# Analyzing Statistical Timing Behavior of Coupled Interconnects Using Quadratic Delay Change Characteristics

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## Abstract

*With continuing scaling of CMOS process, process variations in the form of die-to-die and within-die variations become significant which cause timing uncertainty. This paper proposes a method of analytically analyzing statistical behavior of multiple coupled interconnects with an uncertain signal arrival time at each interconnect input (aggressors and the victim). The method utilizes delay change characteristics due to changes in relative arrival time between an aggressor and the victim. The results show that the proposed method is able to accurately predict delay variations through a coupled interconnect.*

## 1 Introduction

With continuing scaling of CMOS process, die-to-die and within-die variations have a significant impact on chip performance and power consumption [1]. Such variations come from process variations such as Le and Vt [2, 3, 4] as well as supply voltage and temperature variations. Process variations cause timing uncertainty. Current design methods for routs and wires use pessimistic approaches where designs are assumed at their worst-case corners. Typically, an initial design solution is simulated. Monitoring the critical nets, an incremental technique is used with a number of iterations until the design meets its specification [5, 6]. The worst-case scenarios in measuring coupling noise are also assumed, i.e. when the aggressor noise peaks matches the victim switching time in the same or opposite direction. Such an approach often leads to over-designing circuits causing unnecessary elevation of power and other reliability problems. Statistical design methods have been proposed in the past to model the impact of process variations. However, all the existing methods deal almost exclusively with modelling delay variations of logical gates [7] or physical variations of interconnect wires [8, 9]. This paper deals with a method of analytically analyzing statistical behavior of multiple coupled interconnects with an uncertain signal arrival time at each interconnect input. The goal is to determine the statistical behavior of signal transmission from one point to another point in a circuit under the influence of uncertainty multiple coupling sources.

The statistical behavior of such a delay can be obtained by Monte Carlo simulations of the circuit involved. However, Monte Carlo simulations are expensive, faster analytical methods with enough accuracy is needed to deal with complex VLSI designs. The method discussed in this paper achieves both goals of having an analytical-based faster method and high enough accuracy by calculating delay change characteristics with respect to relative signal arrival times between the aggressors and the victim. The statistical behavior of signal delay through the coupled interconnect can then be easily obtained through analytical methods rather than Monte Carlo simulations. The paper is organized as follows: Section 2 reviews the existing work on delay change characteristics under the influence of coupling. Section 3 presents a new set of noise response curves that are easier to handle and more accurate. Section 4 describes the proposed quadratic delay change curve. Experimental results are presented in Section 5. Finally, conclusions and discussions are given in Section 6.

## 2 Existing Delay Change Models

Sato, *et al.* [10] proposed an In-situ model that relates the delay change in a victim line to the relative arrival time of the its aggressor switching signal. The model assumes an exponential waveform at the victim output node when the victim line switches without noise as:

$$g(t) \;=\; V_{dd}(1 - e^{-\frac{t}{\tau_r}})\;\; t \geq 0 \qquad (1)$$

where $\tau_r$ is related to the rise time of the victim output signal without noise. When the aggressor switches with a relative time difference $k$ to the victim input, it produces a noise signal at the victim output node with the following

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waveform:

$$f(t,k) \;=\; \begin{cases} 0 & t<k \\ \frac{V_p}{t_a}(t-k) & k \le t < t_a + k \\ V_p\, e^{-\frac{t-t_a-k}{\tau_d}} & t \ge t_a + k \end{cases} \tag{2}$$

The delay change at the victim output is then the solution of:

$$g(t) + f(t,k) = \frac{V_{dd}}{2} \tag{3}$$

There is no closed form solution to the delay change curve of Equation (3). In [10] authors expanded Equation (3) to its first order Taylor series at $t_0 = \tau_r \ln 2$, which is the original victim delay without noise, and solved for $t$. The resulting In-situ delay change curve, $dcc$, as a function of the relative arrival time of aggressor, $k$, is then:

$$dcc(k) \;=\; \begin{cases} 0 & t_0 < k \\ \dfrac{1}{1+\dfrac{t_a}{2\rho\,\tau_r}}(k-t_0) & k_0 \le k < t_0 \\ \dfrac{1}{1-\dfrac{\tau_d}{2\rho\tau_r}\,e^{-\frac{k+t_a-t_0}{\tau_d}}} & k \le k_0 \end{cases} \tag{4}$$

