AMPLIFICATION OF ULTRAWIDEBAND SIGNALS

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ABSTRACT

One of the main implementation challenges in the ultrawide-band (UWB) radio is the design of efficient amplifiers. The diffi-culty in amplifying an UWB signal stems from its bandwidth being a large fraction of the amplifier gain-bandwidth product. This paper describes a methodology and the tradeoffs associated with the design of UWB amplifiers. The amplifiers are designed to mini-mize a new performance metric, which we refer to as the effective noise figure (NF). The effective NF measures the degradation caused by the amplifier in the achievable receiver performance after the digital decoding process, which is ultimately the most relevant measure of performance.

1. INTRODUCTION

The ultrawideband (UWB) radio is a relatively new technol-ogy that is being pursued for both commercial and military pur-poses [1][2]. The rationale for deploying the UWB radio lies in the benefits of exceptionally wide bandwidths, thereby achieving a combination of very fine time/range resolution, high data rates, robustness to narrowband interferers, and ability to resolve multi-path components [3].

One of the main implementation challenges in the UWB radio is the design of efficient UWB amplifiers. The difficulty arises from the finite gain-bandwidth product of most existing amplifiers. Since the UWB signal bandwidth is a large fraction of the gain-bandwidth product, achieving a reasonable gain with enough bandwidth to pass the wideband received signal largely undistorted is difficult.

The performance of an amplifier is generally quantified using the noise factor (or noise figure in dB), which is defined as the ratio of the signal-to-noise ratio (SNR) at the input of the amplifier to the SNR at the output of the amplifier. Although the use of the noise figure (NF) metric is straightforward in narrowband systems, its use becomes more difficult in UWB systems. The main difficulty arises in defining the SNR. In a narrowband system, where both the input signal and noise are assumed to be a single tone at the carrier fre-quency, the SNR is obtained by simply dividing the signal power by the noise power. In an UWB system, however, the input signal is broadband and the additive noise may be colored. The SNR obtained by simply dividing the signal power by the total noise power (whose bandwidth must also be defined) is less meaningful,

since a higher SNR value defined in this manner does not necessar-ily translate to a higher receiver performance. This is because the performance of the receiver after the digital decoding process does not depend on the total signal and noise power but on the power spectrum density (PSD) of the additive noise and the impulse responses of the propagation channel and the transmit pulse.

Because of the difficulty in defining the SNR, existing work on broadband amplifier defines the NF as the weighted average of the single-tone NF [4]. Although such definition of NF is an exten-sion of a single-tone NF, minimizing such arbitrary performance metric does not necessarily improve the overall receiver perfor-mance.

For the NF of the amplifier to be a meaningful metric in an UWB receiver, the SNR at the input and output of the amplifier should measure the achievable performance after the eventual digi-tal decoding process, as it is ultimately the most relevant measure of performance. Hence, we define the SNR as the matched filter bound (MFB) [5], which represents an upper limit on the perfor-mance of data transmission systems. The MFB is obtained when a noise whitened matched filter is employed to receive a single trans-mitted pulse. By defining the SNR as the MFB, the NF measures the degree of degradation in the achievable receiver performance caused by the amplifier. We subsequently refer to this NF as the effective NF.

This paper describes a methodology and the tradeoffs associ-ated with the design of UWB amplifiers with a finite gain-band-width product. The amplifiers are designed by minimizing the effective NF, so that the performance after the digital signal pro-cessing is maximized. For ease of explanation, we assume a base-band UWB signal with bandwidth B, although similar analysis and conclusions can be made for modulated UWB signals.

The organization of this paper is as follows. Section 2 intro-duces the effective noise figure. The optimization of a single-stage amplifier is described in Section 3, and the cascaded stages in Sec-tion 4. Conclusions are drawn in Section 5.

