A Sum-over-Paths Impulse-Response Moment-Extraction Algorithm for IC-Interconnect Networks: Verification, Coupled *RC* Lines

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ABSTRACT
We have created a stochastic impulse-response (IR) moment-extraction algorithm for *RC* circuit networks. It employs a newly discovered Feynman Sum-over-Paths Postulate. *Full* parallelism has been preserved. Numerical verification results for coupled *RC* lines confirmed rapid convergence. We believe this algorithm may find useful application in massively coupled electrical systems, such as those encountered in high-end digital-IC interconnects.

Categories and Subject Descriptors
[**Verification, Modeling and Simulation**]: 3.1 Interconnect-parameter extraction and circuit-model generation. 3.2 Signal-integrity analysis. Power/Ground network analysis.

General Terms
Algorithms, Interconnect Performance.

Keywords
Feynman Sum over Paths, IC-Interconnect Modeling, Impulse-Response Moment Extraction, *RC* Circuit Networks, Stochastic Algorithms.

1. INTRODUCTION
Understanding and predicting the multi-GHz behavior of IC interconnects is critical to meeting objectives of the semiconductor design industry for the present decade. Essentially, we need solve Maxwell's equations. There are two ways in which this can be done: (*i*) one-step, direct solution of the underlying field equations; (*ii*) two-step, lumped-element parasitic extraction followed by solution of the resulting circuit equations. Both methods are viable; however, the second has a certain appeal, since it conveniently separates the physics of circuit-element extraction from the mathematics of circuit solution.

We will presume that two-step Maxwell solution has been selected above. It is the second step, efficient solution of complex *RC* interconnect-circuit equations, that we consider our primary motivating factor. We suggest, in fact, direct evaluation of IC-interconnect delay characteristics to support high-level timing and verification CAD tools encountered in the typical industrial design flow—thus bypassing time-consuming circuit simulation.

Our aim here is to create an impulse-response (IR) moment-extraction algorithm for arbitrary *RC* interconnect networks. We take a novel approach, inspired by Feynman's well-known path-integral method [2] in theoretical physics. This approach employs diagrammatic expansion and summation of a perturbation series for solution of the appropriate "field equations". It is hoped that we will benefit from the proven advantages of diagrammatic expansion: computational efficiency, mathematical organization, and intrinsic full parallelism.

We have mentioned lumped-element parasitic extraction in connection with "two-step" Maxwell solution of IC interconnects. This extraction process traditionally uses the method of partial-element equivalent circuits (PEEC), developed by Ruehli [3,9]. In this approach, one segments interconnect conductors into individual lumped-element resistors, capacitors, and self- and mutual inductors. Electrical-component values derive, conveniently, from quasi-static solution of the Maxwell equations. Unfortunately, for large numbers of interconnects, the PEEC method can produce a dense circuit matrix requiring prohibitive amounts of computer memory and execution time. To use PEEC in realistic, structurally complex problems, one should first sparsify the circuit equations, as an approximation, if at all possible. In addition, model-order reduction algorithms, such as asymptotic waveform evaluation (AWE) [6,7], can achieve improved circuit-solution efficiency with little loss of accuracy.

It is important to note, by the way, that AWE, and many other model-order-reduction schemes, derive an approximate transfer function by matching desired temporal IR moments to those of the original, complicated "full-order" circuit. In fact, IR-moment extraction in delay estimation for linear *RC* cir-

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cuits has origins with Elmore's [1] observation that the temporal unit-step response is, essentially, an integrated probability density. Elmore proposed to characterize electrical propagation delay, the so-called "Elmore Delay", with the first IR moment.

Later, Penfield *et al*. [5,8] devised a deterministic algorithm involving "tree walks" to extract realistic lower and upper delay bounds in *RC* tree networks. Others, like Lin and Mead [4], have further emphasized the usefulness of Elmore Delay in *RC* networks. They generalized its definition to include non-monotonic step responses. They, as well, proposed a delay-analysis methodology for arbitrary *RC*-networks, by decomposition into tree sub-networks. Most recently, Yu and Kuh [10] derived an Elmore Delay formula for coupled *RC* trees, along with a recursive algorithm for general IR-moment computation.

