Approximate Formulae Approach for Efficient Inductance Extraction

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Abstract— In this paper, we present a new and effective approach to the extraction of on-chip inductance, in which we apply approximate formulae. The equations are based on the assumption of filaments or bars of finite width and zero thickness and are derived through Taylor's expansion of the exact formula for mutual inductance between filaments. Despite the assumption of uniform current density in each of the bars, the model is sufficiently accurate for the interconnections of current and future LSIs, in which most of the wires are not affected by the skin and proximity effects. Expression of the equations in polynomial form provides a balance between accuracy and computational complexity. These equations are mapped according to the geometric structures for which they are most suitable in minimizing runtime in the calculation of inductance while remaining accurate to within 3%. Within the geometrical constraints, the wires are of arbitrary specification.

From a comprehensive evaluation on the ITRS-specified global wiring structure for 2003, the values for inductance extracted through the proposed approach are within 3% of the values obtained by commercial three-dimensional (3-D) field solvers. The efficiency of the proposed approach is also demonstrated by extraction from a real layout design that has 300-k interconnecting segments.

I. INTRODUCTION

The effects of on-chip inductance on wiring delays have become increasingly significant with the increasing clock frequencies of VLSI circuits. The difference between the delay times of RC and RLC interconnects can reach more than 10% [1-3]. The extraction of the parasitic inductance is becoming increasingly important for accurate timing calculation.

The idea of partial inductance [4-6] has been developed and applied in partial equivalent element circuit (PEEC) analysis. The partial inductance is used because determining the current-return path in the complicated wiring of a modern VLSI is difficult.

To calculate the inductances of LSI interconnects, three-dimensional (3-D) field solvers [7-8] are widely used. These approaches are capable of capturing both the skin and proximity effects, which appear as increases in clock frequency. These tools are effective for the very accurate calculation of inductance values for individual wires but are in general too CPU-intensive to be applied as parasitic extraction tools for the thousands of wires in an actual process.

The analytical formulae have been proposed as a faster way of calculating the inductance [9-11]. In these approaches, the self-inductance formula provides an approximation of the inductance in which the geometric mean distance (GMD) of the rectangular cross-section is taken into account. The mutual inductance formula is the exact or approximate equation for the inductance between two filaments — the cross-sections of the wires are ignored. The authors have previously reported [12] that even the inductance between wires that are inclined relative to each other can be calculated by using the analytical formula. However, the equations for mutual inductance between filaments become inaccurate when the wires are very wide or short, since the infinitely long filament with zero cross-sectional area is no longer a good approximation.

Grover has proposed a table look-up model for the calculation of mutual inductance [9]. The wires are treated as having rectangular cross-sections, and the GMD is used to take this into account. There are exact formulae for the different cross-sectional rectangles [13], but they are still too complicated and large amounts of computational time are required to extract the inductance values.

Recently, numerous reports on forms of analysis for and results of evaluation of wiring inductance have been published. In most of these papers, long straight wires have been assumed; this is not a general representation of the wires of practical layouts. A method that fully applies analytical formula to interconnects in general has not been presented yet. In this paper, we focus on mutual inductance, because the computational costs for self- and mutual inductances are $O(n)$ and $O(n^2)$ respectively, where n is the number of segments. The purpose of this paper is to present a formula-based approach that is efficient with arbitrary VLSI interconnect structures. These equations can be used in floorplan processes, or even in parasitic extraction for delay calculation.

The remainder of this paper is organized as follows. In Section II, we review the inductance formula and the idea of GMD. In Section III, we present new approximate equations for use in calculating inductance values for VLSI interconnects. In Section IV, we evaluate the accuracy and applicable ranges of these equations and give an example of their application to a real physical layout. Finally, we conclude the paper in Section V.

II. General Formulae

In this section, we review several formulae for inductance and GMD that are used in calculating the mutual inductance between two conductors [9].

