Discharge Current Steering for Battery Lifetime Optimization

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ABSTRACT
Recent work on battery-driven power management has demonstrated that sequential discharge is suboptimal in multibattery systems, and lifetime can be maximized by distributing (steering) the current load on the available batteries, thereby discharging them in a partially concurrent fashion. Based on these observations, we formulate multi-battery lifetime maximization as a continuous, constrained optimization problem, which can be efficiently solved by non-linear optimizers. We show that great lifetime extensions can be obtained with respect to standard sequential discharge, as well to previously proposed battery allocation schemes.

Categories and Subject Descriptors

General Terms
Design, Performance

Keywords
Energy consumption, battery lifetime optimization

1. INTRODUCTION
Supporting multi-battery power supplies is becoming standard for modern electronic products, as this option enables the user to trade battery lifetime for weight upon needs. From the manufacturing standpoint, there is clearly a number of issues that must be faced when multiple batteries have to be accommodated into the case of an electronic product, such as a laptop or a cell phone. They range from the selection of battery capacities and shapes, to the design of the power supply circuitry (including the switching regulator that interfaces the various batteries to the current load).

One degree of freedom that, so far, has not been fully exploited, is the policy to be used for discharging the available batteries. In other words, the approach adopted in existing products consists of fixing, once and for all during system design, the order in which batteries have to be discharged. A battery is not disconnected from the current load until it is exhausted.

The rationale for this solution stands in the assumption that batteries well approximate ideal charge storage. In other words, the amount of charge (i.e., the capacity) a battery can deliver is independent of the way the charge is extracted. Unfortunately, the behavior of a real battery is far different from the ideal case. In particular, at higher current loads, a battery is less efficient in converting its chemically stored energy into available electrical energy; thus, its actual capacity deviates more sensibly from the nominal value. This effect has been discussed in a number of previous works [1, 2, 3, 4]. Accurate battery modeling is a complex task, because a number of additional non-idealities need to be taken into account (e.g., battery recovery, internal resistance, thermal effects). However, load-dependent capacity is the most significant non-ideality in real-life batteries and all battery manufacturers provide quantitative data on this effect in their data-sheets (discharge curves, plotting capacity vs. current load), while other, more complex effects, such as charge recovery require additional, extensive experimental characterization.

In multi-battery systems, the load-dependent capacity of batteries has profound implications. First and foremost, the commonly accepted sequential discharge schedule is a very inefficient policy from a battery lifetime viewpoint, as observed in [5, 6]. Pedram, Wu and Qiu [5] propose an improved battery discharge policy that selects the battery to be connected to the load based on the absorbed current level. This strategy is effective when batteries are highly asymmetric (e.g., one battery is very efficient at low currents, while the other is much better at high currents), and when the current load has high variance. Unfortunately, load-based battery switching degenerates to sequential discharge for multiple cells that respond similarly to the load (e.g., multiple equal batteries, similar batteries with different size) and for constant loads.

An alternative approach to multi-battery scheduling was proposed in [6]. One of the main results obtained in that work was that, given a simple two-battery system made of two identical battery cells, significant lifetime improvements can be achieved by alternatively connecting the two battery cells to the load. The higher the frequency at which bat-

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2. Problem Formulation

2.1 Capacity vs. Lifetime

Because of the non-idealities of real batteries, the relation between discharge time $T$ (the lifetime, hereafter) and battery capacity $C$ cannot be simply derived by the ideal battery capacity formula $C = T \cdot I$, where $I$ is the discharge current.

This fact is at the basis of the well-known Peukert’s equation [8], that models non-idealities for the case of a constant current load by introducing a penalty value that decreases the actual capacity (i.e., the battery efficiency) for larger current loads. Peukert’s equation relates $C$ and $T$ as follows:

$$C = T \cdot I^\alpha$$  \hspace{1cm} (1)

where $\alpha > 1$ is called the Peukert’s value. Typical values of $\alpha$ are between 1.2 and 1.4. The value $\alpha = 1$ represents the ideal case.

For our purposes, we model this non-ideality in a slightly different way, similar to the approach followed by Pedram and Wu in [9]. This alternative solution consists of expressing the dependency between capacity and discharge current as follows:

$$C = T \cdot I \cdot \rho_1$$  \hspace{1cm} (2)

In Equation 2, $\rho_1 > 1$ is the current scaling factor, which accounts for the fact that the battery is less efficient in using its capacity for larger current values. $\rho_1$ actually expresses the ratio of the nominal capacity of the battery $C_0$, and the discharge characteristics of the battery versus the load current. In formula, $\rho_1 = \frac{C}{C_0 I}$. Re-arranging the equation for $C$, we get:

$$T = \frac{C(I)}{C_0 I}$$  \hspace{1cm} (3)

2.2 Load Characterization

The constant current load at the basis of Peukert’s equation is not a realistic assumption for real-life systems, that are typically characterized by variable loads. One possibility to model a variable load is to assume a set of $M$ current levels ($I_1, \ldots, I_M$); their distribution over time is described by a set ($x_1, \ldots, x_M$), where $x_i$ denotes the percentage of total operation time spent with current $I_i$.

