A 3-D Minimum-order Boundary Integral Equation Technique to Extract Frequency-dependent Inductance and Resistance in ULSI

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Abstract

The frequency-dependent resistance and inductance can be calculated by solving an eddy current problem. In this paper, a model to describe such 3-D eddy current problem is proposed, which is called 3-D Omni-A model because both of the conducting and non-conducting regions are described in terms of magnetic vector potential $A$. Therefore, the induced voltages of the conductors may appear as the unknowns directly in the boundary integral equations (BIE). Compared with popular coupled circuit methods, the computational method based on 3-D Omni-A model has two advantages. First, it does not fix the current direction along the axis of conductor, so in this method the perpendicular conductors may have mutual impedance. It could be more accurate in deep submicron ($0.1 \mu m$) chips at high speed (10G Hz). Second, it only discretizes the surfaces of the conductor, so it could be more efficient.

1. Introduction

It is very important to calculate frequency-dependent resistance and inductance when the density and speed of integrated circuits increase rapidly. Especially in applications of deep submicron chips working at high frequency, we need a more accurate method with high efficiency.

The present dominant method to extract frequency-dependent resistance and inductance is the coupled circuit method [1]. In this type of method, each of the conductors is divided into parallel filaments, in which the current density is assumed to be constant. By adopting the partial inductance concept, the inductance of filament can be calculated only based on geometrical parameters. The inductance and resistance of filaments are actually DC parameters. All of the filaments form a circuit, in which the resistances and inductances of these filaments are the devices. One conductor is driven by known voltage, and the voltages of the rest conductors are set to be zero. Solving this circuit problem, the current in each filament can be known, and then the total current in each conductor. Thus, the self and mutual impedance can be obtained to form a column in impedance matrix of the conductor system. To reduce the number of unknowns, one filament has only one unknown, which is the current density along the filament axis direction, assuming there is not current flowing across the sides of the filament.

This kind of method has two problems. First, it is a finite volume method, the number of unknowns is too large. Second, the assumption that the current flows along the axis direction of filament will be far away from reality of eddy current problem. The eddy current can flow in arbitrary directions. Now, because of very high working frequency and very complex interconnect structures, it is necessary to consider the effects of the eddy current in the resistance and inductance extraction.

In this paper, a 3-D Omni-A model is proposed to describe the 3-D electromagnetic field. It consists of a set of boundary integral equations (BIE), and the three-component boundary element method (minimum-order BEM) is used to find their solutions [2]. This method can avoid the problems mentioned above. Therefore, it could be more accurate and efficient.

In the next section, we will show the 3-D Omni-A model in detail. In section III, we briefly describe how to discretize the BIE of 3-D Omni-A model by the minimum-order BEM. In section IV, we discuss the different assumptions of the current directions and different results of the mutual impedance in the perpendicular conductors between the coupled circuit method and the new method. The numerical results are
given in section V. Finally, we reach the conclusions.

2. 3-D Omni-A model

The problem space is divided into a non-conducting subregion and many subregions of the conductors. The electromagnetic field is time harmonic. The material properties, such as the conductivity, permeability and permittivity, are assumed to be constant in different subregions. The Maxwell-Ampere law is

\[ \nabla \times H = j \omega \varepsilon_0 \varepsilon_r E + \sigma E \]  

(1)

In most practical cases, such as the submicron chips, the conductivity \( \sigma \) can be \( 3 \times 10^7 \) mho/m (aluminum), the permittivity of free space \( \varepsilon_0 \) is \( \frac{1}{36 \pi} \times 10^{-9} \) F/m, and \( \varepsilon_r \) usually is less than 100 (it should be as small as possible in the ULSI). Even though the frequency is as high as \( 10^9 \) Hz, the displacement current density can still be ignored, compared with the conduction current density. But this does not mean that the capacitative effects do not exist, because the absolute value of displacement current density is quite large (about \( 10^8 \)). It can be also seen that the free charge can only distribute on the surface of the conductor when we take the divergence on both sides of equation (1).

For the region inside a conductor \( i \), the time-harmonic diffusion equation in terms of the magnetic vector potential \( A_i \), whose divergence should be zero, is [3]

\[-\nabla^2 A_i + j \omega \sigma \mu A_i = \mu J_i^s\]  

(2)

where \( j \), \( \omega \), \( \sigma \), \( \mu \), and \( J_i^s \) are imaginary unit, angular frequency, conductivity, permeability and source current density, respectively.