2. THE EFFECTIVE NOISE FIGURE

A general system model of a communication channel includ-ing the amplifier is shown in Fig. 1(a). The kth transmit symbol x_k is filtered by the equivalent pulse response then corrupted by the additive noise $n_i(t)$. The equivalent pulse response (whose fre-quency response is $P(f)$) represents the combination of both the transmit pulse and the propagation channel. The resulting corrupted signal is the input to the amplifier, which has a transfer function given by ($G^{1/2}(f)$) and internally additive noise $n_g(t)$

Amplifier

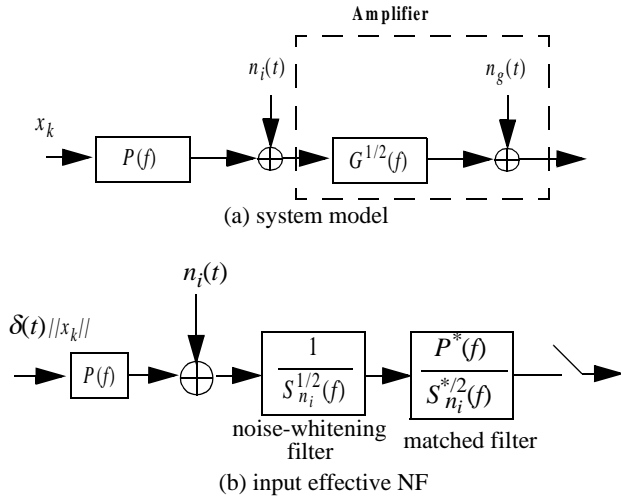(a) system model

(b) input effective NF

Fig. 1. General communication channel model.

The MFB, also called the "one-shot" bound, is an upper limit on the performance of data transmission systems with intersymbol interference (ISI). As an example, the computation of the MFB at the input of the amplifier is illustrated in Fig. 1(b). An impulse is transmitted through the equivalent pulse response, which is then corrupted by $n_i(t)$. The input to the receiving system is noise whitened followed by a matched filter that is matched to both the pulse response and the noise whitening filter. $S_{n_i}(f)$ represents the PSD of $n_i(t)$. The matched filter output is then sampled when the output signal is at its maximum. The resulting SNR is the MFB.

The MFB at the input and output of the receiving system is [5]

$$SNR_{in} = \int \frac{|P(f)|^2}{S_{n_i}(f)} df \tag{1}$$

$$SNR_{out} = \int \frac{|P(f)|^2 G(f)}{S_{n_i}(f)G(f) + S_{n_g}(f)} df \tag{2}$$

Assuming, as is commonly done, that the input noise $n_i(t)$ is white with a PSD of -174dBm/Hz, the effective NF of the receiving system can be written as a function of the spot NF by dividing (2) from (1) then rearranging, i.e.,

$$F_{eff} = \frac{1}{\int \left(\frac{|P(f)|^2}{P_T} \right) \frac{1}{F_s(f)} df} \tag{3}$$

where $P_T = \int |P(f)|^2 df$ and $F_s(f)$ denotes the spot NF as given by

$$F_s(f) = \frac{S_{n_i}(f)G(f) + S_{n_g}(f)}{S_{n_i}(f)G(f)} . \tag{4}$$

For a cascade of the multiple-stage receiving systems, $F_s(f)$ can be determined by the well-known Friis formula [6] i.e.,

$$F_s(f) = F_{s1}(f) + \frac{F_{s2}(f) - 1}{G_1(f)} + \dots + \frac{F_{sN}(f) - 1}{G_1(f)\dots G_{N-1}(f)} \tag{5}$$

where $F_{si}(f)$ and $G_i(f)$ denote the spot NF and gain of the ith cascaded receiving system.

To simplify the effective NF measurement, we assume that $P(f)$ is constant over the frequency band of interest. This is a reasonable assumption since the uncertainty in the propagation channel response makes the pulse response $P(f)$ generally unknown at design time. The effective NF given in (3) can then be approximated as

$$F_{eff} \approx \frac{N}{\displaystyle\sum_{i=0}^{N-1} \frac{1}{F_s(f_i)}} \tag{6}$$

where $\{f_0, f_1, \dots, f_{N-1}\}$ represent the center frequencies for each of the spot NF measurements in the frequency band of interest and N is the total number of measured values. The effective NF equation in (6) is analogous to computing the normalized effective resistance of N parallel resistors each with resistance $N \cdot F_s(f_i)$.

