

3.1 Moments of a General Piece-Wise Function

Figure 3 shows the response of an *RLCK* circuit at a receiver node. This signal starts rising at time t=0 and can be considered settled at time t=t_n. The figure also shows a piece-wise function approximation $x(t)$ with n pieces between $t_0,t_1,\ldots,t_n$ and a last horizontal piece at v=V_{DD} from t_n to t=∞. The equation of a general piece-wise function is given by

$$x(t) = u(t-t_n) + \sum_{k=1}^{n} x_k(t) \cdot \left[u(t-t_{k-1}) - u(t-t_k) \right] \qquad (3)$$

where $u(t)$ is the unit-step function and the function $x_k(t)$ describes the k^{th} piece.

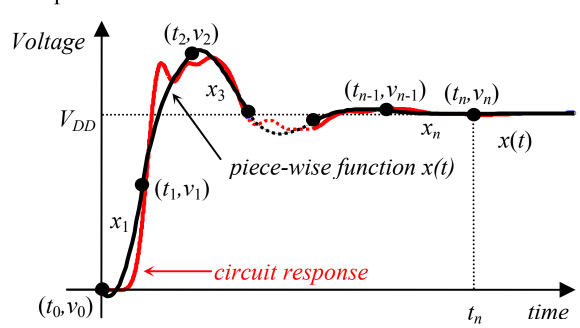

Figure 3. Representing a circuit response as a piece-wise function *x*(*t*).

The s-domain moments of the piece-wise function $x(t)$ can be computed easily using the definition of Laplace transform. The Laplace transform $X(s)$ of $x(t)$ is

$$X(s) = \frac{L[x'(t)] + x(0^-)}{s} = \frac{L[x'(t)]}{s} = \frac{1}{s}\int_0^\infty x'(t)e^{-st}\,dt \tag{4}$$

since $x(0^-)$=0. Expanding the exponent in (4) around s=0 results in

$$X(s) = \frac{1}{s}\left[\int_0^\infty x'(t)\left(1 - st + \frac{1}{2!}s^2t^2 - \frac{1}{3!}s^3t^3 + \cdots\right)dt\right]. \tag{5}$$

Since $x'(t) = 0$ for $t > t_n$, equation (5) can be rewritten as

$$X(s) = \frac{1}{s}\sum_{i=0}^{n}\frac{(-1)^i s^i}{i!}\int_0^{t_n} t^i x'(t)dt. \tag{6}$$

The integral in equation (6) can be further expanded in terms of the pieces of the function $x'(t)$ as

$$X(s) = \frac{1}{s}\sum_{i=0}^{n}\frac{(-1)^i s^i}{i!}\sum_{k=1}^{n}\int_{t_{k-1}}^{t_k} t^i x_k'(t)dt. \tag{7}$$

The frequency domain response $X(s)$ is assumed to be in the form

$$X(s) = \frac{m_0^X}{s} + m_1^X + m_2^X s + \cdots + m_n^X s^{n-1} \tag{8}$$

where m_i^X, i=0 to n, are the s-domain moments of $X(s)$ for a voltage signal $x(t)$. Comparing equation (7) with (8) results in the following formula for moments m_i^X of a general piece-wise function:

$$m_i^X = \frac{(-1)^i}{i!}\sum_{k=1}^{n}\int_{t_{k-1}}^{t_k} t^i x_k'(t)dt. \tag{9}$$

As long as the pieces $x_k(t)$ are polynomials in t, the integral in (9) can be evaluated quickly and the piece-wise description of $x(t)$ results in much simpler moment matching equations than the system (2) for the SOE representation. Polynomials form a large class of functions and a lot of freedom is available to design the pieces to approximate very complex *RLCK* responses.

3.2 Calculating a Piece-Wise Linear (PWL) Approximation from Circuit Moments

Figure 4 shows the piece-wise linear version of the function $x(t)$. Each piece $x_k(t)$ is defined by

$$x_k(t) = a_k \cdot t + b_k. \tag{10}$$

Substituting the PWL $x(t)$ from (10) into (9), the s-domain moments m^X of the PWL function $x(t)$ have a compact analytical representation given by

$$m_i^X = \frac{(-1)^i}{(i+1)!}\sum_{k=1}^{n} a_k\,(t_k^{i+1} - t_{k-1}^{i+1}) \text{ for } i = 0 \text{ to } n. \tag{11}$$