In 2-D applications, \( J_i^s \) in each conductor is replaced by a constant magnetic vector potential \( A_i^s \) [3-5]

\[ J_i^s = \sigma E_i^s = -j \omega \sigma A_i^s \]  

(3)

In 3-D applications, we also adopt this idea to the straight conductor with arbitrary cross-section in both static and time-harmonic cases. Since the surface charge keeps \( J_i^s \) constant and parallel to the axis of the conductor [6], we can say the effect of surface charge to the conducting region is incorporated in the unknown \( A_i^s \). Now, equation (2) is changed to

\[-\nabla^2 (A_i + A_i^s) - j \omega \sigma \mu (A_i + A_i^s) = 0\]  

(4)

This is a homogeneous Helmholtz equation. In Mayergoyz’s paper [7], its solution can be very elegantly expressed in terms of imaginary source current density \( J_i \) on the surface of this conductor \( S_i \)

\[ A_i(\xi) + A_i^s(\xi) = \frac{\mu}{4\pi} \int_{S_i} e^{-(\omega k R(\xi, \eta))} J_i(\eta) \, dS \]  

(5)

where \( \xi \) is the observation point in the conductor, \( \eta \) is the integration point on the conductor surface, \( S_i \) is the surface of the conductor \( i \), \( R(\xi, \eta) \) is the distance between \( \xi \) and \( \eta \).

Next, we consider the outside region of all conductors, and denote it with \( 0 \) subscript. The magnetic vector potential \( A_0 \) whose divergence should be also zero, satisfies vector Laplace equation

\[ \nabla^2 A_0 = 0 \]  

(6)

We apply the second order vector potential concept, which is discussed very fully by Morse, Feshbach [8] and Smythe [9]. \( A_0 \) can be expressed as

\[ A_0 = \nabla \times W \]  

(7)

where \( W \) is called the second order vector potential. If \( A_0 \) satisfies the vector Helmholtz equation, \( W \) should be defined by two scalar parameters which both satisfy the scalar Helmholtz equation. Because formula (6) is a vector Laplace equation, just one scalar parameter, which satisfies scalar Laplace equation, is sufficient. This idea was discussed by Hammond [10] based on the work of Kriezis and Xypteras, who very elegantly applied the method to an eddy current problem. They expressed \( W \) in free space as [11]

\[ W = e \psi \]  

(8)

where \( e \) is unit vector parallel to suitably chosen coordinate axis. \( \psi \) obeys scalar Laplace equation, thus it can be treated as a scalar potential. Therefore, it can be expressed in terms of imaginary charge density \( \sigma_0 \) on the union of all conductor surfaces \( S_{all} \)

\[ \psi(\xi) = \frac{1}{4\pi} \int_{S_{all}} \frac{1}{R(\xi, \eta)} \sigma_0(\eta) \, dS \]  

(9)

where \( \xi \) is the observation point in free space, \( \eta \) is the integration point on \( S_{all} \). Then we get
\[ A_0(\xi) = \nabla_\xi \times (e \psi) = -\frac{1}{4\pi} \int_{S_{all}} e \times \nabla_\xi \cdot \left( \frac{1}{R(\xi, \eta)} \right) \sigma_0(\eta) dS \]  
\text{(10)}

We impose following boundary conditions on the surface between the conductor \( i \) and free space \( 0 \)
\[ t^k \cdot (E_i - E_0) = 0 \quad k = 1, 2 \]  
\text{(11)}
\[ n \cdot J_i^f = i\omega \rho = i\omega n \cdot (\xi E_i - \epsilon_0 E_0) \]  
\text{(12)}

where \( n \) is the outward normal unit vector on conductor surface, \( t^1 \) and \( t^2 \) are unit vectors on the tangential plane of conductor surface and perpendicular to each other. \( J_i^f \) is real current density in the conductor. \( \rho \) is the free charge density on the surface.

Following relationship exists in the conducting and non-conducting regions
\[ E = -j\omega A + E^c \]  
\text{(13)}

\( E^c \) is the electric field caused by the surface charge. Because the surface charge can not change the tangential component of \( E^c \) across the surface between the conducting and non-conducting regions, we have
\[ t^k \cdot (E_i^c - E_0^c) = 0 \quad k = 1, 2 \]  
\text{(14)}

Considering (13) and (14), the boundary condition (11) becomes
\[ t^k \cdot (A_i - A_0) = 0 \]  
\text{(15)}

The right hand side in (12) is much less than \( J_i^f \), even though the frequency is as high as 10G Hz (\( \omega \epsilon < \sigma \)). \( E^c \) on the conducting side of the surface is -\( j\omega A_i^f \). So, the boundary condition (12) becomes
\[ n \cdot (A_i + A_0^f) = 0 \]  
\text{(16)}

It should be noted that \( A_i^f \) appears in (16) only when the boundary is one of the two equipotential end surfaces of the straight conductor. Only at that time, the direction of \( A_i^f \) is not perpendicular to \( n \), and it is actually parallel to \( n \).