where $\rho = \frac{V_p}{V_{dd}}$, and $k_0 \approx t_0 - t_a - \tau_r \ln(2\rho+1)$. The linear approximation of the noise signal near the peak value is found erroneous. The authors modified $dcc$ when $k = k_0$ as suggested in [11] to be:

$$\Delta t_{peak} \;=\; -\,\tau_r \ln(2\rho+1) \tag{5}$$

One of the drawbacks of the approach in [10] is that it does not handle multiple aggressors. The authors improved the efficiency of the above approach by using a more general analytical approach involving 2-pole RC models rather than SPICE simulations to obtain noise waveform [12]. As a result of that, multiple aggressors can be handled in the improved framework by lumping the coupling capacitance between an aggressor and the victim to the middle of the aggressor. For global interconnects, such an approximation may not be valid and the structural variations of the RC model can be wide ranging depending on numerous combinations between drivers, aggressor locations, and wire lengths. Therefore, we will rely on SPICE simulations to fit our noise waveforms to maintain accuracy.

## 3 Proposed Noise and Without-Noise Waveforms

Utilizing the superposition property of electronic circuits, one can decompose the output waveform of a given victim line derived by a number of aggressors into two sets: a without-noise waveform caused by the switching at the victim input, and a set of noise signals caused by the aggressors of the line when the victim line is dead. Let $f_i(t)$ be the noise signals, and $g(t)$ be the without-noise signal. We propose the following shape functions for $f$ and $g$:

$$g(t) \;=\; V_{dd}\, 2^{-\,e^{-\left(\frac{t-\nu}{\tau_r}\right)}} \tag{6}$$

$$f_i(t,k_i) \;=\; V_{p_i}\, e^{-\beta_i \ln^2\left(\frac{t-k_i}{\mu_i}\right)} \tag{7}$$

where $V_{p_i}$ is the peak voltage of the noise signal occurring at time $\mu_i$ when aggressor $i$ switches, $\beta_i$ is a shaping factor of the noise waveform describing its wideness. $\tau_r$ is the rise time of the victim output without-noise signal (defined to be the time between the 10% to the 90% of the amplitude value), and $\nu$ is the delay of the without-noise signal. In [10], authors suggested other waveform shapes for both the victim without-noise signal and the noise signal. In [12], the authors also suggested using exponential type waveforms in the future, but the results in [12] were still based on waveform shapes in [10]. Figures 1 and 2 show the amount of improvement our proposed waveform shapes have in terms of error compared to the one proposed in [10]. We also include SPICE results for comparison.

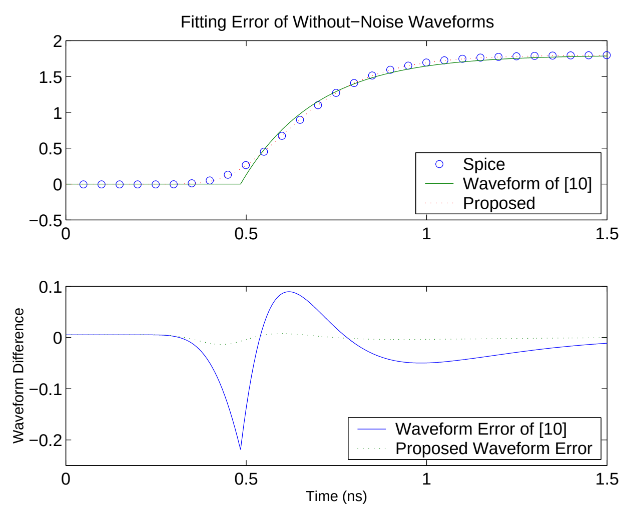


**Figure 1.** Without-Noise Signal Error, $g(t)$

Let the switching at the victim output node be our time-zero reference of all signals (i.e. when $g(t=\nu) = \frac{1}{2}V_{dd}$). Let $k_i$ be the difference of the input switching times between aggressor $i$ and the input switching of the victim line. Therefore, the time variable of Equation (7) is modified to $t \rightarrow (t-k_i)$. Employing the superposition property at the output node of the victim line, the output voltage signal, $O(t)$, becomes:

$$O(t) \;=\; g(t) \;+\; \sum_{i=1}^{n} f_i(t,k_i) \tag{8}$$

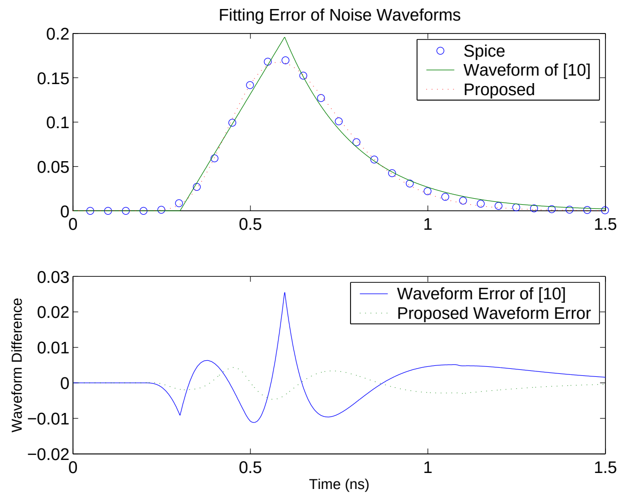


**Figure 2.** Noise Signal Error, $f(t)$

with a reference time-zero at $t = \nu$.

## 4 Quadratic Delay Change Curve (qDCC)

From Equation (8), one can solve:

$$O(t) = \frac{V_{dd}}{2} \tag{9}$$

for $t$ and get the delay change curve (due to aggressors noise) as a function of $k_i$, $dcc(\vec{k})$. However, Equation (9) has no closed form solution. In [10], authors expanded the equation to the first order Taylor series around $t = \nu$ using the simpler shape functions for $g(t)$ and $f(t)$ illustrated in Equations (1) and (2). The resulting In-situ $dcc$ equation has a linear part of $k$ and an exponential part, Equation (4), given one aggressor is affecting the victim line. At the peak of the $dcc$ function, the authors used the worst-case delay change suggested in [11] to overcome the error difference in $dcc$ due to bad approximation around the flat peak of the noise signal. Deriving statistical expectations out of In-situ $dcc$ function of [10] would be a great deal erroneous because of errors of the delay especially around the peak value. However, since the peak region of the noise signal, $f(t)$, curved like a parabola, we suggest to expand the proposed noise function of Equation (8) to its second order Taylor series around $t = \nu$ and solve the quadratic equation for $(t-\nu)$ and get a better $dcc$ function. The first and second derivatives of the proposed $f(t)$ and $g(t)$ at $t = \nu$ are:

$$g'(\nu) = \frac{V_{dd} \ln 2}{2\tau_r} \tag{10}$$

$$g''(\nu) = \frac{V_{dd}\, \ln 2\, (\ln 2 - 1)}{2\tau_r^2} \tag{11}$$

For the noise signals, $f_i(t, k_i)$, we have:

$$f_i'(\nu, k_i) = -2\, \beta_i\, B_i\, \frac{f_i(\nu, k_i)}{\nu - k_i} \tag{12}$$

$$f_i''(\nu, k_i) = \frac{\left(1 - B_i - 2\beta_i\, B_i^2\right)}{B_i} \frac{f_i'(\nu, k_i)}{\nu - k_i} \tag{13}$$

where $B_i = \ln\left(\frac{\nu - k_i}{\mu_i}\right)$. Therefore, the second order Taylor expansion of Equation (8) around $t = \nu$ can be expressed as:

$$O(t) \simeq G_0(\vec{k}) + G_1(\vec{k})\,(t-\nu) + \frac{G_2(\vec{k})}{2}\,(t-\nu)^2 \tag{14}$$

where,

$$G_0(\vec{k}) = g(\nu) + \sum_{i=1}^{n} f_i(\nu, k_i) \tag{15}$$

$$G_1(\vec{k}) = g'(\nu) + \sum_{i=1}^{n} f_i'(\nu, k_i) \tag{16}$$

$$G_2(\vec{k}) = g''(\nu) + \sum_{i=1}^{n} f_i''(\nu, k_i) \tag{17}$$

Solving $O(t) = \frac{V_{dd}}{2}$ for $(t-\nu)$ gives the proposed *quadratic delay change curve* $qdcc(\vec{k})$ as:

$$qdcc(\vec{k}) = -\frac{G_1 \pm \sqrt{G_1^2 - 2\,\left(G_0 - \frac{V_{dd}}{2}\right)\, G_2}}{G_2} \tag{18}$$