3. SINGLE-STAGE AMPLIFIER

An amplifier can be accurately modeled using the general model of the receiving system in Fig. 1(a). The following assumptions are made: $P(f)$ is an ideal brickwall filter with bandwidth B, which represents the bandwidth of the UWB signal; the input noise $n_i(t)$ and the internally generated noise $n_g(t)$ are both white with PSD N_i and N_g, respectively; and the amplifier power gain is

$$G(f) = \frac{K^2}{f^2 + f_{3dB}^2} \tag{7}$$

where K denotes the gain-bandwidth product (GBP) and f_{3dB} is the 3dB bandwidth of the amplifier.

Substituting (7) into (4), the spot NF can be determined after straightforward algebraic manipulations as

$$F_s(f) = 1 + \frac{N_g}{K^2 N_i}[f^2 + f_{3dB}^2] \tag{8}$$

As evident in (8), the spot NF is minimized at all freqencies by making f_{3dB} as small as possible. This occurs because the power gain given in (7) increases as f_{3dB} is decreased. The larger power gain suppresses the effects of the internally generated noise $n_g(t)$, which in turn results in smaller spot NF values.

The effective NF is readily determined by substituting (8) into (3) and integrating:

$$F_{eff} = \frac{\sqrt{F_s(0)\frac{N_g}{N_i}\frac{B^2}{K^2}}}{\text{atan}\left\{ \sqrt{\frac{1}{F_s(0)}\frac{N_g}{N_i}\frac{B^2}{K^2}} \right\}} \tag{9}$$

$$\equiv \frac{\sqrt{\gamma + \gamma^2\left(\frac{f_{3dB}}{B}\right)^2}}{\mathrm{atan}\left\{\sqrt{\dfrac{\gamma}{1 + \gamma\left(\frac{f_{3dB}}{B}\right)^2}}\right\}} \qquad (10)$$

where $F_s(0)$ is the spot NF in (8) when $f = 0$, and γ is defined as $N_g B^2 / (N_i K^2)$. Note that F_{eff} is fully characterized by f_{3dB}/B and γ. In Fig. 2, the effective NF (in dB) is determined as a function of f_{3dB}/B for different γ values. The effective NF improves as f_{3dB}/B is reduced, suggesting that the best performance is achieved by making the amplifier as narrowband as possible. This result is expected since, as described above, the spot NF is minimized at all frequencies by reducing f_{3dB}. However, the drawback of employing a small f_{3dB} is that the signal bandwidth is also greatly reduced by the amplifier, which in turn diminishes some of the benefits of the UWB radio, such as the ability to resolve multipaths. As a compromise between these conflicting objectives, f_{3dB} corresponding to an effective NF that is slightly greater (e.g., 1dB) than the minimum effective NF can be chosen for a given γ value.
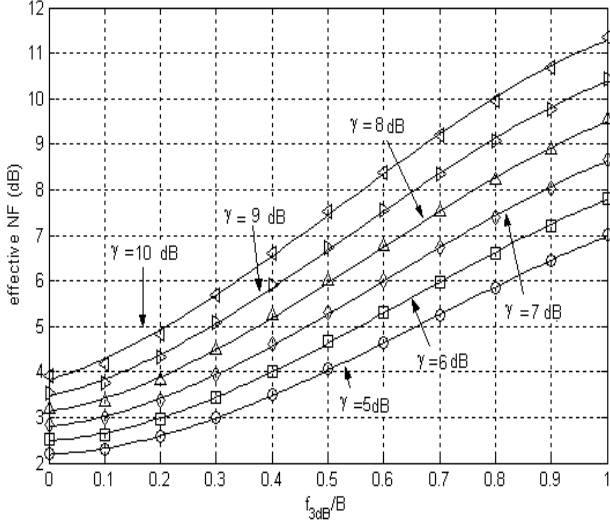

Fig. 2. The effective NF relating to f_{3dB}/B and γ for the single amplifier sytem.