The system of the equations (4)(6)(15)(16) implies the Coulomb gauge [12], and the expression of (5) is the solution to this system of the equations. Therefore, \( \nabla \cdot (A_i + A_0^f) = 0 \). This is consistent with the fact that no free charge exists in the conductors.

Moving the observation point \( \xi \) in (5) and (10) to the boundary, we obtain the expressions of \( A_i \) and \( A_0 \) on the surface of conductor. The detail process is shown in the Appendix. Substituting these expressions into (15) and (16), we obtain
\[ t^k (\xi) \cdot (\mu \int_{S_i} K(\xi, \eta) J_j(\eta) dS_j - A_i^f(\xi)) + \int_{S_{all}} e \times \nabla G(\xi, \eta) \sigma_0(\eta) dS - \frac{\sigma_0(\xi) e \times n(\xi)}{4} = 0 \]  
\text{(17)}
\[ n(\xi) \cdot (\mu \int_{S_i} K(\xi, \eta) J_j(\eta) dS_j - A_i^f(\xi)) = 0 \]  
\text{(18)}

\( A_i^f \) is an extra unknown associated to one conductor, and this needs an extra constrain. Applying Ampere’s law can provide an extra equation about one conductor [4]
\[ \oint_{l_i} \nabla \times A_i \cdot dl = \mu I_i \]  
\text{(19)}

where \( l_i \) is a close contour of cross-section of the conductor \( i \). \( I_i \) is the measurable current flowing in the conductor.

3. Discretization of 3-D Omni-A model

The three-component boundary element method (minimum-order BEM) can be applied [2]. The boundary integral equations (17) (18) (19) of the 3-D Omni-A model can be discretized into the boundary element equations. Assuming there are total \( m \) elements on all of the surfaces, and a conductor \( l \) has \( m \) surface elements, we have
\[ \sum_{j=1}^{m} e \times \nabla G_{ij} \sigma_{0j} dS_j - \frac{\sigma_{0j} e \times n_j}{4} = 0 \]  
\text{(20)}
\[ n_j \cdot (\sum_{j=1}^{m} K_{ij} J_j dS_j - A_i^f) = 0 \]  
\text{(21)}
\[ \sum_{i=1}^{m} \Delta l_i (\sum_{j=1}^{m} J_j \times \nabla K_{ij} dS_j) = -n_{\text{contour}} \mu I_i \]  
\text{(22)}

where the subscript \( i (i=1, \ldots, m) \) denotes the observation point which is the center of an element located on the surface of conductor \( l \). The subscript \( j (j=1, \ldots, m \text{ or } 0, \ldots, m) \) denotes the source elements on which the integration
is performed. $\Delta l_i$ are the lines which can be connected each other to form $n_{\text{contour}}$ contours of the cross sections of discretized conductor $l$. $I_l$ is the measurable current in conductor $l$.

When we adopt the constant element, the imaginary surface current density can be expressed as

$$J_j = J_{uj} u_j + J_{vj} v_j$$ (23)

where $u_j$ and $v_j$ are any two perpendicular unit vectors lying on the $j$th element. So, on each element, we have three unknowns: $J_{uj}$, $J_{vj}$, and $\sigma_0 j$. We also have three equations (20) (21) for three perpendicular directions at each element’s center. In each conductor, there is one extra unknown: $A'_m$, still we have one measurable current equation (22) to balance the unknowns.

To calculate the impedance matrix, one conductor $n$ is driven by known measurable current $I_n$, and the measurable currents of the rest conductors are set to be zero. After solving linear equations system from (20), (21), and (22), we obtain the constant magnetic vector potential $A'_m$ of conductor $m$, the induced voltage in this conductor $V_m$ can be calculated by

$$V_m = E \cdot A'_m = -j\omega A'_m \cdot L$$ (24)

where $L$ is the length of the straight conductor. Therefore, one component of $n$th column in the impedance matrix is

$$Z_{mn} = \frac{V_m}{I_n} = R_{mn} + j\omega L_{mn}$$ (25)

4. The conductors perpendicular to each other can have mutual impedance

The coupled circuit method like the one implemented in [1] calculates mutual inductance between filaments $i$ and $j$ by

$$L_{ij} = \frac{\mu}{4\pi a_i a_j} \int V_i \int V_j \frac{l_i \cdot l_j}{l_i - r_j} dV_i dV_j$$ (26)

where $l_i$, $l_j$ are unit vector along axes of filaments $i$, $j$. $a_i, a_j$ are the areas of the cross section of these filaments. $V_i, V_j$ are the volumes of these filaments.