To compare the quadratic $qdcc$ waveform proposed in Equation (18) with the one suggested in [10], we ran a Monte Carlo spice simulation with 1000 random arrival times for the aggressor input with a fixed arrival time for the victim input and obtained the simulated $dcc$ points. Figure 3 shows the high accuracy of the proposed quadratic $dcc$ function over the existing one in terms of delay difference.

## 5 Experimental Results

In our experimental analysis, we examined a portion of a 0.18 $\mu m$ technology design containing a victim line coupled with 10 aggressors. An approximate layout of the coupled lines is shown in Figure 4.

We extracted the equivalent distributed RC components of the coupled interconnects. To account for non-linearity of drivers during switching, a piece-wise linear source approximating an exponential waveform derived from SPICE simulations was used. The goal of this study is to analytically derive the delay change statistics to the switching statistics of the design at multiple nodes and check the validation of the noise and the without-noise waveform generated by the coupled transmission lines alone.

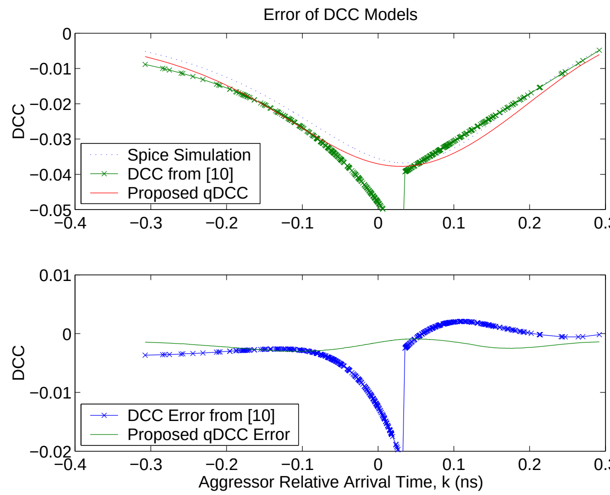


**Figure 3.** Delay Change Curve Error

**Figure 4.** Experimented Design Layout

## 5.1 DCC Errors

We show in this section how different the $dcc$ values of the proposed models from that of the Spice model. Since $dcc$ is a function of $\vec{k}$ (all the aggressors relative arrival times), we applied 1000 random $\vec{k}$ values from their normal distribution functions of zero means and $0.1\ ns$ standard deviation and sorted the $dcc$ values in ascending order. We compared the analytical $dcc$ values of the proposed model with the Spice runs. Results are illustrated in Figure 5. On average, the quadratic $dcc$ function has an average error of less than 5% from the Spice $dcc$ values, while applying the In-situ $dcc$ function of [10] yields an average error of 19%.

## 5.2 Standard Deviation Results

In this set of experiments, we checked the error in estimating the delay variation of the victim line given that the statistics of its aggressors relative arrival times, $\vec{k}$, are known, i.e. when $\mu_k$'s (mean arrival time difference) and $\sigma_k$'s (standard deviation of arrival time difference) are all known. We chose a relative arrival time difference, $\mu_k$'s, between 0ns to $0.20ns$. With each choice of $\mu_k$, we ran four different $\sigma_k$'s: $10ps$, $15ps$, $20ps$, and $30ps$. Having the statistics of the relative arrival times, we generated 1000 random data sets for each choice of $\mu_k$ and $\sigma_k$ and ran Monte Carlo simulations to determine $\sigma_{dcc}$. The 1000 generated $\vec{k}$ values are then fed to the proposed quadratic $dcc$