A. *Mutual Inductance Formula*

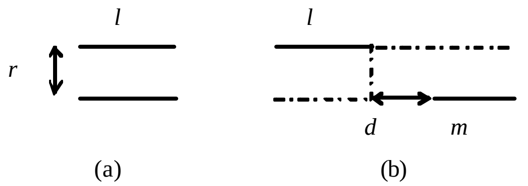

Fig. 1. (a) Two straight parallel filaments of equal length. (b) Parallel filaments of unequal length.

The exact formula for the calculation of mutual inductance for the parallel filaments of equal length shown in Fig. 1(a) is

$$dM_l = \frac{m_0 l}{2p}\left(\ln\left(\frac{l}{r}+\sqrt{1+\left(\frac{l}{r}\right)^2}\right)-\sqrt{1+\left(\frac{r}{l}\right)^2}+\frac{r}{l}\right) \qquad (1)$$

where l is the length of each wire and r is the vertical distance between the wires in the plane. The formula for filaments of different length, as shown in Fig. 1(b), is expressed as a combination of four variants of (1):

$$dM = \frac{1}{2}\left(\left(dM_{l+m+d} + dM_d\right) - \left(dM_{l+d} + dM_{m+d}\right)\right). \qquad (2)$$

Here, dM expresses the mutual inductance equation for filaments, where d is the horizontal distance between the wires in the plane.

B. *Geometric Mean Distance*

The equation for mutual inductance between filaments is simple as long as the length is much greater than the distance ($l >> r$):

$$dM \cong \frac{m_0 l}{2p}\left(\ln\left(\frac{2l}{r}\right)-1\right). \qquad (3)$$

The mutual inductance for wires of finite cross-sectional area is

$$\begin{aligned} M &= \frac{m_0}{2p}\frac{1}{S_1 S_2}\int_{S_1} dS_1 \int_{S_2}\left(\ln\left(\frac{2l}{r}\right)-1\right)dS_2 \\ &= \frac{m_0}{2p}\left(\left(\ln(2l)-1\right)-\frac{1}{S_1 S_2}\int_{S_1} dS_1 \int_{S_2} \ln r\, dS_2\right) \end{aligned} \qquad (4)$$

$$\ln R = \frac{1}{S_1 S_2}\int_{S_1} dS_1 \int_{S_2} \ln r\, dS_2 \qquad (5)$$

where S_1 and S_2 are the cross-sectional areas of the respective conductors. Here, GMD, $\ln R$ is the average distance of the natural logarithm. Once GMD has been obtained, the equation (4), which is a modification of the approximate equation (3) to include consideration of the rectangular cross-section, can be obtained.

Mutual inductance is the average of the mutual inductances between filaments when the cross-section of each of the conductors is divided into infinitely many filaments. Let the indices of the filaments in the rectangular sections of the two conductors be i and j respectively; the mutual inductance is then expressed as

$$M = \lim_{m\to\infty,\, n\to\infty} \frac{1}{m \cdot n}\sum_{i=1}^{m}\sum_{j=1}^{n} dM_{ij}\ . \qquad (6)$$

The equation (3) considered GMD is accurate as long as the condition $l >> r$ is met. Otherwise, the error becomes too great, for the method to be used as a replacement for 3-D extractors. In the next section, we investigate when the equations become accurate and inaccurate. We also give more accurate equations that are valid for structures to which the filament assumption is not applicable.

III. Proposed Equations

We present new approximate equations for calculating mutual inductances between VLSI interconnects to different degrees of accuracy and complexity. These equations are intended to cover geometrical ranges where the filament assumption becomes inaccurate.

In recent CMOS technology, the following approximations and relations hold:

$$\begin{aligned} &w_{\min} \cong s_{\min} \cong 0.5t \cong 0.5h_{\min} \\ &w \ge w_{\min},\quad s \ge s_{\min},\quad h \ge h_{\min} \end{aligned} \qquad (7)$$

where $w_{\min}$ is the minimum width of wiring, $s_{\min}$ is the minimum spacing, t is the wiring thickness, and $h_{\min}$ is the dielectric height between the metal layers.

