Multi-level Approach for Integrated Spiral Inductor Optimization

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ABSTRACT

The efficient optimization of integrated spiral inductors remains a fundamental barrier to the realization of effective analog and mixed-signal design automation. In this paper, we develop a scalable multi-level optimization methodology for spiral inductors that integrates the flexibility of constrained global optimization using Mesh-Adaptive Direct Search (MADS) algorithms with the rapid convergence of local nonlinear convex optimization techniques. Experimental results indicate that our methodology locates optimal spiral inductor geometries with significantly fewer function evaluations than current techniques.

Categories and Subject Descriptors

B.7.2 [**Integrated Circuits**]: Design Aids

General Terms

Design, Algorithms

Keywords

Spiral Inductor, Inductor Optimization, Analog Synthesis

1. INTRODUCTION

Increasing levels of integration and complexity in mixed-signal and system-on-chip (SoC) designs have spurred the need for innovative design automation techniques to improve reliability and time-to-market. Within the analog realm, accurate optimization and synthesis of spiral inductors continues to be a major roadblock on the path to design automation. Analog circuits such as Low Noise Amplifiers, Voltage Controlled Oscillators and bandpass filters depend on inductors with optimized inductance values and quality factors. Consequently, accurate design and optimization of spiral inductors is critical to a successful and cost-effective realization of analog systems in mixed-signal SoC.

Spiral inductor optimization continues to be primarily a manual process using either pre-characterized inductor designs or *ad hoc* optimization techniques. The most common programmatic approach for inductor optimization is exhaustive enumeration where each combination of design parameters over a discretized range of values is simulated, which is intractable for computationally-intensive spiral inductor models based on field solvers [5]. Other proposed optimization techniques based on geometric programming [2] or analytical formulas [6] require specific model formulations that can potentially limit their usefulness as modeling techniques are improved. Recently, Zhan and Sapatnekar developed a spiral inductor optimization technique based on Sequential Quadratic Programming (SQP) [11]. Their technique yielded two orders of magnitude speedup over exhaustive enumeration. However, substrate effects were neglected. Furthermore, since the objective functions and constraints were never proven to be convex, SQP may not approach a globally optimal solution.

In this paper, we develop a scalable multi-level optimization methodology for integrated spiral inductors that supports any generalized modeling technique. We first demonstrate that the spiral inductor optimization problem potentially employs non-convex objective and constraint functions. We then develop a multi-level optimization approach that first utilizes a Mesh-Adaptive Direct Search (MADS) algorithm to locate an approximate global solution in the non-convex design space [1]. We then refine the approximate solution obtained by MADS using gradient-based nonlinear constrained convex optimization [8]. Our results indicate that our methodology provides a reliable means for finding optimal spiral inductor designs with significantly fewer function evaluations than current techniques.

2. DESIGN SPACE CHARACTERIZATION

2.1 Integrated Spiral Inductor Modeling

In order to characterize the inductor's design space and test our proposed optimization methodology, we utilize accepted analytical spiral inductor modeling techniques based on a frequency dependent pi-model. Previous spiral inductor optimization studies have considered number of inductor turns (n) to be a discrete parameter with quarter turn multiples [5, 2, 11]. In contrast, we consider n to be a continuous variable in the optimization problem and therefore avoid resorting to mixed-integer optimization techniques that are typically less efficient than their continuous counterparts [8]. Furthermore, by not restricting n to discrete values, we have the flexibility to generate designs with higher quality factors.

To model the inductor's free-space resistance, we combine the formulation for proximity effect losses presented in [7] with a fitted formula for the effective thickness used in the spiral inductor resistance calculations, $t_{eff} = \delta_{eff}(1 - e^{-t/\delta_{eff}})(1 + 1.715t/w)$, where $\delta_{eff} = 1.195\delta$ and t, w and δ are the conductor's thickness, width and skin depth, respectively. The formula is valid for copper conductors with typ-