We will present, here, a novel stochastic algorithm for IR-moment extraction in *arbitrary RC* networks. The algorithm has a unique theoretical basis: a "Sum-over-Paths Postulate". Unlike previously developed recursive algorithms, ours is fully parallel, requiring little inter-processor communication, even in massively coupled networks.

In Section 2 we present the theory of our new IR moment-extraction algorithm for *RC* circuit networks—by example. Section 3 establishes our general algorithm for the first three IR moments, using an efficient stochastic evaluation technique. Section 4 gives numerical verification results for coupled *RC*-line circuit networks. We conclude with Section 5, summarizing our achievements and indicating future directions.

2. THEORY

Let us now consider the problem of solving linear *RC* circuit networks. The voltages throughout such a network satisfy a system of coupled ordinary differential equations. For simplicity, we confine ourselves to problems with a single impulsive input voltage source, for which we seek a single nodal voltage output. The associated transfer function $H(s)$ and temporal moments of impulse response $h(t)$ are

$$H(s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} m_n s^n, \qquad (1)$$

and

$$m_n = \int_{0^-}^{\infty} dt\, t^n h(t), \qquad (2)$$

respectively. The above equations establish the important, well-known mathematical connection between transfer-function expansion coefficients and IR moments.

We now propose a Sum-over-Paths Postulate. This Postulate will define our new IR moment-extraction algorithm; it will provide an efficient means of carrying out expansion (1) to find the IR moments m_n:

> *"The order-s^n term in an RC-circuit transfer-function expansion is a sum of weight-factor products over <u>all possible</u> unique paths within the associated network transition diagram. The product of weight-factors, constructed over each path, is similarly of order s^n."*

Note, transition-diagram weight factors, for order s^n, derive from order-s^n (Laplace-transformed) circuit-equation coupling coefficients between network voltage nodes. If, for example, a given voltage node "0" has an s-domain circuit equation $v_0 = (a_0 + a_1 s + a_2 s^2)v_1 + (b_0 + b_1 s + b_2 s^2)v_2$, which couples to nodes "1" and "2"; then, for example, the order-s^2 weight factor for a transition from node "0" to node "2" is simply b_2.

To further clarify matters, let us apply our Postulate to a three-stage *RC* line IR-moment extraction problem for m_2. Figure 1 is an order-s^2 transition diagram for a three-stage *RC* line. Shown, also, is the associated *full RC*-circuit network. Large black dots are voltage nodes. Arcs with direction arrows represent order s^0, s^1, and s^2 coupling coefficients between adjacent nodes. Note, also, here and hereafter, the quantity $\tau = RC$, where R and C are respective constant-component resistance and capacitance.

The leftmost node in Fig. 1, is the input source; the rightmost, the output. We define a so-called "path" as any series of virtual "jumps" following transition arcs that connect adjacent nodes. All paths, by definition, start at the output node, propagate to adjacent nodes along arrow directions, and terminate at the source node. Note, importantly, a path may create cyclical loops, where, for example, repeated traversals of the same transition arc may occur. Moreover, some paths may not differ by the number of repeated transition-arc traversals, but, instead, by the sequence in which these repetitions occur. Paths with a

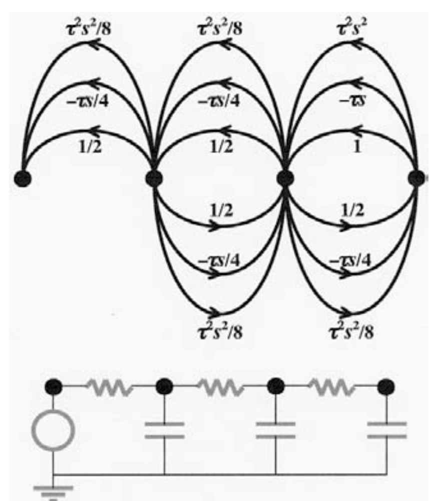

Figure 1. An order-s^2 transition diagram for a three-stage *RC* line, and the associated *RC*-circuit network. Here, and in all subsequent illustrations, black-dot circuit nodes correspond to transition-diagram nodes; the leftmost node is voltage input, the rightmost, voltage output. Arcs with direction arrows represent order s^0, s^1, and s^2 coupling coefficients between adjacent nodes.