The inductance formula for filaments is accurate when the distance between filaments is great enough to make their cross-sectional areas negligible. In other words, the worst-case accuracy condition for the filament equation (1) is the case where the wiring is broad and separated by the minimum spacing. Of course, as wiring becomes shorter, the accuracy of the approximation $l >> r$ used to obtained equation (3) is also lost.

The authors have elsewhere reported [12] that the exact formula for filaments is practical (providing results that have an error within 10%) under this condition: the wiring width is up to about 10 times its minimum value. The calculation becomes at least 60 times faster than with a 3-D solver. In other cases, the error is too large to use this approach as part of the parasitic extraction tool. To make full use of the efficiency of the analytical equations, we present a way to use combinations of the approximate equations for mutual inductance. The exact formula for a rectangular cross section is long and complicated, and thus requires huge amounts of computational resources. Since the skin and proximity effects are not significant, we assume uniform distributions of the current over the cross-sections of the wires.

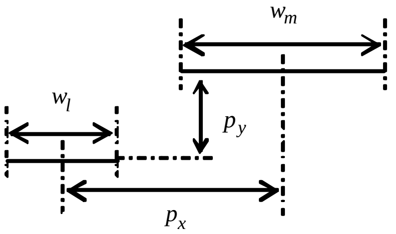

Fig. 2. Cross-sectional view of thin wires.

Fig. 2 shows the wire cross-sections for the thin thickness approximation. The function of the distance between the filaments is $g(r)$ and the function $f(R)$ of the average related to the distance between the cross sections of thin thickness approximation is expressed as

$$f(R) = \frac{1}{w_l \cdot w_m} \int_{a_{21}}^{a_{22}} dx_2 \int_0^{w_l} g(r)\, dx_1 \tag{8}$$

where w_l and w_m are the widths of wires l and m, respectively, p_x and p_y are the x- and y-direction distances between the centers of the respective cross-sections; the a_{ij} are then $a_{11} = p_x - \frac{w_l}{2} - \frac{w_m}{2}$, $a_{12} = p_x - \frac{w_l}{2} + \frac{w_m}{2}$, $a_{21} = p_x + \frac{w_l}{2} - \frac{w_m}{2}$ and $a_{22} = p_x + \frac{w_l}{2} + \frac{w_m}{2}$.

In the following subsections, we present a set of approximate equations for calculating mutual inductance without the shortcomings of the filament equations.

A. Approximate Form for Filaments

Polynomial equations can be truncated to provide simple but approximate equations with different degrees of accuracy. These can be derived from the Taylor-series expansion of the exact equation (1) for inductance between filaments.

For $l > r$,

$$dM = \frac{m_0 l}{2p}\left(\ln\left(\frac{2l}{r}\right) - 1 + \frac{r}{l} - \frac{1}{4}\frac{r^2}{l^2} + \cdots\right) \tag{9}$$

and for $l \le r$,

$$dM = \frac{m_0 l}{2p}\left(\frac{1}{2}\cdot\frac{l}{r} - \frac{1}{24}\cdot\frac{l^3}{r^3} + \cdots\right). \tag{10}$$

The second-, third-, and fourth-term approximations of (9) are

$$dM \cong \frac{m_0 l}{2p}\left(\ln\left(\frac{2l}{r}\right) - 1\right), \tag{11}$$

$$dM \cong \frac{m_0 l}{2p}\left(\ln\left(\frac{2l}{r}\right) - 1 + \frac{r}{l}\right), \text{ and} \tag{12}$$

$$dM \cong \frac{m_0 l}{2p}\left(\ln\left(\frac{2l}{r}\right) - 1 + \frac{r}{l} - \frac{1}{4}\frac{r^2}{l^2}\right). \tag{13}$$

Both (11) and (12) are well known as simple inductance equations. The greater the ratio l/r, the more accurate the equations. The relationship between the accuracy and computational cost of each equation will be described in a later section.