Such characterization can be achieved by statistical profiling of the typical behavior of the system under analysis over a significant period of time. The value of $M$ determines the quantization interval used to characterize the load. Although somehow simplistic, this model can be tuned with arbitrarily fine accuracy, and is general enough.

Under this load model, Equation 2 generalizes to:

$$C = T \cdot \sum_{i=1}^{M} x_i \cdot I_i \cdot \rho_{i}$$  \hspace{1cm} (4)

Equation 4 is at the basis of our formulation of the load optimization problem described in the next subsection.

2.3 Load Optimization

We assume that $N$ batteries are available, each one characterized by its capacity equation $C_i(I_i)$, $i = 1, \ldots, N$. Our formulation of the load optimization problem is based on the following two assumptions:

- All the $N$ batteries are discharged concurrently, and the load current is partitioned, in general not equally, among them. This is equivalent to assuming round-robin switching policy which connects each battery to the load for a time proportional to the fraction of the load current we wish to absorb from the battery. The cycle frequency of the round-robin schedule is fast enough to let batteries perceive only its time-averaged effect (i.e., a constant current equal to a fraction of the load current).

- Since we are discharging the $N$ batteries concurrently, we wish to fully discharge all of them at the same time $T$, that represents the lifetime of the overall battery pack. Any differences in time-to-total discharge among batteries can be seen as an inefficiency in the current
steering policy, and we want to eliminate the inefficiency by construction.

The load optimization problem can be formulated as a non-linear optimization problem as follows:

Maximize $T$, such that:

\[
\begin{align*}
     I_1 &= I_{1,1} + \ldots + I_{1,N} \\
     \vdots \\
     I_M &= I_{M,1} + \ldots + I_{M,N} \\
    1 &= T \cdot \sum_{i=1}^{M} \frac{s_i I_{i,1}}{C_i(t_{i,1})} \\
    \vdots \\
    1 &= T \cdot \sum_{i=1}^{M} \frac{s_i I_{i,N}}{C_N(t_{i,N})}
\end{align*}
\]  

subject to:

\[0 \leq I_{i,j} \leq I_i, \quad i = 1, \ldots, M \quad j = 1, \ldots, N\]  

The unknowns (decision variables) are:

- The lifetime $T$.
- The currents $I_{i,j}$, that define what fraction of the current $I_i$ is to be extracted from battery $j$. For $M$ current levels and $N$ batteries, there are $N \cdot M$ such currents.

The problem is subject to three types of constraints:

- $M$ current equality constraints (Equation 5): These express the fact that all the battery-loading currents $I_{i,j}, j = 1, \ldots, N$, in each load condition, must sum up to the corresponding total load current $I_i$ and must sum up to $I_i$.

- $N$ battery equality constraints (Equation 6): These constraints are obtained by Equation 3, applied to each battery. They express that, for each battery $j, j = 1, \ldots, N$, the various current loads $I_{i,j}$ allocated to it must discharge the battery at time $T$. This condition must hold for all the batteries, which discharge at the same time $T$.

- $M \cdot N$ bound constraints (Equation 7): These simply express the fact that each of the sub-currents on each battery $I_{i,j}$ must be (i) nonnegative quantities, and (ii) must not exceed the corresponding total current $I_i$. Since the upper bound of this set of constraints is implicitly contained in Equation 5, they can be simplified to $I_{i,j} \geq 0$.

In spite of the simplicity of the objective function, the problem is far from having a trivial solution, because the equality constraints of Equation 6 are in general non-linear. A local minimum can be found using standard continuous non-linear optimizers [10] (e.g., quasi-Newton, gradient).

An important observation about the above formulation concerns the battery models. It is important to emphasize that reducing the battery behavior to a capacity equation only approximately models the complex behavior of real-life batteries; other effects such as charge recovery due to battery idleness are not included in this model. Notice, however, that the proposed allocation scheme is insensitive to the recovery effect, because it is based on a high-frequency alternation of the various battery packs.

The following is a simple instance of the problem that shows how the current allocation is superior, for instance, to a sequential discharge scheme.