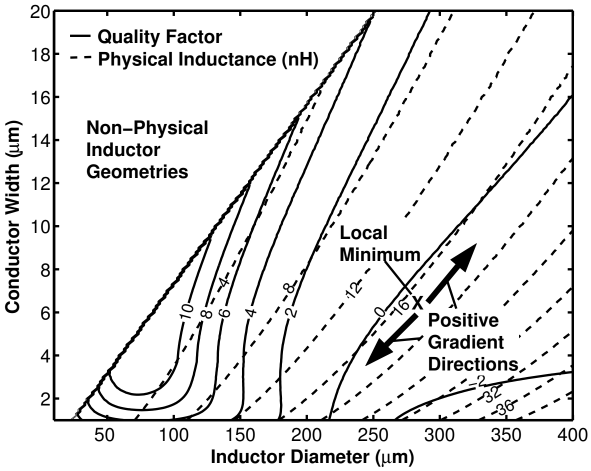

Figure 1: Non-convex quality factor and physical inductance functions

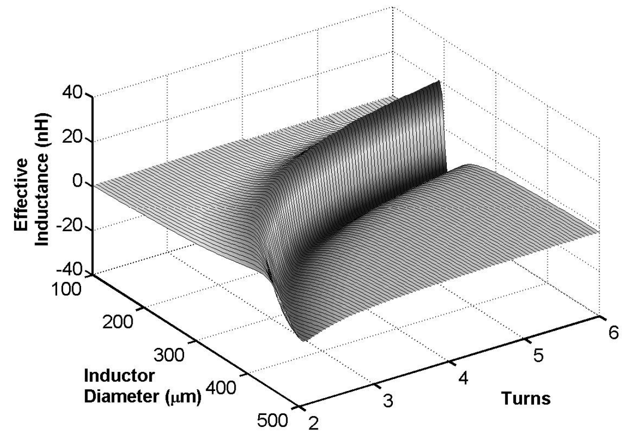

Figure 2: Non-convex effective inductance constraint function

ical dimensions. The model's series inductance is calculated using the Greenhouse method [3]. To capture the impact of single layer substrate eddy currents, which were not modeled in [6, 11], we utilize the method based on complex image theory presented in [4]. The model's series capacitance due to the inductor's conductors sidewall capacitance is calculated using the distributed capacitance model presented in [9]. We use the techniques presented in [10] to model the oxide capacitance, substrate capacitance and conductance elements in the pi-model of the spiral inductor.

2.2 Optimization Problem Formulation

A typical goal for spiral inductor optimization is to provide the inductor design with the highest quality factor for a given inductance value for common circuits such as VCOs, LNAs and bandpass filters. The optimization problem can be formalized using the following expressions:

$$\begin{aligned} \text{Maximize} \quad & Q(n,s,w,d) \\ \text{Subject to} \quad & L(n,s,w,d) \leq L_{wanted}(1+tol) \\ & L(n,s,w,d) \geq L_{wanted}(1-tol) \\ & \text{... other constraints ...} \\ & [n_{min}, s_{min}, w_{min}, d_{min}] \leq [n,s,w,d] \\ & [n_{max}, s_{max}, w_{max}, d_{max}] \geq [n,s,w,d] \end{aligned}$$

where L_{wanted} is the inductor's desired inductance value, tol is the tolerance on the allowed inductance values, and $Q(n,s,w,d)$ and $L(n,s,w,d)$ define the inductor's quality factor and inductance as a function of the spiral inductor's number of turns (n), conductor spacing (s), conductor width (w) and outer diameter (d). In this work we define the quality factor as $\text{Im}(Z_{in})/\text{Re}(Z_{in})$ where Z_{in} is the inductor's input impedance. In addition, other inequality constraints such as limits on the inductor's area and minimum self-resonant frequency (SRF) can be imposed to optimize inductor designs for specific applications.

2.3 Inductor Design Space Characterization

In order to develop efficient optimization techniques, the spiral inductor design space must be analyzed to determine what properties can be exploited. Many popular gradient-based nonlinear constrained optimization techniques such as SQP rely upon the convexity of the objective and constraint functions in order to find the optimal solution. A function is said to be convex if it satisfies the following property:

$$f\left[\frac{1}{2}\left(\overrightarrow{x_1}+\overrightarrow{x_2}\right)\right] \leq \frac{1}{2}\left[f(\overrightarrow{x_1})+f(\overrightarrow{x_2})\right], \quad \forall\ [\overrightarrow{x_1},\overrightarrow{x_2}]. \tag{1}$$

Therefore, a convex function always satisfies the property, $f''(\overrightarrow{x}) \geq 0$, which implies that a convex function has a single global minimum. Gradient-based nonlinear optimization techniques require the objective and constraint functions to be convex since they search in directions with negative gradients [8]. Consequently, if the function contains multiple local minima or maxima, gradient based optimization techniques will not always converge to the function's global minimum value or locate feasible solutions.