4. CASCADED AMPLIFIER STAGES

A system model of M cascaded amplifier stages is shown in Fig. 3. $G_l(f)$ denotes the power gain of the l-th amplifier stage, where $l \in \{1, 2, \ldots, M\}$, and $n_{gl}(t)$ is the internally generated noise of the lth amplifier stage, whose PSD is assumed white and denoted as N_{gl}. The total power gain is

$$G(f) = \prod_{l=1}^{M} \frac{K_l^2}{f^2 + f_l^2}, \qquad (11)$$

where K_l and f_l denote respectively the GBP and the 3dB bandwidth of the lth amplifier.
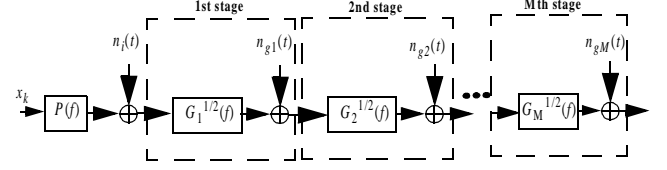

Fig. 3. M-stage amplifier system model.

By definition, the effective f_{3dB} of the M-stage amplifier can be obtain by solving the following implicit equation

$$\prod_{l=1}^{M} [f_{3dB}^2 + f_l^2] = 2 \prod_{i=l}^{M} f_l^2 . \qquad (12)$$

Substituting (11) into (5), the spot NF of the M-stage amplifier becomes

$$F_s(f) = 1 + \gamma_1[f^2 + f_1^2] + \gamma_2[f^2 + f_1^2][f^2 + f_2^2] + \ldots , \qquad (13)$$

$$+ \gamma_M \prod_{l=1}^{M} [f^2 + f_l^2]$$

where

$$\gamma_l \equiv \frac{N_{gl}}{N_i} \prod_{j=1}^{l} \frac{B^2}{K_j^2} , \quad l \in \{1, 2, \ldots, M\} \qquad (14)$$
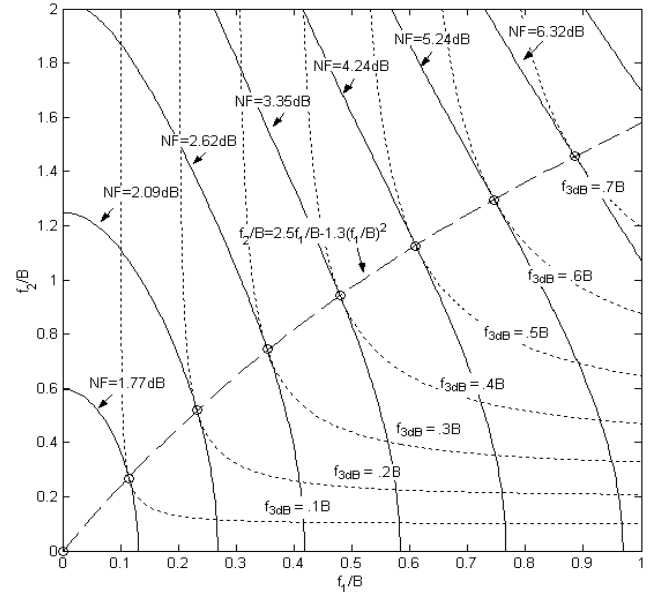

Fig. 4. Contours of effective NF (solid line) and f_{3dB} (dash line) as a function of f_1/B and f_2/B for a 2-stage amplifier system with $N_{g1}/N_i = N_{g2}/N_i = 5$dB and $K_1/B = K_2/B = 5$dB. Preferred pairs (f_1/B, f_2/B) are denoted as circles and the dash-dot line is the corresponding curve fit.
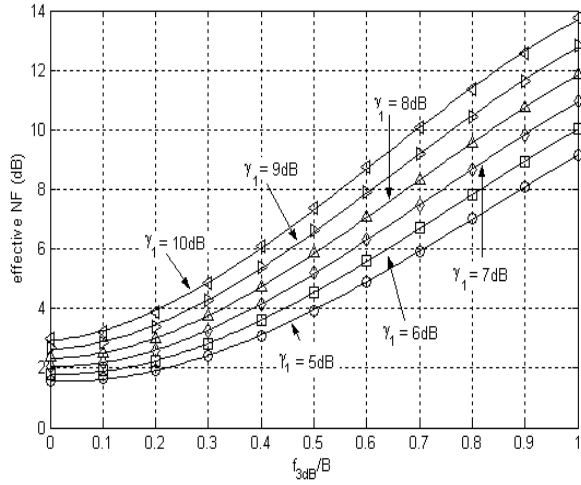
Fig. 5. The effective NF related to f_{3dB}/B and γ_1 for the 2-stage amplifier sytem, assuming $K_1/B = K_2/B = 5$dB.