Circuit response moments m^C can be computed easily using any of the existing moment calculation techniques ([17]). Matching the circuit response moments with equation (11) yields the system of equations

$$m_i^C = \frac{(-1)^i}{(i+1)!}\sum_{k=1}^{n} a_k\,(t_k^{i+1} - t_{k-1}^{i+1}) \quad \text{for } i = 0 \text{ to } n. \tag{12}$$

There are two types of unknowns to be determined in the PWL approximation. The first set of unknowns are the time-points t_k (k=1,2,…,n) where the consecutive pieces of $x(t)$ intersect (t_0=0 without any loss of generality). The second set consists of the parameters that define each of the pieces $x_k(t)$. For the PWL function, only (a_1,a_2,…,a_n) have to be calculated. If a_k's are known,

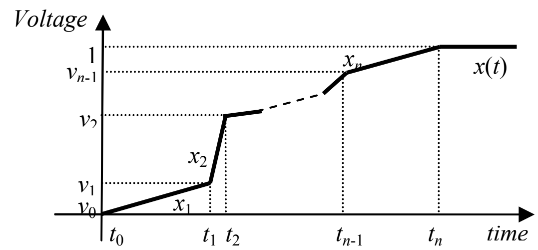

Figure 4. Representing a circuit response as a piece-wise linear (PWL) function $x(t)$.

then the parameters (b_1,b_2,…,b_n) can be calculated easily by matching the end-points of the pieces. Hence b_k's are not considered as unknowns. Thus, there are $2n$ unknowns to be calculated for an n^{th} order PWL approximation.

Equation (12) is non-linear in the time-points t_k for k=1 to n. However, if the values of time-points t_k are fixed, there are only n unknowns (a_1,a_2,…,a_n). Moreover, the system (12) becomes linear and can be solved quickly. Now only n circuit moments are necessary to calculate the n unknowns (a_1,a_2,…,a_n). The set of time-points are known to be real and positive and they can be easily estimated from the moments as will be shown in section 3.5. The set of parameters (a_1,a_2,…,a_n) can thus be obtained by solving the following linear system where t_k are known

$$\begin{bmatrix} m_1^C \cdot (-2) \\ m_2^C \cdot (6) \\ \vdots \\ m_n^C \cdot \dfrac{(n+1)!}{(-1)^n} \end{bmatrix} = \begin{bmatrix} (t_1^2 - t_0^2) & (t_2^2 - t_1^2) & \cdots & (t_n^2 - t_{n-1}^2) \\ (t_1^3 - t_0^3) & (t_2^3 - t_1^3) & \cdots & (t_n^3 - t_{n-1}^3) \\ \vdots & \vdots & \ddots & \vdots \\ (t_1^{n+1} - t_0^{n+1}) & (t_2^{n+1} - t_1^{n+1}) & \cdots & (t_n^{n+1} - t_{n-1}^{n+1}) \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}. \tag{13}$$

The values of a_k are always real numbers if the time-points t_k are real non-negative numbers, which is guaranteed. The constants b_k for equation (10) can be calculated recursively using

$$b_k = \begin{cases} 0 \text{ for rising, } 1 \text{ for falling} & k = 1 \\ x_{k-1}(t_{k-1}) - a_k t_{k-1} = a_{k-1}t_{k-1} + b_{k-1} - a_k t_{k-1} & k > 1 \end{cases} \tag{14}$$

where the formula for k>1 results from equating $x_{k-1}(t)$ and $x_k(t)$ at t=t_{k-1}. The results of approximating circuit responses using the PWL function are discussed in section 4.

3.3 Calculating a Piece-Wise Quadratic (PWQ) Approximation from Circuit Moments

The PWQ function offers a lot of variety in the possible waveshapes that it can capture. The pieces of $x_k(t)$ of the piece-wise quadratic version of the function $x(t)$ are defined by

$$x_k(t) = a_k \cdot t^2 + b_k \cdot t + c_k. \tag{15}$$

Substituting (15) into (9), the s-domain moments of the PWQ function are given by

$$m_i^X = \frac{(-1)^i}{i!}\sum_{k=1}^{n}\left[\frac{2a_k\,(t_k^{i+2} - t_{k-1}^{i+2})}{i+2} + \frac{b_k\,(t_k^{i+1} - t_{k-1}^{i+1})}{i+1}\right] \tag{16}$$

for i = 0 to n. Since $x_k(t)$ is a piece-wise quadratic function, there are n extra unknowns as compared to the PWL function. As with the PWL function, the time-points t_k for the PWQ function can be calculated based on the method presented in section 3.5.