To explore the convexity of $Q(n,s,w,d)$, we examined the quality factor as a function of inductor diameter and conductor width for a typical inductor geometry. As displayed in Figure 1, $Q(n,s,w,d)$ has a local minimum value along a given physical inductance contour. Therefore, gradient-based optimization methods will converge to two different function values depending on the algorithm's initial start point. Standard gradient-based constrained optimization techniques also require the constraint functions to be convex. Figure 2 displays the inductor's effective inductance, $L_{eff} = \text{Im}(Z_{in})/\omega$, as a function of a spiral inductor's diameter and number of turns. L_{eff} clearly lacks convexity, which may potentially cause gradient-based constrained optimization techniques to breakdown when the optimization algorithm starts at certain locations in the design space. Consequently, gradient-based optimization techniques alone may fail to provide optimal solutions.

3. MULTI-LEVEL SPIRAL INDUCTOR OPTIMIZATION METHODOLOGY

3.1 Optimization Methodology

In order to optimize the spiral inductor's non-convex quality factor function with non-convex constraint functions, robust optimization strategies must be utilized. Multi-level

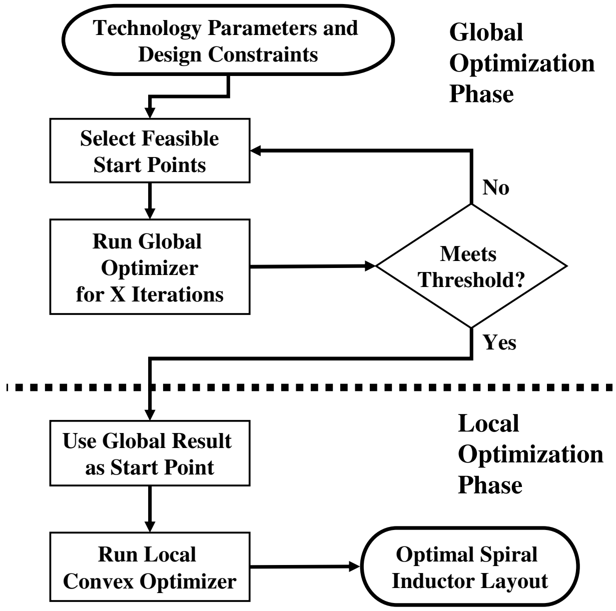

Figure 3: Multi-level inductor optimization flow

optimization strategies typically employ several different optimization techniques in tandem that compensate for the weaknesses of each technique individually. Multi-level optimization provides the speed and flexibility necessary to optimize the complex spiral inductor design space.

Our multi-level spiral inductor optimization methodology, which is depicted in Figure 3, consists of two distinct phases: the global optimization phase and the local optimization phase. Initially, objective and constraint functions are defined for the spiral inductor optimization problem based on application requirements. We utilize a Mesh-Adaptive Direct Search (MADS) algorithm to globally search the spiral inductor's design space. Other popular global optimization techniques such as Simulated Annealing and Genetic Algorithms typically employ penalty functions for constrained optimization problems that may require tens of thousands of function evaluations to converge [8]. In contrast, MADS is specifically designed for rapid convergence on nonlinear constrained global optimization problems [1].

Since MADS is a pattern search algorithm, the convergence rate depends on the optimizer's initial start point in the design space. In order to improve convergence, we deterministically sample the design space in order to identify possible start points for the global optimizer. We then run MADS for a certain number of function evaluations to approach a local minimum. If the optimizer finds a feasible solution with a quality factor that meets certain criteria, the output of the MADS optimizer is used as the start point for the local optimization phase. During the local optimization phase, we employ gradient-based non-linear constrained optimization to exploit the design space's local convexity in order to quickly converge to the optimal inductor geometry. Once optimization is complete, spiral inductor layout can be synthesized based on the optimization results. For patterned ground shield optimization, two successive runs of the algorithm for inductors both with and without the ground shield can be implemented with the second iteration

	L_{wanted} (nH)	F (GHz)	t (μm)	σ_{sub} ($1/(\Omega \cdot m)$)
Example 1	16	4	2	0.1
Example 2	4	1	1	0.1
Example 3	10	2.4	1	7000
Example 4	6	0.9	0.5	7000

Table 1: Design examples for optimization

using the result of the first as a start point in order to accelerate convergence. Our methodology exploits both the robustness of global optimization and the rapid convergence of the local optimization to locate optimal designs in the inductor's non-convex design space.