In the same way, taking the first and second terms of (10) yields

$$dM \cong \frac{m_0 l}{2p}\left(\frac{1}{2}\cdot\frac{l}{r}\right) \text{ and} \tag{14}$$

$$dM \cong \frac{m_0 l}{2p}\left(\frac{1}{2}\cdot\frac{l}{r} - \frac{1}{24}\cdot\frac{l^3}{r^3}\right). \tag{15}$$

B. New Formulae for the Finite-Width and Zero-Thickness Assumption

New approximate equations are now presented for wires with finite width but zero thickness. The finite width is achieved by lining up filaments side by side. Averaging the mutual inductances between all filaments in the respective conductors provides an approximate equation. For wires of finite width, (11) to (15) are replaced by the following equations.

For $l > r$,

$$M \cong \frac{m_0 l}{2p}\left(\ln(2l) - 1 - \ln R_1\right), \tag{16}$$

$$M \cong \frac{m_0 l}{2p}\left(\ln(2l) - 1 - \ln R_1 + \frac{R_2}{l}\right), \text{ and} \tag{17}$$

$$M \cong \frac{m_0 l}{2p}\left(\ln(2l) - 1 - \ln R_1 + \frac{R_2}{l} - \frac{1}{4}\frac{R_3{}^2}{l^2}\right) \tag{18}$$

and for $l \le r$,

$$M \cong \frac{m_0 l}{2p}\left(\frac{1}{2}\cdot\frac{l}{R_4}\right), \text{ and} \tag{19}$$

$$M \cong \frac{m_0 l}{2p}\left(\frac{1}{2}\cdot\frac{l}{R_4} - \frac{1}{24}\cdot\frac{l^3}{R_5{}^3}\right). \tag{20}$$

The variables concerned with distance, R_1 to R_5, can be obtained by solving (8) for the zero-thickness assumption. Here, $\ln R_1$ is the average of the natural logarithm of distance and is obtained by substituting $\ln\sqrt{(x_2 - x_1)^2 + p_y{}^2}$ for $g(r)$ of (8).

$$\begin{aligned} f(R_1) = \ln R_1 &= \frac{1}{w_l \cdot w_m}\int_{a_{21}}^{a_{22}} dx_2 \int_0^{w_l} \ln\sqrt{(x_2 - x_1)^2 + p_y{}^2}\, dx_1 \\ &= -1.5 + \frac{1}{w_l w_m}\sum_{i=1}^{2}\sum_{j=1}^{2}\Bigg((-1)^{i+j} \\ &\left(\frac{1}{4}\left(a_{ij}{}^2 - p_y{}^2\right)\ln\left(a_{ij}{}^2 + p_y{}^2\right) + a_{ij} p_y \tan^{-1}\frac{a_{ij}}{p_y}\right)\Bigg). \end{aligned} \tag{21}$$

If $p_y = 0$,

$$\ln R_1 = -1.5 + \frac{1}{2 w_l w_m} \sum_{i=1}^{2} \sum_{j=1}^{2} \left((-1)^{i+j} a_{ij}^2 \ln a_{ij} \right). \tag{22}$$

The average distance, R_2 is

$$R_2 = \frac{1}{2 w_l w_m} \sum_{i}^{2} \sum_{j}^{2} \Bigg((-1)^{i+j} \bigg(\frac{1}{3} \left(a_{ij}^2 - 2 p_y^2 \right) \sqrt{a_{ij}^2 + p_y^2} + a_{ij} p_y^2 \ln \left(a_{ij} + \sqrt{a_{ij}^2 + p_y^2} \right) \bigg) \Bigg). \tag{23}$$