The system of $2n$ equations can be formed as follows. The first n equations can be obtained by matching m_1^X to m_n^X with the circuit response moments m_1^C to m_n^C. The next (n-1) equations can be obtained by matching the first time derivative of $x_k(t)$ with the first time-derivative of $x_{k+1}(t)$ at time-points t_k, k=1,2,…,(n-1). This not only guarantees the smoothness of response where the pieces of $x(t)$ meet but it also uses only n circuit response moments. The last equation results from setting $x_n'(t{=}t_n)$ equal to zero. This guarantees that the response is steady at $t{=}t_n$. Thus, the linear system of $2n$ equations in $2n$ unknowns is given by

$$\begin{aligned} m_i^C \cdot \frac{i!}{(-1)^i} &= \sum_{k=1}^{n}\left[\frac{2a_k(t_k^{i+2}-t_{k-1}^{i+2})}{i+2}+\frac{b_k(t_k^{i+1}-t_{k-1}^{i+1})}{i+1}\right] && \text{for } i=1 \text{ to } n \\ 0 &= 2a_kt_k+b_k-2a_{k+1}t_k-b_{k+1} && \text{for } k=1 \text{ to } (n-1) \\ 0 &= 2a_kt_k+b_k && \text{for } k=n \end{aligned} \tag{17}$$

and it can be solved easily by Gaussian elimination.

Again, the values a_k and b_k obtained by solving (17) are real numbers. A tradeoff can be made between matching moments and matching the first derivatives of $x_k(t)$ and $x_{k+1}(t)$ at time-points t_k to form the system of equations. If more than n moments are matched, then less than n derivatives should be matched.

The general recursive formula for calculating c_k is given by

$$c_k = \begin{cases} 0 \text{ for rising, } 1 \text{ for falling} & k=1 \\ x_{k-1}(t_{k-1}) - a_k t_{k-1}^2 - b_k t_{k-1} & k>1 \end{cases} \tag{18}$$

where $x_{k-1}(t_{k-1}) = a_{k-1}t_{k-1}^2 + b_{k-1}t_{k-1} + c_{k-1}$.

The results of approximating circuit responses using the PWQ function are presented in section 4.

The method presented in this section for PWQ functions can be extended easily for higher order polynomials (*e.g.* cubic). Higher order polynomials require calculation of more parameters and this can be done by either matching more moments and/or by matching higher order derivatives of $x_k(t)$ and $x_{k+1}(t)$ at t_k, (k=1,2,…,n-1) and solving the resulting linear system of equations.

3.4 Calculating a Hybrid Piece-Wise (HPW) Approximation from Circuit Moments

In addition to polynomials in powers of t, polynomials in powers of $1/t$ also result in linear equations for moment matching. Based on many experiments and observations, it was determined that a hybrid piece-wise function defined by

$$x_k(t) = \begin{cases} a_k \cdot t^2 + b_k \cdot t + c_k & k=1 \\ \dfrac{a_k}{t^2} + \dfrac{b_k}{t} + c_k & 1<k\le n \end{cases} \tag{19}$$

produces the best results for *RLCK* as well as *RC* circuits. The first piece of the HPW function is a quadratic function with respect to time t and the remaining n-1 pieces are quadratic functions with respect to inverse time $1/t$.

The *s*-domain moments of the HPW function can be determined by using equation (9) and are given by

$$m_i^X = \frac{(-1)^i}{i!}(m_i^Q + m_i^{IQ}) \quad \text{where}$$

$$m_i^Q = \left[\frac{2a_1(t_1^{i+2}-t_0^{i+2})}{i+2} + \frac{b_1(t_1^{i+1}-t_0^{i+1})}{i+1}\right] \quad \text{and}$$

$$m_i^{IQ} = \begin{cases} \sum_{k=2}^{n}\left[\dfrac{-2a_k(t_k^{i-2}-t_{k-1}^{i-2})}{i-2} + \dfrac{-b_k(t_k^{i-1}-t_{k-1}^{i-1})}{i-1}\right], & i=0, i\ge 3 \\ \sum_{k=2}^{n}\left[2a_k(t_k^{-1}-t_{k-1}^{-1}) - b_k \log_e\left(\dfrac{t_k}{t_{k-1}}\right)\right], & i=1 \\ \sum_{k=2}^{n}\left[-2a_k \log_e\left(\dfrac{t_k}{t_{k-1}}\right) - b_k(t_k - t_{k-1})\right], & i=2 \end{cases} \tag{20}$$

where m^Q refers to the integral evaluation of the quadratic piece $x_1(t)$ from time $t{=}t_0$ to $t{=}t_1$ and m^{IQ} refers to the integral evaluation of the pieces $x_2(t)$ to $x_n(t)$ which are quadratic functions of inverse time $1/t$.