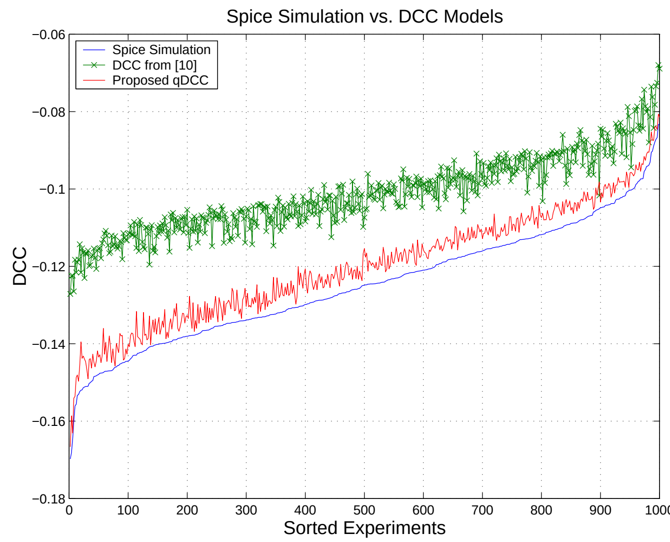


**Figure 5.** DCC Differences of Spice and Proposed Models

model and to the In-situ model to get the modelled $\sigma_{dcc}$'s. The simulated and computed $\sigma_{dcc}$ of all the runs are shown in Table 1 and illustrated Figure 6.

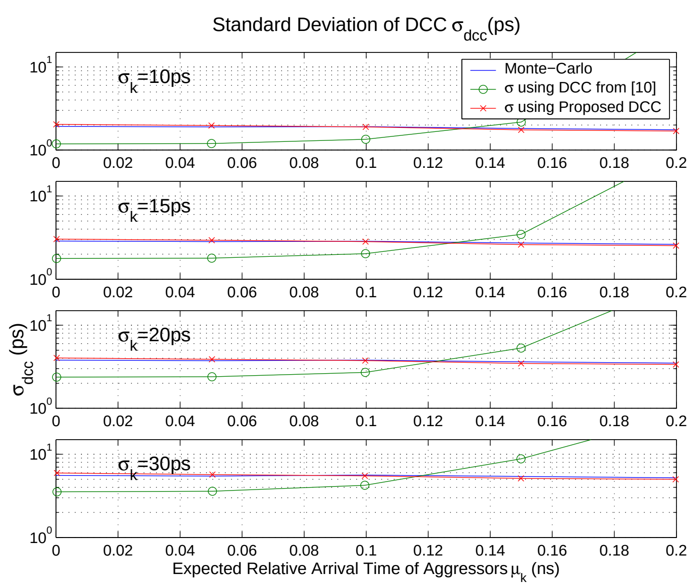


**Figure 6.** DCC Standard Deviation Errors

In general, the $\sigma_{dcc}$ values of the proposed model are much closer to the Monte Carlo results than that using the method in [10]. Table 2 shows that the average error in estimating the standard deviation in $dcc$ is about 4% across all the mean values of $\vec{k}$. Table 3 shows that the error in estimating $\sigma_{dcc}$ using the method in [10] exceeds 25% in