If $p_y = 0$,

$$R_2 = p_x . \tag{24}$$

The average of distance squared, R_3^2 is

$$R_3^2 = p_x^2 + p_y^2 + \frac{1}{12} \left(w_1^2 + w_2^2 \right). \tag{25}$$

The average of inverse distance, $1/R_4$ is

$$\frac{1}{R_4} = \frac{1}{w_l w_m} \sum_{i}^{2} \sum_{j}^{2} \left((-1)^{i+j} \left(a_{ij} \sinh^{-1} \left(\frac{a_{ij}}{p_y} \right) - \sqrt{a_{ij}^2 + p_y^2} \right) \right). \tag{26}$$

If $p_y = 0$,

$$\frac{1}{R_4} = \frac{1}{w_l \cdot w_m} \sum_{i=1}^{2} \sum_{j=1}^{2} \left((-1)^{i+j} a_{ij} \ln a_{ij} \right). \tag{27}$$

The average of the inverse distance cubed, $1/R_5^3$ is

$$\frac{1}{R_5^3} = \frac{1}{p_y^2} \frac{1}{w_l w_m} \sum_{i}^{2} \sum_{j}^{2} \left((-1)^{i+j} \left(\sqrt{a_{ij}^2 + p_y^2} \right) \right). \tag{28}$$

If $p_y = 0$,

$$\frac{1}{R_5^3} = \frac{1}{2 \cdot w_1 \cdot w_2} \sum_{i=1}^{2} \sum_{j=1}^{2} \left((-1)^{i+j} \frac{1}{a_{ij}} \right). \tag{29}$$

According to our needs in terms of accuracy and speed, we can extract on-chip inductances effectively by using the most suitable formula.

IV. Accuracy and Effective Range

In this section, we clarify the geometric range of accuracy for each of the equations and build a map of this accuracy. The map helps us to select the equation that achieves the required degree of accuracy. The accuracy is mainly evaluated through two wiring structures. One is the simple two-line configuration that comprehensively covers the wiring structures of modern LSIs, and the other is wiring of the form that will be found in future process technologies based on the specification of ITRS 2001. We use a 3-D field solver [8] to provide reference figures for comparison.

A. *Evaluation of Simplified Structures*

Fig. 3 shows the structure used to evaluate the accuracy of the equations. The result of each equation is compared to the DC inductance as obtained by the reference extraction tool. The structures are two parallel conductors of equal length. The number of combinations is more than 50,000.

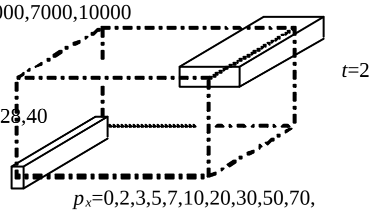

Fig. 3. Simple structure used in accuracy comparison.

Table I summarizes the proportions of geometrical structures of the form shown in Fig. 3 which are covered to within 3% and 10% error. Equation (1) covers the broadest range of r and l values. However, dividing structures at r/l=1 lowers the error of the other equations, which can be calculated faster than (1). Equations (18) and (20), newly presented in the previous section, can cover more than 98% of the structures to within 3% error. The structures that cannot be covered to this accuracy by those equations are lines which are very wide and very close to each other; such a situation rarely appears in an actual design.

Although the exact formula (1) for inductance between filaments covers the widest range, it does not give a good estimate when the wires are very wide. The equations (16)-(20), newly proposed in the previous section, make up for the shortfalls of the well-known simple or exact equation for inductance between filaments.

TABLE I
Ratio Covered by Each Equation

Formula	Coverage (%)	
	≤3% error	≤10% error
(11)	23.7 (43.6 in $l > r$)	35.1 (64.6 in $l > r$)
(12)	36.9 (67.9 in $l > r$)	45.1 (83.0 in $l > r$)
(13)	43.1 (79.2 in $l > r$)	52.0 (90.2 in $l > r$)
(14)	32.2 (70.3 in $l \le r$)	41.3 (88.0 in $l \le r$)
(15)	36.8 (79.5 in $l \le r$)	43.5 (88.0 in $l \le r$)
(1)	79.8	89.2
(16)	28.3 (52.1 in $l > r$)	37.1 (68.3 in $l > r$)
(17)	44.2 (81.5 in $l > r$)	48.8 (89.7 in $l > r$)
(18)	52.9 (97.3 in $l > r$)	56.7 (99.0 in $l > r$)
(19)	38.7 (84.9 in $l \le r$)	46.3 (99.0 in $l \le r$)
(20)	46.2 (98.8in $l \le r$)	49.9 (99.9 in $l \le r$)