Assuming that the time-points t_k are known (see section 3.5), there are $2n$ unknowns ($a_1,b_1,a_2,b_2,\ldots,a_n,b_n$). As with the PWQ function, the system of $2n$ equations for the HPW function can be formed by matching n moments (n equations), by matching the first time-derivatives of $x_k(t)$ and $x_{k+1}(t)$ at time-points t_1 to t_{n-1} (n-1 equations) to guarantee smoothness, and by setting $x_n'(t{=}t_n)$ equal to zero (1 equation) to guarantee a settled value at time t_n. The system of $2n$ equations is given by

$$\begin{aligned} m_i^C \cdot \frac{i!}{(-1)^i} &= m_i^Q + m_i^{IQ} && \text{for } i=1 \text{ to } n \\ 0 &= 2a_1t_k + b_1 + \frac{2a_2}{t_k^3} + \frac{b_2}{t_k^2} && \text{for } k=1 \\ 0 &= -\frac{2a_k}{t_k^3} - \frac{b_k}{t_k^2} + \frac{2a_{k+1}}{t_k^3} + \frac{b_{k+1}}{t_k^2} && \text{for } k=2 \text{ to } (n-1) \\ 0 &= \frac{2a_k}{t_k^3} + \frac{b_k}{t_k^2} && \text{for } k=n \end{aligned} \tag{21}$$

where m^Q and m^{IQ} are defined in equation (20).

As with the PWQ function, the above linear system can be solved easily. Again, a tradeoff can be made between matching more moments and matching the first derivatives.

The general recursive formula for calculating c_k is given by

$$c_k = \begin{cases} 0 \text{ for rising, } 1 \text{ for falling} & k=1 \\ x_{k-1}(t_{k-1}) - \dfrac{a_k}{t_{k-1}^2} - \dfrac{b_k}{t_{k-1}} & k>1 \end{cases} \tag{22}$$

where $x_{k-1}(t)$ is defined by equation (19). The results of approximating circuit responses using the HPW function are presented in section 4.

3.5 Selection of the Time-points t_k

Selection of the time-points t_k is very easy as compared to solving for poles in the model order reduction techniques based on the SOE method. The method presented here selects real non-negative time-points t_k which produce very good results for *RLCK* as well as *RC* circuits.

The algorithm for selecting the time-points t_k proceeds as follows. The initial time-point t_0 is set to zero (t_0=0) without the loss of any generality. Then a guess for the final time-point t_n is made based on the first moment m_1^C. The value $t_n=10\cdot|m_1^C|$ works very well because most *RLCK* and *RC* signals settle down (stop changing) by this time. Then the remaining time-points t_1 to t_{n-1} are calculated such that they divide the range $[0,t_n]$ uniformly. Thus, the formula for t_k is given by

$$t_k = (k/n)\cdot t_n \tag{23}$$

for k=0 to n. Sometimes, using a ratio approach given by

$$t_k = \begin{cases} 0, & k=0 \\ t_n/n, & k=1 \\ r\cdot(t_{k-1}-t_{k-2}) + t_{k-1}, & k>1 \end{cases} \tag{24}$$

where r=1.15 gives very good results for *RC*, *RLC*, and *RLCK* circuits. For the ratio approach, note that the final value of t_n is greater than the initial guess.

The computation of $x(t)$ then proceeds as presented in the previous sections. The result based on initial $t_n=10\cdot|m_1^C|$ works very well but it can be made more accurate by one or two more refinements based on $x(t)$. It can be observed that the transition region where the signal makes initial rise or fall needs at least a couple of pieces for a good approximation of the exact circuit response. If t_1 is set to be roughly in the middle of this region, then t_k can be refined as $t_k=k{\cdot}t_1$ and the parameters a_k, b_k, and c_k can be calculated again based on the new values of t_k.

The theory presented above for selecting the time-points t_k has been tested thoroughly on many different extracted *RLCK* and *RC* netlists. It works very well as will be shown in section 4. Moreover, the calculation of the parameters a_k, b_k, c_k, etc. is not very sensitive on the values of t_k. A rough estimate of t_k is sufficient to get good values of the parameters.