3.2 Results

In order to demonstrate our spiral inductor optimization methodology's improvement over current techniques, we applied several different methodologies to maximize the quality factors for the four typical design problems listed in Table 1 where L_{wanted}, F, t and σ_{sub} are the inductor' effective inductance, operating frequency, conductor thickness and substrate conductivity, respectively. The parameters have the following bounds: $2 \leq n \leq 10$, $1\ \mu m \leq w \leq 30\ \mu m$, $0.5\ \mu m \leq s \leq 3\ \mu m$ and $60\ \mu m \leq d \leq 800\ \mu m$. Each inductor is assumed to have copper conductors that are 6 μm above a 500 μm thick single layer substrate. The four design problems represent a broad range of typical design problems with varying levels of difficulty for the optimization routines. Design examples 1 and 2 stress the optimization of different inductance values while design problems 3 and 4 highlight the impact of substrate eddy currents on optimization.

The maximum quality factors and the number of function evaluations required for several optimization methods are listed in Table 2. For the enumeration methods, we iterate over each of the four inductor design parameters (n, w, s, d) for the total number of function evaluations listed in Table 2. For the "SQP - Random Start Point" method, we optimized the inductor using SQP with 200 random start points that correspond to feasible inductor geometries with positive quality factors. To determine the average number of function evaluations to obtain an optimal result using SQP, we divide the total number of function evaluations for the 200 SQP runs by the number of runs that produce a quality factor value within 5 percent of the inductor's optimal quality factor. Similarly, the quality factors listed in Table 3.2 for SQP are the average of the quality factors that are within 5 percent of the optimal value.

In order to demonstrate our optimization methodology's performance improvement over other potential multi-level approaches, we optimized spiral inductors using a multi-level combination of global enumeration with local SQP optimization. Furthermore, we applied MADS alone to the optimization problem. Finally, we optimized the inductors' quality factors with MADS coupled with SQP using our proposed methodology. We utilized the NOMADm optimization program to implement the MADS algorithm [1]. For our local optimization engine, we utilized a standard version of the SQP algorithm, one of the best nonlinear constrained convex optimization techniques available [8]. For both multi-level approaches, enumeration with SQP and MADS with SQP,

	Example 1		Example 2		Example 3		Example 4	
Optimization Method	Max. QF	Func. Eval.	Max. QF	Func. Eval.	Max. QF	Func. Eval.	Max. QF	Func. Eval.
Enumeration - Large	7.05	15323481	7.26	15323481	6.37	15323481	3.64	15323481
Enumeration - Medium	6.59	179315	7.26	179315	6.27	179315	3.64	179315
Enumeration - Small	6.59	13548	7.01	13548	6.01	13548	3.62	13548
SQP - Random SP	7.11	12207	7.06	691	6.25	4662	3.54	9120
Enumeration (5) + SQP	N/A	N/A	6.88	213	4.01	643	3.39	498
Enumeration (10) + SQP	5.81	866	7.13	1028	6.19	1382	3.55	1136
Enumeration (15) + SQP	6.40	3440	7.19	2710	5.73	2788	3.61	3225
Enumeration (20) + SQP	6.76	6359	7.19	9362	6.37	6268	3.62	8936
MADS	7.08	225	6.91	687	5.84	200	3.55	699
MADS + SQP	**7.16**	**478**	**7.10**	**495**	**6.39**	**380**	**3.55**	**473**

Table 2: Comparison of function evaluations required to obtain maximum quality factor

we only optimized with respect to three design variables (n, w, d) in the global optimization phase since the inductor's quality factor is less sensitive to conductor spacing.