Fig. 4 shows an equation selection map for (11)-(20) and (1), which shows the region where an accuracy of 3% or better is achieved, relative to the results of the reference extractor. Where several equations achieve this degree of accuracy, the simplest is chosen. This map provides a useful way of choosing the most efficient equations, in terms of computational cost, that also fall within the required threshold of accuracy. From the map, we derive the rule of thumb shown in Table II.

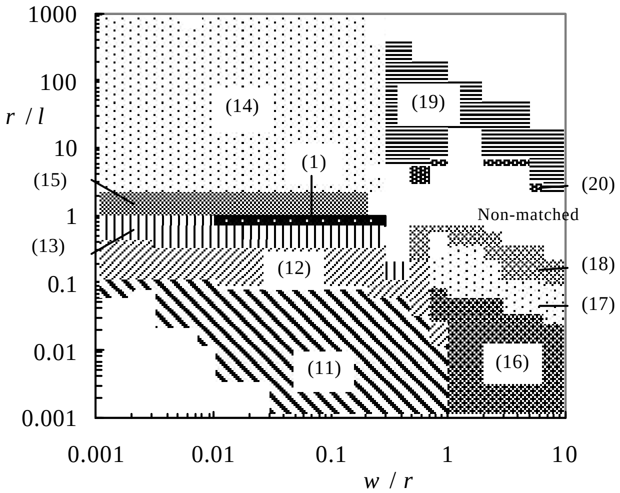

Fig. 4. Map of the areas of coverage by optimal equations for error within 3%.

TABLE II
CRITERIA FOR SELECTING THE OPTIMAL EQUATION

Range			Formula
2≤r/l	&	w/r≤0.3	(14)
1≤r/l<2	&	w/r≤0.3	(15)
0.7≤r/l<1	&	w/r≤0.3	(1)
0.3≤r/l<0.7	&	w/r≤0.3	(13)
0.05≤r/l<0.3	&	w/r≤0.3	(12)
r/l<0.05	&	w/r≤0.3	(11)
7≤r/l	&	0.3<w/r	(19)
1≤r/l<7	&	0.3<w/r	(20) or exact
0.05≤r/l<1	&	0.3<w/r	(18) or exact
0.01≤r/l<0.05	&	0.3<w/r	(17)
r/l<0.01	&	0.3<w/r	(16)

Table III gives the comparative values for the cost of calculating each of these equations. The values are relative to (11) as a reference. The results are of trials on the premise that the start and end points, width, and height of the segments are already known.

TABLE III
RELATIVE TIME TO CALCULATE EACH EQUATION

Equation	Relative cost
(11)	1.0
(12)	1.2
(13)	1.2
(14)	0.7
(15)	0.8
(1)	1.4
(16)	4.3
(17)	7.0
(18)	7.2
(19)	4.5
(20)	4.5
Exact [13]	67.9

B. Applicability to the Future Process

The parameters of the global wiring structure specified for use from 2003 by ITRS 2001 [14] are given in Table IV. We use these as parameters that reflect those of future structures, and verify the accuracy of the selected equations for these parameters. The extraction frequency of the 3-D solver is set at 3.5 GHz.

Table V shows the geometries used in verification. The optimal equations given in Table II match the results of the reference extraction method to within 3% in most of the cases shown in Fig. 5.

The relations of Table II must be effective across generations where most of the relations of (7) apply. However, for still better accuracy, maps of the optimal equations for every process will be useful. The required speeds of calculation can be obtained by adjusting the limit on the error.