3.6 Extraction of Timing Parameters from Piece-Wise Approximations

Timing parameters such as delay and transition time can be extracted very easily from $x(t)$ for PWL, PWQ, and HPW functions, where $x_k(t)$ have analytical inverse expressions.

The time-point t where $x_k(t)$ crosses a pre-specified threshold value v is then given by:

$$PWL: \quad t = \frac{v - b_k}{a_k}$$

$$PWQ: \quad t = \frac{-b_k \pm \sqrt{b_k^2 - 4a_k(c_k - v)}}{2a_k} \tag{25}$$

$$HPW: \quad t = \begin{cases} \dfrac{-b_k \pm \sqrt{b_k^2 - 4a_k(c_k - v)}}{2a_k}, & k = 1 \\ \dfrac{2a_k}{-b_k \pm \sqrt{b_k^2 - 4a_k(c_k - v)}}, & k > 1 \end{cases}$$

The 50% interconnect delay can be calculated by setting $v{=}0.5V_{DD}$ and the transition time can be calculated by computing $t_{10\%}$ when $v{=}0.1V_{DD}$ and $t_{90\%}$ when $v{=}0.9V_{DD}$. These solutions are much simpler than the Newton-Raphson iterative methods that are used for the SOE representation.

4. RESULTS

In this section, simulation results will be presented to show that the piece-wise functions perform much better than the SOE formulation for many different circuits.

Figure 1 in the introduction shows the signal at a receiver node of an industrial *RLCK* netlist driven by a step input. The response is very complex and the SOE form cannot capture it accurately using four moments as they are unable to produce complex poles. However, the PWL approximation of this signal produced sufficiently accurate results with only four moments. The delay and the transition time are captured accurately and the PWL function mimics the overall shape of the complex *RLCK* response.

Using the PWQ and HPW functions produce even better results than the PWL function. It was shown in the background section that the SOE function requires as many as 40 moments to capture the response of an industrial *RLCK* netlist. Figure 5 shows that the same response is captured well by the PWQ and HPW functions using only four moments.

The piece-wise functions also perform very well on small but hard to model circuits such as a transmission line. Figure 6 shows the response of an *RLC* transmission line with five ladder sections. The line was driven by a step input and the results show that the HPW function performs very well as compared to the SOE function with only four moments. The 50% delay was predicted accurately by the HPW with only 3.5% error whereas the delay error for the SOE function was an unacceptable 65%. Moreover, the HPW function predicted overshoot with only 6.6% error whereas the SOE function does not even capture the overshoot with four moments.

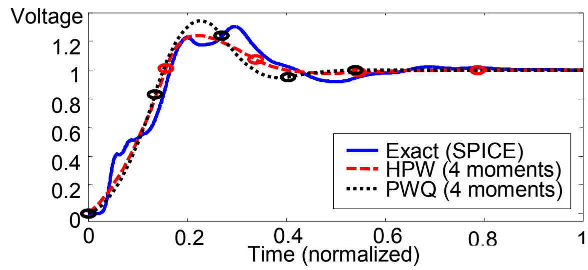

Figure 5. The PWQ and HPW functions capture *RLCK* response shown in Figure 2 very well with only 4 moments.

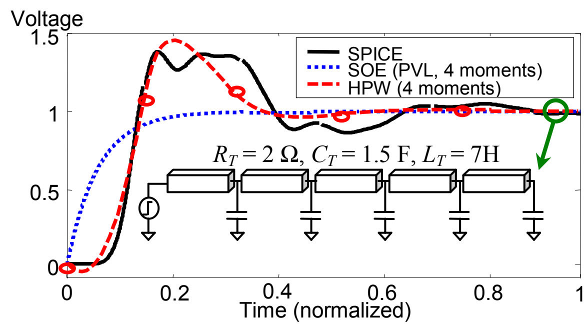

Figure 6. Comparison of HPW function with SOE for an *RLC* transmission line signal. HPW captures delay and transition time very accurately.

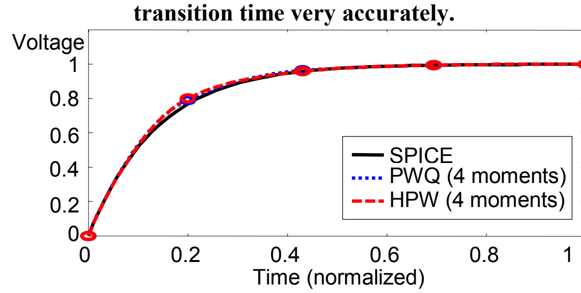

Figure 7. Piece-wise functions capture *RC* responses well.