If filament $i$ and $j$ are located in two conductors perpendicular to each other, $L_{ij}$ will be zero, and the $R_{ij}$ is always zero in this method when $i \neq j$. This will cause the mutual impedance between these two perpendicular conductors to be zero.

In the new method, the current density in conductor $i$ is expressed as

$$J_i' (\xi) = -j\omega \sigma \mu \int_{S_i} K(\xi, \eta) J_j (\eta) dS_j + j\omega \sigma A'_j (\xi)$$ (27)

It can be along arbitrary direction. We can conclude from the above expression that directions of the currents, which flow in two perpendicular conductors, are not always perpendicular to each other. Therefore, the mutual impedance may exist. When the frequency increases and the gaps between the conductors decrease, we can’t ignore these mutual effects as before. These mutual effects become more important when we consider the fact that a conductor may cross many other conductors in real world, such as in the ULSI.

For an empirical evidence, we can consider two square conductor planes, one over another with small gap. It can be said they are perpendicular to each other, but certainly they have mutual effect throughout the magnetic field. The coupled circuit method will face the problem how to divide the conductors into the filaments to describe the eddy currents.

5. Results

5.1 Analysis of a conductor with round cross-section to validate the new model

We analyze the first example in [5], because there exists an analytical solution when the conductor has infinite length. The conductor is made of aluminum with a conductivity of $3 \times 10^7$ mho/m. It has a radius of 1 cm. The conductor is supplied with a 60 Hz measurable current of 1000 A. Table 1 shows the surface current density, comparing our result with the analytical solution. Our result approaches to the analytical solution when the length of conductor increases.

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Disk 15</th>
<th>Disk 20</th>
<th>Disk 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretization (perimeter x axis)</td>
<td>8x8</td>
<td>8x8</td>
<td>8x12</td>
</tr>
<tr>
<td>Our result (kA/m²)</td>
<td>3125</td>
<td>3159</td>
<td>3278</td>
</tr>
<tr>
<td>Analytical result (kA/m²)</td>
<td>3265 (infinite length)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen that the difference between the solutions is within 0.4% when the length is 25 cm.

5.2 Analysis of two round cross-section conductors perpendicular to each other
To illustrate that the mutual impedance between two perpendicular conductors may exist, and it can not be ignored when the frequency increases and the gap between them decreases, we calculate the impedance matrix of two conductors perpendicular to each other.

These two conductors have radius of 0.1 µm, length of 0.5 µm. Table II gives the impedance matrix when the frequency is 1G Hz, and the gap between them is 0.1 µm. Table III gives the impedance matrix when the frequency is 10G Hz, and the gap between them is 0.05 µm.

Table 2 Impedance matrix when \( f = 1 \text{G Hz} \) and gap = 0.1 µm

<table>
<thead>
<tr>
<th>( Z_{ij} )</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.75e-4 + 9.52e-3 j</td>
<td>7.14e-7 - 2.46e-7 j</td>
</tr>
<tr>
<td>2</td>
<td>7.14e-7 - 2.46e-7 j</td>
<td>8.75e-4 + 9.52e-3 j</td>
</tr>
</tbody>
</table>

Table 3 Impedance matrix when \( f = 10 \text{G Hz} \) and gap = 0.05 µm

<table>
<thead>
<tr>
<th>( Z_{ij} )</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.93e-2 + 7.32e-2 j</td>
<td>7.46e-4 + 7.57e-4 j</td>
</tr>
<tr>
<td>2</td>
<td>7.46e-4 + 7.57e-4 j</td>
<td>2.93e-2 + 7.32e-2 j</td>
</tr>
</tbody>
</table>

It can be seen that although the mutual resistance in Table 2 is about 0.08% of self-resistance, in Table 3 the mutual resistance is about 2.55% of self-resistance. Considering that a conductor may cross many other conductors, we can imagine that ignoring mutual impedance may cause great error in the deep submicron chips working at high frequency.

5.2 Some discussions

The skin depth in fist case is \( \frac{1}{f \pi \mu \sigma} \), where \( f \) is the frequency, \( \mu \) is the permeability, and \( \sigma \) is the conductivity. The ratio of the skin depth to the radius is 1.2 cm. The skin depth in second case is about 0.9 µm, the ratio of the skin depth to the radius is 9. It can be seen that although the frequency in first case is only 60 Hz, the skin effect is much stronger than the second case. This fact means the skin effect in the deep submicron chips is not so strong as we imagined. Therefore, the proximity effect is worthy of more attentions. In other words, the mutual impedance between the conductors should be considered very carefully, even though the conductors are perpendicular to each other.