\textbf{Example 1.} Consider a system with $N = 2$ batteries, and a constant current load of 2A. This corresponds to the case $M = 1$, with $I_1 = 2$, and $x_1 = 100\%$. For simplicity, let us assume that the two batteries have the following linear capacity equations (in some unit of charge):

\[C_1(I) = 10 - I\]

\[C_2(I) = 15 - 2 \cdot I\]

The unknowns of the problem are the battery lifetime $T$, and the two sub-currents $I_{1,1}$ and $I_{1,2}$ that specify which fraction of $I_1$ is allocated to battery 1 and 2, respectively. The problem formulation is the following:

Maximize $T$, such that:

\[
\begin{align*}
2 &= I_{1,1} + I_{1,2} \\
1 &= T \cdot \frac{I_{1,1}}{I_{1,1}} \\
1 &= T \cdot \frac{I_{1,2}}{15 - 2 \cdot I_{1,2}}
\end{align*}
\]

subject to:

\[
\begin{align*}
0 &\leq I_{1,1} \leq 2 \\
0 &\leq I_{1,2} \leq 2
\end{align*}
\]

The above formulation admits one solution only, since there are three equalities for three unknowns; solving the system yields $T = 10.919$. This optimum corresponds to the values of $I_{1,1}$ and $I_{1,2}$ of 0.839A and 1.161A, respectively. In practice, given the characteristics of the two batteries, the best choice is to allocate 41.95\% = (0.839/2) of $I_1$ to the first battery, and 58.05\% = (1.161/2) to the second one.

Let us now compare this value with the sequential discharge of the two batteries. Since the load current is constant, the order of discharge is roughly irrelevant. The first battery, when discharged with $I = 2$A has an effective capacity $C_1(I) = C_1(2) = 10 - 2 = 8$. Under current $I$, this effective capacity corresponds to a duration $T_1 = \frac{C_1(I)}{I} = \frac{8}{2} = 4\text{ s}$. Using the same calculations, the second battery has an effective capacity $C_2(I) = C_2(2) = 15 - 2 \cdot 2 = 11$, corresponding to a duration $T_2 = \frac{C_2(I)}{I} = \frac{11}{2} = 5.5\text{ s}$, for a total duration of $T_s = 4 + 5.5 = 9.5$, a 13\% shorter battery lifetime than the optimal value.

\textbf{3. EXPERIMENTAL RESULTS}

We have used a standard commercial package to solve the current allocation algorithm, namely, the nonlinear constrained optimizer of the Matlab Optimization Package. It uses the so-called Sequential Quadratic Programming (SQP) methods, consisting of an iterative solution of several quadratic programming sub-problems, that have been
shown to represent the state of the art in nonlinear programming methods [10].

To validate the effectiveness of the proposed solution we have run several experiments to compare the lifetime achievable with proportional current allocation (i.e., the method of this paper) with respect to the uniform current splitting approach of [6] (referred to in the sequel as fixed current allocation). Comparison to lifetimes provided by sequential battery discharge (which his the policy adopted by existing electronic products) is also provided for the sake of completeness.

3.1 Explorative Analysis

In a first experiment, we have evaluated the impact of proportional current allocation for a number of variants of a reference workload applied to a system. The system consists of two batteries, whose capacity equations are:

\[ C_1(I) = 10 \cdot (1 - 0.04 \cdot I^{1.4}) \]
\[ C_2(I) = 15 \cdot (1 - 0.08 \cdot I^{1.3}) \]

These models have been obtained by fitting the discharge profile of two real-life batteries to a generic equation template of the form \( C_0 \cdot (1 - \alpha \cdot I^p) \).

The workload consists of two currents of \( I_1 = 1A \) and \( I_2 = 3A \). We have then analyzed the discharge of the two batteries for different distributions of the workload; in particular, according to the formulation of Section 2, we have applied various workloads consisting of \( I_1 \) and \( I_2 \) and different values of \( x_1 \) and \( x_2 \). Figure 1 plots, for different values of the ratio \( x_1 / x_2 \), the lifetime of the battery system obtained by solving the proportional current allocation problem (Proportional) versus the lifetime corresponding to fixed current allocation (Fixed).

![Figure 1: Comparing Lifetimes of Proportional and Fixed Current Allocation.](image)

The plot shows that proportional current allocation roughly provides a fixed amount of lifetime extension, that has a more significant impact on heavier workloads (\( x_1 / x_2 \leq 0.5 \)), signifying that in case of higher currents there is higher margin for current allocation than for smaller currents. This fact is also shown in Figure 2, where the percentage of lifetime extension is plotted for three different workloads: The first one consists of the same current levels as in Figure 1 (\( I_1 = 1, I_2 = 3 \)), whereas the second and the third have \( (I_1 = 2, I_2 = 4) \), and \( (I_1 = 4, I_2 = 8) \), respectively. In the plot we notice how the lifetime extensions increase as the average value of the current drawn increases, and are also more sensitive to the ratio \( x_1 / x_2 \).