The piece-wise functions not only perform well for *RLCK* circuits but they also produce good results for *RC* circuits. Figure 7 shows the response of a uniform 10x10 *RC* mesh circuit. Both PWQ and HPW return good results for this circuit. The error in delay was 3.0% and 1.5% for PWQ and HPW, respectively, and the error in transition time was 3.8% and 5.1% for PWQ and HPW, respectively.

The method presented in this paper was also tested on a large industrial clock distribution network (*RLCK*). The error in delay at receiver nodes is summarized in Table 1. The results clearly show that the HPW function is performing well consistently as compared to the SOE functions with only 3.14% maximum error in delay using only four moments as compared to unacceptable 68% and 30% minimum errors in delay for the SOE function using four and eight moments, respectively.

Table 1. Delay errors at receivers of an industrial *RLCK* netlist. The table gives a comparison between HPW and SOE.

50% delay (% error) at receiver nodes (time is scaled)				
Node	SPICE	**HPW 4 moments**	SOE (PVL) 4 moments	SOE (PVL) 8 moments
1	2.15	**2.19 (1.86)**	0.67 (-68)	1.48 (-31)
2	2.38	**2.34 (-1.68)**	0.74 (-69)	1.67 (-30)
3	2.11	**2.08 (-1.42)**	0.61 (-71)	1.43 (-32)
4	1.91	**1.97 (3.14)**	0.55 (-71)	1.17 (-39)
5	2.12	**2.15 (1.41)**	0.64 (-70)	1.37 (-35)

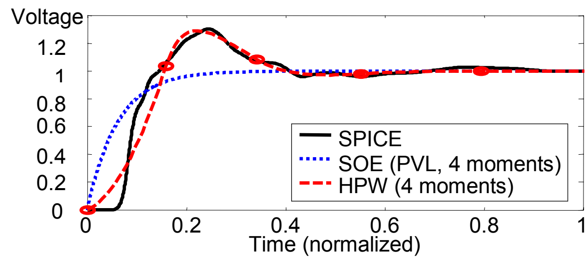

Figure 8. HPW vs. SOE, only 4 moments.

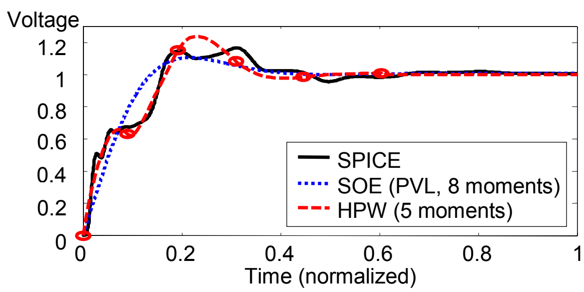

Figure 9. Another signal from an industrial *RLCK* circuit.

The HPW function performed very well compared to SOE for many different signals and circuits. The results for a sample of the signals are shown in Figure 9 and Figure 10. All the complex responses are captured very accurately using only four to six moments by the HPW function. The SOE function does not perform well with as many as eight moments.

All the results clearly indicate that the HPW function performs much better than the SOE function. Although all the details about the signal are not obtained by the HPW function, there is enough information in the approximate signal for accurate static timing analysis. Very complex *RLCK* responses are captured well by the HPW function with only four or five moments. To the authors' knowledge, there is no existing method that can extract as much information about *RLCK* circuits with only four or five moments.

5. CONCLUSIONS

A new family of piece-wise model order reduction techniques is presented in this paper to approximate *RC* and *RLCK* circuit responses accurately using moment matching. The proposed piece-wise functions PWL, PWQ, and HPW perform very well as compared to the sum of exponential representation using only four moments. Results show that delay and transition time errors are very small for many different circuits and signals. The test on the industrial netlist shows that delay can be captured accurately using the proposed technique with less than 5% error. The method is general enough to be extended for a variety of other piece-wise functions. For PWL, PWQ, and HPW, timing parameters such as 50% delay and transition time can be found in closed form using the resulting reduced order function. The method presented here fills the gap between direct metrics for *RC* circuit analysis and the expensive model order reduction techniques by introducing a stable and fast technique to perform *RLCK* circuit analysis with only a few moments. To the authors' knowledge, there is no existing method that can extract as much information about *RLCK* circuits with only four or five moments.

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