In design example 1, enumeration required 15 million function evaluations to achieve optimal results, which is unacceptable even using analytical modeling techniques. SQP performed well on design example 2, where eddy current effects were minimal and a low inductance value was desired. On the other more difficult design problems that either experienced eddy current effects or required large inductance values, the SQP algorithm often either failed to find a feasible solution due to the non-convexity of the effective inductance constraint function or converged to a sub-optimal quality factor due to the non-convexity of the quality factor function. Multi-level optimization based on enumeration coupled with SQP achieved near optimal results when the enumeration technique discretized the design space into a 20x20x20 grid. However, this required on average over 6000 function evaluations. For lesser discretization during the enumeration step, the design problems either did not obtain a feasible value or achieved sub-optimal results for at least one of the design examples. Since the optimal quality factor values for a particular inductor design problem are typically not known *a priori*, 20x20x20 enumeration is the minimum discretization necessary to achieve consistent results. The inductors optimized with MADS alone only required several hundred function evaluations, but failed to reach an optimal value in design example 3.

In contrast, our methodology yielded near-optimal quality factors for each of the four design examples with an average of 457 function evaluations, resulting in up to a 40000x speedup over enumeration and up to a 25x speedup over SQP to locate optimal designs. Our multi-level approach provides a tractable spiral inductor optimization solution even when expensive field-solver based modeling techniques are employed. By coupling the global optimization capabilities of MADS with the local optimization strength of SQP, the proposed multi-level spiral inductor optimization methodology provides an efficient means to construct optimal spiral inductor designs.

4. CONCLUSION

Our multi-level optimization methodology couples the global optimization capabilities of MADS with the rapid local convergence of standard nonlinear convex optimization techniques to provide an efficient, model-independent spiral inductor optimization methodology. We have shown that the spiral inductors' quality factor and effective inductance functions can be non-convex, which may lead to the non-convergence of standard convex optimization techniques. Our methodology overcomes this difficulty and locates optimal spiral inductor designs with with significantly fewer function evaluations than current techniques.

5. REFERENCES

[1] C. Audet and J.E. Dennis. Mesh Adaptive Direct Search Algorithms for Constrained Optimization. *Les Journees de l'optimisation*, 2004.

[2] M. del Mar Hershenson, S. Mohan, S. Boyd, and T. Lee. Optimization of Inductor Circuits via Geometric Programming. In *Proceedings IEEE/ACM Design Automation Conference*, 1999 June.

[3] H. Greenhouse. Design of Planar Rectangular Microelectronic Inductors. *IEEE Trans. on Parts, Hybrids, and Packaging*, pages 101 – 109, June 1974.

[4] D. Melendy and A. Weisshaar. A New Scalable Model for Spiral Inductors on Lossy Silicon Substrate. In *2003 IEEE MTT-S International Microwave Symposium Digest*, pages 1007 – 1010, June 2003.

[5] A. Niknejad and R. Meyer. *Design, Simulation and Applications of Inductors and Transformers for Si RF ICs*. Kluwer Academic Publishers, 2000.

[6] J. Post. Optimizing the Design of Spiral Inductors on Silicon. *IEEE Trans. on Circuits and Systems II*, 47(1):15 – 17, Jan. 2000.

[7] J. Sieiro, J. Lopez-Villegas, J. Cabanillas, J. Osorio, and J. Samitier. A Physical Frequency-Dependent Compact Model for RF Integrated Inductors. *IEEE Trans. on Microwave Theory and Techniques*, pages 384 – 392, Jan. 2002.

[8] P. Venkataraman. *Applied Optimization with Matlab Programming*. Wiley, 2002.

[9] C. Wu, C. Tang, and S. Liu. Analysis of On-Chip Spiral Inductors Using the Distributed Capacitance Model. *IEEE Journal of Solid-State Circuits*, 38(6):1040 – 1044, June 2003.

[10] C. Yue and S. Wong. Physical Modeling of Spiral Inductors on Silicon. *IEEE Trans. on Electron Devices*, pages 560 – 568, March 2000.

[11] Y. Zhan and S. Sapatneker. Optimization of Integrated Spiral Inductors Using Sequential Quadratic Programming. In *Proceedings of the Design, Automation and Test in Europe Conference and Exhibition*, January 2004.