![Figure 2: Lifetime Extensions using Proportional Current Allocation for Different Current Levels.](image)

3.2 Synthetic Workloads

Another type of validation has been carried out over a set of synthetic, i.e., artificially generated workloads, characterized by different current levels and time-domain behaviors. More specifically, we have considered a total of 6 types of current load stimuli, characterized as follows:

- Type CC: 2 constant current loads of magnitude 0.1 and 1.0A.
- Type SSW: 2 symmetric square waves (50% duty-cycle), with average value of 0.5A, and different current levels: (0.4A, 0.6A) and (0.2A, 0.8A).
- Type ASW: 2 asymmetric square waves (20-80% duty-cycle), with average value of 0.5A, and different current levels: (0.4A, 0.6A) and (0.2A, 0.8A).

These workloads have been applied to the case of two, three and four batteries, whose capacity vs. current equations are shown in the following table (in mAh):

<table>
<thead>
<tr>
<th>Type</th>
<th>Capacity Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>( C_1(I) = 10000 \cdot (1 - 4 \cdot 10^{-5} \cdot I^{1.4}) )</td>
</tr>
<tr>
<td>B2</td>
<td>( C_2(I) = 15000 \cdot (1 - 2 \cdot 10^{-4} \cdot I^{1.3}) )</td>
</tr>
<tr>
<td>B3</td>
<td>( C_3(I) = 20000 \cdot (1 - 5 \cdot 10^{-5} \cdot I^{1.2}) )</td>
</tr>
<tr>
<td>B4</td>
<td>( C_4(I) = 15000 \cdot (1 - 1 \cdot 10^{-5} \cdot I^{1.5}) )</td>
</tr>
</tbody>
</table>

The two battery case consists of the combination of \( B_1 \) and \( B_2 \), while the three-battery case consists of batteries \( B_1, B_2 \) and \( B_3 \).

Tables 1- 3 compare the lifetime \( T \) achieved by proportional current allocation (Column Prop) to that achieved through fixed current allocation (Column Fixed), as well as to that given by sequential battery discharge (Column Seq), for the various workloads and for the three battery configurations.
Table 1: Lifetime Comparison for Synthetic Workloads (2 Batteries)

<table>
<thead>
<tr>
<th>Workload</th>
<th>Prop</th>
<th>Fixed</th>
<th>(\Delta %)</th>
<th>Seq</th>
<th>(\Delta %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1</td>
<td>88886</td>
<td>88852</td>
<td>0.26</td>
<td>88365</td>
<td>0.74</td>
</tr>
<tr>
<td>CC2</td>
<td>6670</td>
<td>6126</td>
<td>8.98</td>
<td>3556</td>
<td>87.74</td>
</tr>
<tr>
<td>SSW1</td>
<td>16348</td>
<td>15864</td>
<td>3.06</td>
<td>13545</td>
<td>20.69</td>
</tr>
<tr>
<td>SSW2</td>
<td>15726</td>
<td>14415</td>
<td>9.09</td>
<td>9617</td>
<td>63.52</td>
</tr>
<tr>
<td>ASW1</td>
<td>14248</td>
<td>13818</td>
<td>3.11</td>
<td>11413</td>
<td>24.84</td>
</tr>
<tr>
<td>ASW2</td>
<td>9875</td>
<td>9556</td>
<td>5.52</td>
<td>6052</td>
<td>63.14</td>
</tr>
</tbody>
</table>

Table 2: Lifetime Comparison for Synthetic Workloads (3 Batteries)

<table>
<thead>
<tr>
<th>Workload</th>
<th>Prop</th>
<th>Fixed</th>
<th>(\Delta %)</th>
<th>Seq</th>
<th>(\Delta %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1</td>
<td>161400</td>
<td>161143</td>
<td>0.16</td>
<td>158760</td>
<td>1.66</td>
</tr>
<tr>
<td>CC2</td>
<td>14150</td>
<td>12622</td>
<td>12.10</td>
<td>5443</td>
<td>159.96</td>
</tr>
<tr>
<td>SSW1</td>
<td>31393</td>
<td>30428</td>
<td>3.17</td>
<td>25176</td>
<td>24.69</td>
</tr>
<tr>
<td>SSW2</td>
<td>30489</td>
<td>28466</td>
<td>7.11</td>
<td>19734</td>
<td>81.34</td>
</tr>
<tr>
<td>ASW1</td>
<td>27707</td>
<td>26374</td>
<td>5.05</td>
<td>21387</td>
<td>20.55</td>
</tr>
<tr>
<td>ASW2</td>
<td>20239</td>
<td>18589</td>
<td>8.88</td>
<td>10517</td>
<td>92.44</td>
</tr>
</tbody>
</table>