TABLE IV
ASSUMED GLOBAL METAL GEOMETRY

Parameter	Value
Minimum width, w_{min} (nm)	237.5
Metal thickness, t (nm)	498.75
Dielectric height, h_{min} (nm)	451.25

TABLE V
PARAMETER VARIATION IN VERIFICATION

Parameter	Value
l	w_{min}×1,2,3,5,7,10,20,30,50,70,100, 200,300,500,700,1000, 2000,3000,5000,7000,10000
w_m, w_l	w_{min}×1,2,3,5,7,10,20,30,50,70,100
Spacing in the same layer	w_{min}×1,2,3,5,7,10, ,20,30,50,70,100, 200,300,500,700,1000
p_x for diff. layers (μm)	0,1,2,3,5,7,10,20,30,50,70,100, 200,300,500,700,200
p_y for diff. layers	(t+h_{min}) ×1,2,3,5,7,10

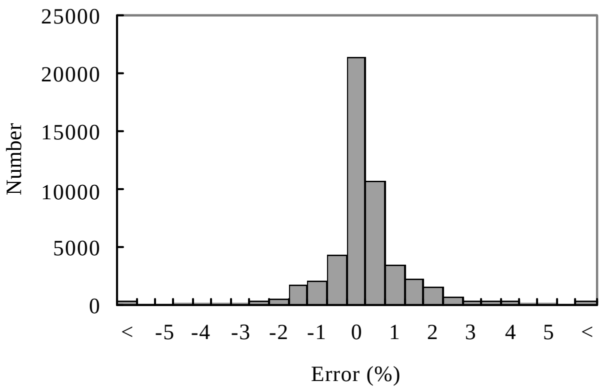

Fig. 5. Distribution of error relative to results of the reference 3D extractor.

C. Application Example

The proposed methodology based on the optimal equations given in Table II was implemented and applied to the real physical layout of 0.13-µm CMOS technology in 6 layers. A 7-mm-square chip contains more than 3 million wiring segments. There are more than 10 billion mutual inductances between these segments. Here, we calculate the mutual inductances for 10, 100 and 300 thousand segments. The results are given in Table VI.

More than 80% of all of the segments are short (<10-µm-long). There are several thousands of wide (>2-µm-wide) segments in the chip. For such segments, a conventional equation such as (12) is not practical because of the large errors it entails. The combinational use of proposed equations according to Table II achieved an error of less than about 3% in a reasonable runtime. The proposed method is 60 or more times as fast as the exact formula [13] with rectangular cross-sections. The proposed methodology is thus very efficient in accurately extracting the inductances of VLSI interconnects.

TABLE VI
RESULTS OF APPLICATION: CPU TIMES ON A SUN BLADE1000

# of segments	CPU time (hour:min:sec)	
	Exact [13]	Proposed
10,000	01:42:22	00:01:44
100,000	99:03:16	01:31:39
300,000	-	15:08:17

V. CONCLUSIONS

We have proposed an efficient and reasonably accurate approach to the extraction of on-chip inductances, based on a set of approximate equations that are derived through Taylor's expansion. The equations are for arbitrary wiring structures, which cover those seen in both current and future process nodes. Starting from the well-known exact equation for filaments, the new equations are obtained in polynomial forms by considering the width but ignoring the thickness of the wire. This allows us to select a solution that has sufficient accuracy by changing the numbers of terms. The equations support relatively wide and short wires to which the filament assumption does not apply. The equations have been mapped so that we are able to minimize runtime while maintaining the required degree of accuracy.

The proposed equations have been evaluated on a comprehensive set of wiring structures of the form specified in ITRS 2001. Our approach achieves a result within 3% of that obtained by a commercial 3-D field solver executed at 3.5 GHz. We applied both methods to a real physical layout that has 300-k interconnect segments. Our method provides a way of calculating parasitic inductances within practical runtimes. The proposed methodology is very useful as a way of efficiently extracting the inductances of VLSI interconnects.

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