different number or sequence of cyclical loops we all consider unique—they all need be included, in principle, as a possible path-sum contribution.

Our Postulate states that we need sum over all possible unique paths within the transition diagram of Fig. 1. In constructing this sum, we multiply all weight factors encountered across each arc in the path. We then sum this product of weight factors with other products so constructed, for each unique path. When establishing a path, for any desired order of s, we must be careful to include only weight factors that multiply to the desired order. For example, an order-s^2 path may include any number of s^0 arcs; in addition, two s^1 arcs, or, instead, one s^2 arc.

The diagram of Fig. 1 is order s^2. We could, in principle, use the Postulate, along with the diagram of Fig. 1, to evaluate IR moments m_0, m_1, and m_2. We simply need sum weight-function products over paths of corresponding order s^0, s^1, and s^2; consistent with Eq. (1).

Now, refer to Fig. 2. Taken together, the black, dashed, and dotted arcs compose a portion of the original second-order transition diagram of Fig. 1. They arise only from the dark-gray resistors and capacitors. Light-gray solid arcs, and corresponding light-gray capacitors, are only shown for reference to Fig. 1 here (and in all subsequent similar diagrams hereafter). *Two additional independent diagrams also exist, involving distinct pairs of dark gray capacitors and associated diagrams (though not shown here).* Black, dashed, and dotted s and s^2 arcs, emanating from any particular transition-diagram node, arise from a single dark-gray capacitor connected to the associated electrical-circuit node.

Figure 2 also shows some order-s^2 transition-diagram paths for m_2—identical to some of those in the full-circuit diagram of Fig. 1. Only paths with any combination of black, dashed, and dotted arcs that traverse, in any given path—either *two* order-s^1 weight-factor arcs or *one* order-s^2 weight-factor arc—contribute to the sum for m_2. Note, incidentally, the two paths, made either solely of dashed arcs or solely of dotted arcs, are, in fact, the simplest possible ones in the sum for m_2. Of course, as noted before, more complicated, possibly cyclical, paths involving combinations of black, dashed, and dotted arcs exist in the sum, as well.

Figure 2 shows, in addition, a corresponding *reduced circuit* for any combination of black, dashed, and dotted arcs. The reduced circuit contains only dark-gray resistors, with two dark-gray capacitors. In summary, the black, dashed, and dotted arcs in the Fig. 2 diagram are identical to those of the corresponding reduced-circuit transition diagram, to order s^2.

Observe, importantly, we could attempt to conveniently sum *all* the second-order paths for m_2 in Fig. 1 analytically, simply by adding weight-factor products for m_2 paths over the reduced-circuit transition diagram of Fig. 2. We need add also path contributions from two *other* independent, related reduced circuits. Each such related, reduced circuit has simply a different pair of dark-gray capacitors (see italics, in the Fig. 2 introductory paragraph). In other words, one might guess that the full-circuit transition diagram of Fig. 1, and the collection of Fig. 2 and two other related reduced-circuit diagrams, yield the same result, to order s^2, when summing over paths for m_2. Unfortunately, this approach incorrectly double counts certain s^2-path contributions.

Figures 3(a) and 3(b) illustrate double-counted path contributions when using the procedure suggested above. The order-s^2 paths for m_2 here we call "self interactions", since they involve traversals, in any given path, of either one order-s^2 weight-factor arc; or of two order-s^1 weight-factor arcs, each emanating from the *same* node (by cyclical path loops). All self-interactions are associated with a single dark-gray-capacitor circuit node.