Table 3: Lifetime Comparison for Synthetic Workloads (4 Batteries)

<table>
<thead>
<tr>
<th>Workload</th>
<th>Prop</th>
<th>Fixed</th>
<th>(\Delta %)</th>
<th>Seq</th>
<th>(\Delta %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1</td>
<td>197495</td>
<td>197135</td>
<td>0.15</td>
<td>193860</td>
<td>1.27</td>
</tr>
<tr>
<td>CC2</td>
<td>18148</td>
<td>16449</td>
<td>10.33</td>
<td>7608</td>
<td>138.54</td>
</tr>
<tr>
<td>SSW1</td>
<td>38981</td>
<td>37771</td>
<td>3.29</td>
<td>31233</td>
<td>24.81</td>
</tr>
<tr>
<td>SSW2</td>
<td>38118</td>
<td>36133</td>
<td>5.49</td>
<td>22147</td>
<td>72.11</td>
</tr>
<tr>
<td>ASW1</td>
<td>34429</td>
<td>33326</td>
<td>3.31</td>
<td>26565</td>
<td>20.69</td>
</tr>
<tr>
<td>ASW2</td>
<td>25675</td>
<td>23655</td>
<td>8.53</td>
<td>13982</td>
<td>83.63</td>
</tr>
</tbody>
</table>

3.3 Real-Life Example

The third experiment we have carried out consists of evaluating a real-life workload extracted from an actual system, and perform some exploration about the opportunities offered by various battery configurations.

The system is a digital audio recorder described in [6], that can operate in four active states with sensibly different current absorptions, summarized in the following table:

<table>
<thead>
<tr>
<th>State</th>
<th>Current [mA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleep</td>
<td>(I_1 = 15)</td>
</tr>
<tr>
<td>Idle</td>
<td>(I_2 = 220)</td>
</tr>
<tr>
<td>RawSound</td>
<td>(I_3 = 460)</td>
</tr>
<tr>
<td>FineSound</td>
<td>(I_4 = 760)</td>
</tr>
</tbody>
</table>

A typical usage of the system consists of an alternate, aperiodic sequence of active (playing sound) and idle (silence) intervals. We have taken a sample usage trace of the system over a significant amount of time, and we translated into our abstract representation of a workload, namely, a set of pairs (current level, percentage of time). The resulting current profile is \([ (I_1, 17\%), (I_2, 14\%), (I_3, 24\%), (I_4, 45\%) ]\). We have tested this workload against three different battery system configurations; batteries have been picked from the four packs described in the previous section. The details of the various configurations we consider are the following:

- **BS1**: Two instances of battery \(B_3\).
- **BS2**: One instance of battery \(B_3\), and one instance of battery \(B_1\).
- **BS3**: One instance of battery \(B_3\), and one instance of a small, backup battery, whose capacity vs. current equation \(C_{\text{C}}(I) = 1200 \cdot (1 - 7c - 5 \cdot I^{1.4})\) is totally dominated by that of \(B_3\).

The following table again compares the lifetime of the battery system obtained by using current steering to the one obtained with fixed current allocation.

<table>
<thead>
<tr>
<th>Battery</th>
<th>Lifetime [s]</th>
<th>(\Delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop</td>
<td>Fixed</td>
<td>(%)</td>
</tr>
<tr>
<td>BS1</td>
<td>29419</td>
<td>29419</td>
</tr>
<tr>
<td>BS2</td>
<td>29017</td>
<td>19118</td>
</tr>
<tr>
<td>BS3</td>
<td>21728</td>
<td>19194</td>
</tr>
</tbody>
</table>

Table 4: Lifetime Comparison for Real-Life Workloads.
4. CONCLUSIONS

Battery management has shown to be a promising approach to extend lifetime of portable electronic appliances. This is particularly true when the devices are equipped with multi-battery power supplies.

In this paper, we have formulated and solved the problem of optimally allocating current loads to the various cells of a multi-battery system in order to achieve battery lifetime maximization.

The proposed solution consistently outperforms the results given by a current allocation policy that equally partitions the current load to all the batteries available in the power supply. Obviously, the new policy is also greatly superior to sequential battery discharge, the latter being the battery discharge policy adopted by modern electronic products.

5. REFERENCES