Substituting (13) into (3) and integrating over the signal bandwidth, the effective NF can be obtained by numerically computing for

$$F_{eff} = B / \int_0^B \frac{1}{F_s(f)} df \qquad (15)$$

Our design objective is to select $\{f_1/B, f_2/B, ..., f_M/B\}$ given $\{\gamma_1, \gamma_2, ..., \gamma_M\}$, so that the effective NF in (15) is optimally traded with the M-stage amplifier bandwidth of f_{3dB}. This design objective is illustrated in Fig. 4 for a two-stage amplifier. Assuming $N_{g1}/N_i = N_{g2}/N_i = 5$dB and $K_1/B = K_2/B = 5$dB, Fig. 4 plots contours of constant effective f_{3dB}/B and effective NF as a function of f_1/B and f_2/B. Solid contour lines closer to the origin represent decreasing effective NF values, whereas the dotted contour lines further away from the origin represent increasing effective f_{3dB}/B values. Since the amplifiers should be designed with f_1/B and f_2/B that maximize the effective f_{3dB}/B for a given effective NF, or equivalently, that minimize the effective NF for a given f_{3dB}/B, the optimal design points, which are represented graphically using circles in Fig. 4, occur when the direction of the gradients of the effective f_{3dB}/B and NF contours are equal. More generally, these optimal design points for an M-stage amplifier can be determined by solving the following constrained optimization problem given a desired f_{3dB}/B value:

$$minimize \qquad F_{eff} \qquad (16)$$

$$subject \ to \qquad \prod_{l=1}^{M} [f_{3dB}^2 + f_l^2] = 2 \prod_{l=1}^{M} f_l^2$$

where F_{eff} is provided in (15). The optimization problem given in (16) can be solved numerically using iterative search techniques such as the sequential quadratic programming (SQP) method [7].

Fitting a quadratic curve through the optimal design points in our 2-stage amplifier example in Fig. 4, the relationship between the optimal f_1/B and f_2/B is given by

$$\frac{f_2}{B} = 2.5 \cdot \frac{f_1}{B} - 1.3 \cdot \left(\frac{f_1}{B}\right)^2 \qquad (17)$$

As is clear from (17), f_1/B is smaller than f_2/B. This is because the noise associated with the first stage is more critical to the overall NF. Therefore, a narrower amplifier in the first stage to improve F_{eff} followed by a wider amplifier achieves the minimum effective NF for a given f_{3dB}. Generalizing this observation to an M-stage amplifier, the optimal design is to successively cascade the amplifier stages with widening bandwidths.

Based on the optimal design points, the effective NF can be plotted as a function of f_{3dB}/B as shown in Fig. 5 for our 2-stage amplifier example. This plot can be used to determine the optimal tradeoffs between the effective NF and f_{3dB}/B of the overall amplifier. Since the effective NF monotonically increases with increasing f_{3dB}/B, the amplifiers can be designed with an f_{3dB} that corresponds to an effective NF that is slightly above (e.g., 1dB) the minimum effective NF.

5. CONCLUSIONS

For the NF of an amplifier to be a meaningful metric, the SNR at the input and output of the receiving system should measure the performance after the eventual digital decoding process, as it is ultimately the most relevant measure of performance. By defining the SNR as the MFB, the effective NF measures the degree of degradation in the achievable receiver performance caused by the receiving system.

The difficulty in amplifying the UWB signal stems from its bandwidth being a large fraction of the amplifier gain-bandwidth product. Hence, a design approach for maximizing the amplifier bandwidth for a given effective NF is described. Our analysis suggests that the optimal design is to successively cascade the amplifier stages with widening bandwidths.

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