| Statistics | | Spice | In-situ | | Quadratic | |
|---|---|---|---|---|---|---|
| $\mu_k$(ns) | $\sigma_k$ (ps) | $\sigma_{dcc}$ | $\sigma_{dcc}$ | Er | $\sigma_{dcc}$ | Er |
| 0.00 | 10 | 1.9 | 1.2 | 38.5% | 2.0 | 5.7% |
| 0.00 | 15 | 2.9 | 1.8 | 38.2% | 3.0 | 5.8% |
| 0.00 | 20 | 3.8 | 2.4 | 37.7% | 4.0 | 6.1% |
| 0.00 | 30 | 5.6 | 3.5 | 36.7% | 6.0 | 6.8% |
| 0.05 | 10 | 1.9 | 1.2 | 37.0% | 2.0 | 3.8% |
| 0.05 | 15 | 2.8 | 1.8 | 36.5% | 2.9 | 4.0% |
| 0.05 | 20 | 3.7 | 2.4 | 35.9% | 3.9 | 4.1% |
| 0.05 | 30 | 5.5 | 3.6 | 34.5% | 5.7 | 4.2% |
| 0.10 | 10 | 1.9 | 1.3 | 29.4% | 1.9 | 0.7% |
| 0.10 | 15 | 2.9 | 2.0 | 29.1% | 2.8 | 1.0% |
| 0.10 | 20 | 3.8 | 2.7 | 28.8% | 3.7 | 1.3% |
| 0.10 | 30 | 5.6 | 4.2 | 24.6% | 5.5 | 2.0% |
| 0.15 | 10 | 1.8 | 2.2 | 19.3% | 1.7 | 4.0% |
| 0.15 | 15 | 2.7 | 3.5 | 27.2% | 2.6 | 4.2% |
| 0.15 | 20 | 3.6 | 5.3 | 46.5% | 3.5 | 4.5% |
| 0.15 | 30 | 5.4 | 8.8 | 62.8% | 5.1 | 5.3% |
| 0.20 | 10 | 1.8 | 29.1 | 1565% | 1.7 | 3.3% |
| 0.20 | 15 | 2.6 | 31.6 | 1106% | 2.5 | 3.5% |
| 0.20 | 20 | 3.5 | 31.2 | 792% | 3.4 | 3.7% |
| 0.20 | 30 | 5.2 | 28.5 | 447% | 5.0 | 4.2% |

**Table 1.** $\sigma_{dcc}$ for Different $k$ Statistics (in ps)

the best case.

| | $\sigma_k$ | | | | |
|---|---|---|---|---|---|
| $\mu_k$ | $10ps$ | $15ps$ | $20ps$ | $30ps$ | Average |
| 0.00 $\mu s$ | 5.7% | 5.8% | 6.1% | 6.8% | 6% |
| 0.05 $\mu s$ | 3.8% | 4.0% | 4.1% | 4.2% | 4% |
| 0.10 $\mu s$ | 0.7% | 1.0% | 1.3% | 2.0% | 1% |
| 0.15 $\mu s$ | 4.0% | 4.2% | 4.5% | 5.3% | 5% |
| 0.20 $\mu s$ | 3.3% | 3.5% | 3.7% | 4.2% | 4% |

**Table 2.** DCC Standard Deviation Errors of QDCC

In another set of experiments, we examined the effect of increasing the aggressors arrival times standard deviation on the estimation of the line delay statistics. Here, we fixed the nominal arrival times of the aggressors and set their standard deviations to certain values. Figure 7 shows that the error in estimating the delay standard deviation using our proposed model is almost the same as the result obtained by the expensive Spice simulations. On the average, the error in the estimation does not exceed 5% in the worst case, while using the In-situ model of [10] yields an error of more than 30% in the best case.

### 5.3 In-Phase and Out-Phase Switching

In this set of experiments, we randomly chose the arrival times of the aggressors with random standard deviations but

| | $\sigma_k$ | | | | |
|---|---|---|---|---|---|
| $\mu_k$ | $10ps$ | $15ps$ | $20ps$ | $30ps$ | Average |
| 0.00 $\mu s$ | 39% | 38% | 38% | 37% | 38% |
| 0.05 $\mu s$ | 37% | 37% | 36% | 35% | 36% |
| 0.10 $\mu s$ | 29% | 29% | 29% | 25% | 28% |
| 0.15 $\mu s$ | 19% | 27% | 47% | 63% | 39% |
| 0.20 $\mu s$ | 1564% | 1106% | 792% | 447% | 977% |