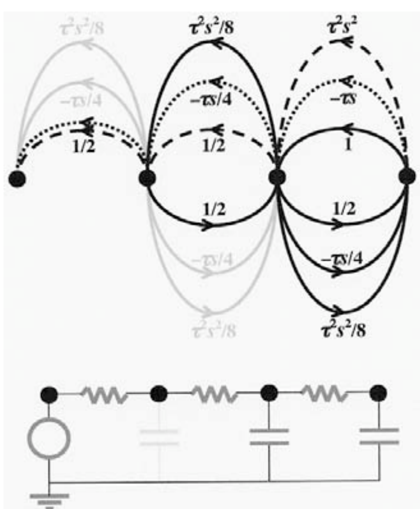

Figure 2. One of three independent order-s^2 transition diagrams for a three-stage *RC* line. The corresponding reduced circuit, with dark-gray *RC* components is shown as well. The other two diagrams arise from networks with either left-and-center, or left-and-right, dark-gray capacitors.

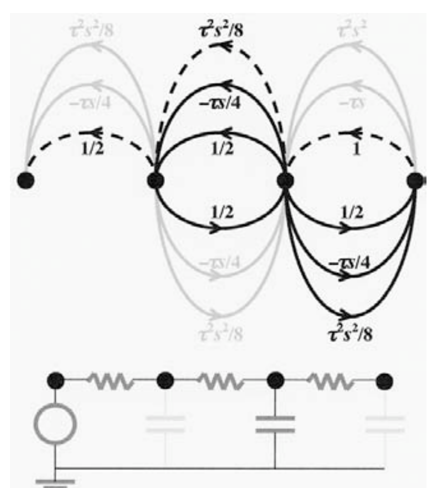

Figure 3(a). One of three order-s^2 "self-interaction" transition diagrams for a three-stage *RC* line. Each involves a single dark-gray capacitor. The other two diagrams arise from networks with either a single left, or right, capacitor.

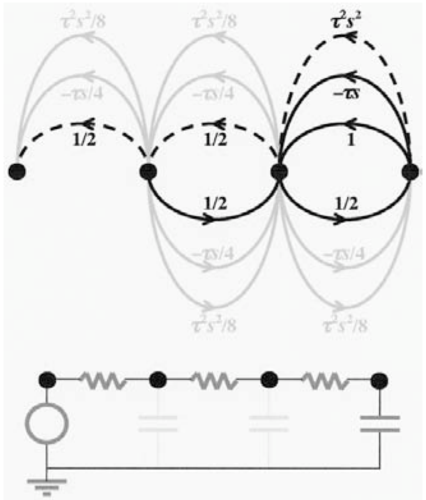

Figure 3(b). Another of three order-s^2 self-interaction transition diagrams for a three-stage *RC*-line.

Observe, importantly, one can construct all the self-interactions for the Fig. 2 reduced circuit as a sum of self-interactions for the *pair* of Fig. 3(a) and Fig. 3(b) reduced circuits. The pair, taken together, has identically placed dark-gray capacitors *vis-a-vis* the Fig. 2 reduced circuit.

Observe, importantly, a *portion* of the transition diagram of Fig. 1, and the *entire* diagram of Fig. 2 are equivalent, to order s^2, in summing over paths for m_2—*as long as one corrects for double counting of self-interaction paths in Fig. 2*. This requirement is fundamentally based on our Sum-over-Paths Postulate. We correct by subtracting, *once*, the sum of Fig. 3(a) and 3(b) order-s^2 self-interactions from *all* the "interactions" (self- and otherwise) in Fig. 2. This properly generates the algorithm we desire for a *portion* of m_2 associated with the circuit of Fig. 1.

The remaining portion of m_2 for Fig. 1 can be found similarly, starting with each of the two other dark-gray capacitor reduced diagrams (like Fig. 2, except a different capacitor pair), and subtracting appropriate self-interactions using the diagrams of Fig. 3(a), 3(b), or a third similar one (like Figs. 3, except containing a sole leftmost dark-gray capacitor).