6. Conclusions

A 3-D Omni-A model to calculate impedance matrix of interconnect is proposed in this paper. The computational method based on this model could be more accurate and efficient, compared with the coupled circuit method. There are two reasons. First, it is a kind of boundary element method, so it could be more efficient. Second, it can describe current density in any direction, so it could be more accurate in the deep submicron applications, which can be classified as one kind of eddy current problem. We show that the mutual impedance between perpendicular conductors can not be ignored in the deep submicron applications.

In this paper, we ignore the effects of the free charge when we calculate the inductance and resistance. This only means that the capacitance has very little influence to the inductance and resistance, even when the frequency is as high as 10G Hz. But its influence to the electric signals can not be ignored.

In the ULSI, although the dielectrics have different permittivities, their permeabilities are nearly the same as the permeability of free space. Therefore, we need to consider multi-media only when we calculate the capacitance. Since the scale of the decoupled problem would be much smaller than that of the coupled one, we should decouple the calculation of the capacitance from the calculation of frequency-dependent inductance and resistance as possible as we can.

Since we mainly show the potentials of 3-D Omni-A model, our future work will focus on the comprehensive comparisons with other methods in the aspects of the accuracy and efficiency.

Appendix

In this section, we show the process to obtain the expression of \( A_0(\xi) \) when \( \xi \) is on \( S_{all} \). We combine the non-conducting region with a sphere with small radius \( e \), which takes \( S_0 \) on \( S_{all} \) as the center. The surface of combined region has two parts: \( S_e \) and \( S_0 \). \( S_0 \) is the partial surface of the sphere, which protrudes into conducting region. When \( \epsilon \rightarrow 0 \), \( S_e \) will approach to a semi-sphere surface. \( S_0 \) is the same as \( S_{all} \) except the small hole caused by the sphere. In the combined region, formula (10) is valid, because \( \xi \) is located inside the combined region. When \( \epsilon \) approach to 0, the combined region and its bounding surface are resumed to the original non-conducting region and \( S_{all} \). Therefore, \( A_0 \) at the surface point \( \xi \) can be expressed as

\[
A_0(\xi) = \lim_{\epsilon \to 0} \frac{1}{4\pi} \int_{S_e} e \times \nabla_\xi \left( \frac{1}{R(\xi, \eta)} \right) \sigma_0(\eta) dS
\]

\[
- \frac{1}{4\pi} \lim_{\epsilon \to 0} \int_{S_0} e \times \nabla_\xi \left( \frac{1}{R(\xi, \eta)} \right) \sigma_0(\eta) dS \tag{A1}
\]
\[
\frac{1}{4\pi} \int_{S_{\epsilon}} \text{Limit}_{\epsilon \to 0} \epsilon x \left( \frac{\xi_x - \eta_x}{R(\xi, \eta)^3}, \frac{\xi_y - \eta_y}{R(\xi, \eta)^3}, \frac{\xi_z - \eta_z}{R(\xi, \eta)^3} \right) \sigma_0(\eta) dS
\]

(A2)

Formula (A2) can be changed into the spherical coordinate system, in which \(\xi\) is taken as the origin, the \(z\) axis is along the normal direction of \(S_{all}\) at \(\xi\), outward to the non-conducting region.

\[
\frac{1}{4\pi} \sigma(\xi) e \times \int_0^{2\pi} \int_0^\frac{\pi}{2} \frac{e^2 \sin \theta \cos \phi}{e^3} \frac{e^2 \sin \theta \sin \phi}{e^3} \frac{e^2 \cos \theta}{e^3} \frac{\pi}{2} d\phi d\theta
\]

\[
= \frac{1}{4\pi} \sigma(\xi) e \times (0,0,\pi) \quad (A3)
\]

Noting that \((0,0,1)\) is the unit normal vector of \(S_{all}\) at \(\xi\), outward to the non-conducting region, we denote it as \(n(\xi)\). Now formula (A1) becomes

\[
A_0(\xi) = \frac{1}{4\pi} \int_{S_{all}} e \times V_\xi \left( \frac{1}{R(\xi, \eta)} \right) \sigma_0(\eta) dS
\]

\[
+ \frac{1}{4} \sigma(\xi) e \times n(\xi) \quad (\xi \in S_{all}) \quad (A4)
\]

Following similar process, the expression of \(A_i\) at the surface point \(\xi\) can be obtained.

References