**Table 3.** DCC Standard Deviation Errors of In-situ

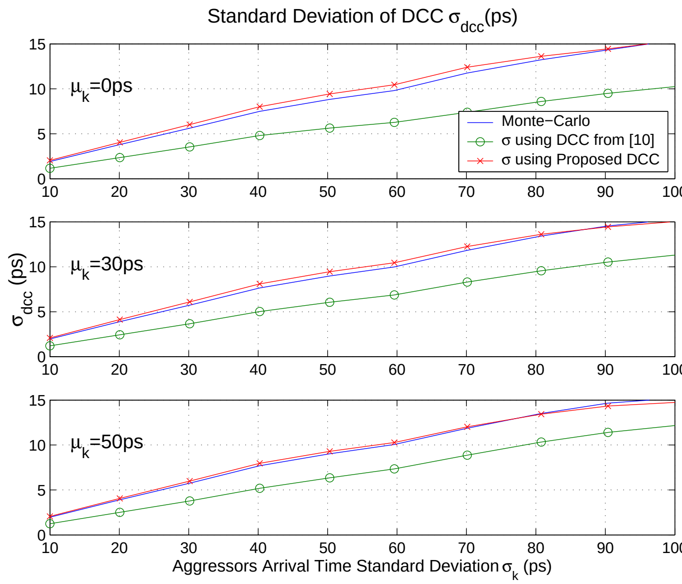


**Figure 7.** DCC Standard Deviation Errors

we forced some aggressors to switch in-phase with the victim and other to switch out-phase. Figure 8 shows two experiments: In the first case, we toggle the direction of the aggressors to be rising (u) and falling (d) as shown. Figure 8 shows the results of comparing the proposed model in estimating $\sigma_{dcc}$ to that using models in [10] and Monte Carlo results. On the average, the error of the proposed model is about 2%, while using the In-situ model yields more than 75% error comparing to Monte Carlo simulations. In the second case, we set the first five aggressors to switch in-phase with the victim, and the rest of the aggressors to switch one out-phase and one in-phase. The results show that the error using the proposed model is about 7%, while the error using the In-situ model is 118%.

## 6 Conclusion

In this paper, we presented a method of deriving statistical timing information for a given coupled interconnect due to the uncertainties of signal arrival times at its aggres-

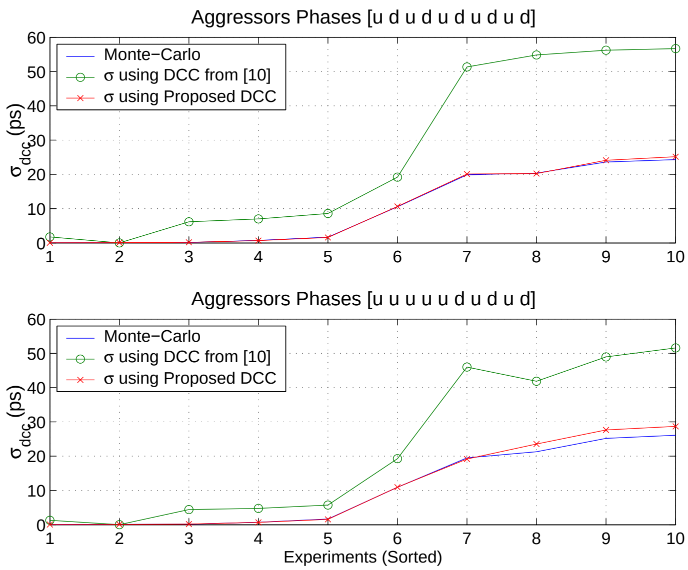


**Figure 8.** In-Phase and Out-Phase Errors

sors and the victim. The proposed approach utilizes delay change characteristics of the coupled interconnect to analytically obtain statistical timing without relying on circuit level Monte Carlo simulations. Deriving delay change characteristics of a given coupled interconnect involves fitting the proposed noise model through SPICE simulations. The errors of $\sigma_{dcc}$ for a given coupled interconnect is an order of magnitude lower than that using an existing delay change curve method in [10]. The higher accuracy of the proposed method is due to

- a better noise waveform shape function proposed in this paper for deriving delay change characteristics, and
- use of quadratic delay change characteristics rather than linear ones to more accurately model delay changes when relative arrival time between aggressors and the victim are close.

One limitation of the proposed method is that it requires SPICE simulations of a given coupled interconnect to fit its noise waveform shape. This can be a lot of simulations for a complex chip even though each simulation is very fast. One method to mitigate this requirement is to set a threshold for the amount of coupling required versus the driver strength to reduce the amount of simulations needed. Another method can be similar to that proposed in [12] to reduce the amount of SPICE simulations. However, the analytical method in [12] is only used for determining the peak noise voltage and the time in which it occurs. Furthermore, a wide range of models still need to be developed to cover arbitrary coupling cases as the authors in [12] pointed out. The extent of such a range of models is unknown for practical applications.

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