It is relatively easy to evaluate reduced-circuit IR moments analytically, since they contain only one or two capacitors. Carrying out the m_2 evaluation of Fig. 2, and adding the remaining m_2 of the other two independent, related two-capacitor reduced circuits; and, appropriately, subtracting self-interactions using m_2 of Fig. 3(a), Fig. 3(b), or, the other independent one-capacitor reduced circuit; we find

$$m_2 = 62\square^2. \quad (3)$$

A check of (3) by analytically expanding $H(s)$ about $s = 0$ for the full circuit of Fig. 1 confirms our result.

3. MOMENT-EXTRACTION ALGORITHM

The algorithm we describe is, in fact, completely general. Extension to arbitrary *RC* electrical networks is straightforward. In realistic applications, such as analysis of complex IC interconnects, one must sum over a relatively large number of paths. Unfortunately, this procedure can require an excessive amount of computation. To facilitate path summing in large *RC* networks, we suggest, therefore, *stochastic* evaluation of the path sums. In this way, we preserve full parallelism, and allow efficient IR-moment extraction by means of statistical sampling.

What follows are extraction algorithms for the first three IR moments. The first two originate from simpler arguments than the one for m_2 presented in this work:

—Zeroth IR Moment m_0—

Step 1: *Remove all capacitors from the full circuit, establishing a reduced circuit.*

Step 2: *Evaluate m_0 for the reduced circuit and equate to that of the full circuit.*

—First IR Moment m_1—

Step 1: *Set a running sum to zero.*

Step 2: *With an equal probability, select any particular capacitor in the full circuit.*

Step 3: *Remove all other capacitors from the full circuit, establishing a reduced circuit.*

Step 4: *Evaluate m_1 for the reduced circuit and add to the running sum.*

Step 1: *Repeat Steps 2–4 a given number of times (samples), based on desired statistical accuracy.*

Step 5: *Divide the running sum by the number of times Steps 2–4 have been executed, evaluating m_1 for the full circuit.*

—Second IR Moment m_2—

Step 2: *Set a running sum to zero.*

Step 3: *With an equal probability, select any capacitor in the full circuit.*

Step 4: *Repeat Step 2.*

Step 5: *Remove all other capacitors, not selected in Step 2–3, from the full circuit, establishing a reduced circuit.*

Step 6: *If the capacitors selected in Steps 2–3 are the same ones, evaluate m_2 for the reduced circuit, double the result, and add to the running sum.*

Step 7: *If the capacitors selected in Steps 2–3 are different ones, construct two additional single-capacitor reduced circuits: one for each capacitor in Steps 2–3. Evaluate m_2 for the reduced circuit of Step 4 and subtract m_2 for each of the additional single-capacitor reduced circuits. Add the resulting m_2 to the running sum.*

Step 8: *Repeat Steps 2–6 a given number of times (samples), based on desired statistical accuracy.*

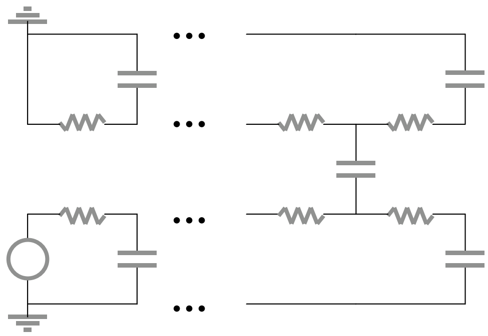

Figure 4. A coupled, multi-stage *RC* line.

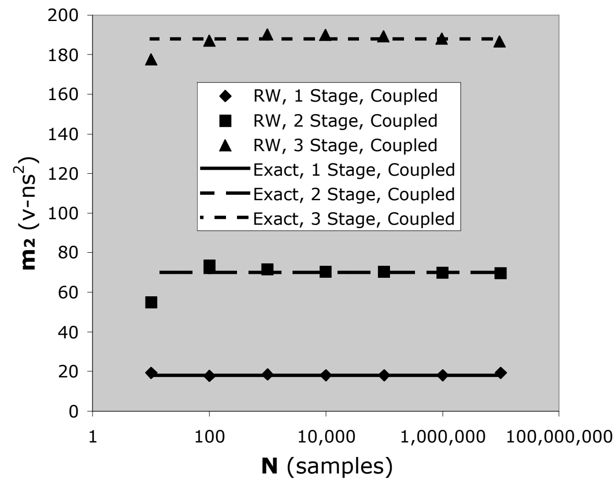

Figure 5(b). Second-moment IR-extraction results versus number of statistical samples *N* for the coupled, multi- stage lines of Fig. 4. Continuous horizontal lines are exact, analytical values; specific data points are numerical values based on our stochastic sum-over-paths algorithm.

4. COMPUTATIONAL RESULTS

To verify our proposed stochastic sum-over-paths algorithm, we present computational results for coupled *RC* lines. All calculations were performed with an IBM T20 PC Laptop™, which uses a 700MHz Pentium III™ microprocessor. Our compiler of choice for numerical programming was Metrowerks Codewarrior C/C++ Professional™, Release 5. We selected compiler settings to achieve a nominal floating-point multiplication rate execution rate of approximately 66.66M/s. All circuits in our study have common respective resistor and capacitor component values, with $\tau = RC$ = 1ns.

Figure 4 depicts a multi-stage coupled *RC* line. We define a *p*-stage line as a succession of p–1 *RC* stages. The last *p*th termination stage is a split-resistor coupling-capacitor network, as indicated in the figure.

Figures 5(a) and 5(b) show coupled-line results for m_1 and m_2. Respective m_1 and m_2 execution times for 10M samples, the rightmost data points, were on the order of 4s and 9s for all circuits. Excellent agreement with analytical values was achieved in the high sample-number limit. We, in fact, estimate a 1-σ statistical error of order 5% for m_1 and 15% for m_2, even after a relatively few 100 samples.

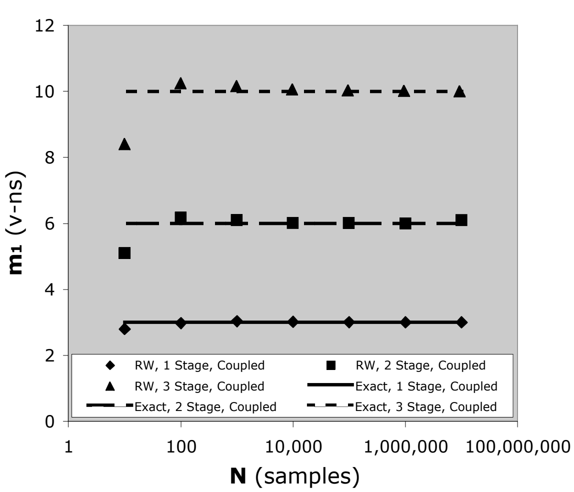

Figure 5(a). First-moment IR-extraction results versus number of statistical samples *N* for the coupled, multi-stage lines of Fig. 4. Continuous horizontal lines are exact, analytical values; specific data points are numerical values based on our stochastic sum-over-paths algorithm.

5. CONCLUSION

In summary, we have created a new IR moment-extraction algorithm for *RC* circuit networks. Our approach employs a Feynman Sum-over-Paths Postulate, diagrammatic expansion of the circuit transfer function, and stochastic evaluation of the path sum. In developing the algorithm, we have maintained computational efficiency and full parallelism. Initial verification studies of coupled *RC* lines furnished promising results: rapid initial convergence and excellent agreement with exact, analytical moment values.

Though the initial verification study presented in this work was confined to *RC* lines, our stated Sum-over-Paths Postulate implies generality for arbitrary *RC*-interconnect networks. The algorithm, of note, is well suited to future application in massively coupled systems, like those encountered in high-end digital-IC interconnects.

6. ACKNOWLEDGMENTS

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