The Enhancing of Efficiency of the Harmonic Balance Analysis by Adaptation of Preconditioner to Circuit Nonlinearity

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Abstract - Krylov subspace techniques in harmonic balance simulations become increasingly ineffective when applied to strongly nonlinear circuits. This limitation is particularly important in the simulation if the circu it has components being operated in a very nonlinear region. Ev en if the circuit contains only a few very nonlinear components, Krylov methods using standard preconditioners can become ineffective. To overcome this problem, we present two adaptive preconditioners that dynamically exploit the properties of the harmonic balance Jacobian. With these techniques we have been able to retain the advantages of Krylov methods even for strongly nonlinear circuits. Some numerical experiments illustrating the techniques are presented.

I. Introduction

Harmonic balance (HB) [1-3] is a widely used method for steady-state analysis of nonlinear circuits in the frequency domain. Krylov subspace iterative methods have greatly increased the usefulness of harmonic balance [3-5]. The main advantage of applying Krylov subspace techniques in HB is due to the use of matrix implicit operations for evaluating of matrix-vector products [3]. Unfortunately, the convergence and therefore efficiency of Krylov subspace techniques critically depends on the proper choice of a preconditioner [6].

The simplest preconditioner applied in HB method uses the block-diagonal portion of the harmonic Jacobian. This block-diagonal (BD) preconditioner is an effective and efficient choice for large number of weakly nonlinear problems arising in practice [3,5]. However, it becomes insufficient for strongly nonlinear problems. This can be explained that BD preconditioner becomes poor approximation of harmonic Jacobian.

The computational efforts of harmonic balance with Krylov subspace techniques and preconditioners are determined by two factors: the number of linear iterations of iterative algorithm and the cost of preconditioner. More complex nondiagonal preconditioner reduces the number of linear iterations. Therefore depending on the relation between the cost of linear iterations and the cost of preconditioner we can expect the reduction of computational efforts in strong nonlinear problems which require many linear iterations.

In this paper we present two techniques for constructing the adaptive preconditioner at every Newton iteration. The first technique dynamically exploits the properties of the harmonic Jacobian at every Newton iteration. The second technique essentially exploits the estimate of circuit nonlinearity at every Newton iteration. We have proposed this estimation and the approach that allows to determine the number of harmonics for all variables at every Newton iteration in [7]. In this paper we apply this approach for constructing an adaptive preconditioner.

.II. Preconditioning in HB Problems

Applying Newton method to solve the nonlinear circuit equation formed in the frequency domain leads to the following linear system

$$Jx = b \qquad (1)$$

to be solved at every Newton iteration [1].

Here J is harmonic Jacobian, b is right hand side (RHS) vector and x is Newton's update.

The harmonic Jacobi matrix in the frequency domain has block structure: each block defined by numbers k, l (block row and block column) contains N by N matrix where N is the number of circuit variables. The diagonal block and the non-diagonal block has the following form [1]:

$$J_{kk} = Y^{(k)} + G^{(0)} + jk\omega C^{(0)} \qquad (2)$$

$$J_{kl} = G^{(k-l)} + jk\omega C^{(k-l)} \qquad (3)$$

respectively. Here G, C, Y are matrices of harmonics of conductances, capacitances and admittances of linear elements respectively.

The linear system (1) is large and therefore the iterative methods are applied to solve (1). To provide acceptable convergence property of iterative methods some preconditioning schemes are used [6]. Applying the right preconditioner P transforms the original system (1) into the preconditioned system

$$JP^{-1}y = b \qquad (4)$$

where $y = Px \qquad (5)$

Preconditioner P is an approximation of Jacobi matrix J that can be easily invertible. The natural block-diagonal preconditioner applied allows to solve the equivalent linear problem with respect to vector y:

$$JD^{-1}y = b \qquad (6)$$

where $y = Dx \qquad (7)$

The block-diagonal preconditioner has been shown to be very effective for large number of weakly nonlinear problems. However, it becomes increasingly ineffective for strongly nonlinear problems because BD preconditioner becomes poor approximation of harmonic Jacobian.

Therefore we can expect that more complex preconditioner reduces the resulting computational efforts due to essential decreasing of linear iterations with slightly increased efforts of factoring the preconditioner. In fact we expect reduction of efforts due to the redistribution of computational efforts between linear iterations and factoring the preconditioner. For this reason constructing an adaptive preconditioner that dynamically exploits the properties of the harmonic Jacobian at every Newton iteration allows to increase computational efficiency of solving of strongly nonlinear HB problems.

Note that to construct an adaptive preconditioner we exploit a matrix implicit form of Jacobian, i. e. only *G, C, Y* components of Jacobian are stored in the memory.

III. Constructing the Adaptive Preconditioners

A. Adaptively Pruning the Harmonic Jacobian (AP1)

We form the preconditioner by neglecting the small harmonics of conductances and capacitances. This process corresponds to retaining, in addition to the block-diagonal entries, only the most essential nondiagonal entries of Jacobian.

To perform forming we determine the indexes of threshold harmonics for conductances and capacitances separately. At every Newton iterate, the threshold harmonics of conductance htg_{mn} and capacitance htc_{mn} are obtained from the following conditions [8]:

$$\left|g_{mn}^{(h)}\right| < \mu \cdot \sqrt{G_{mn}^2 + \left|y_{mn}(0)\right|^2} \quad \text{at} \quad h \ge htg_{mn} \tag{8}$$

$$\left|c_{mn}^{(h)}\right| < \mu \cdot \frac{\sqrt{G_{mn}^2 + \omega^2 K^2 C_{mn}^2 + \left|y_{mn}(K\omega)\right|^2}}{K\omega} \quad \text{at} \quad h \ge htc_{mn} \tag{9}$$

Here ω the fundamental frequency, K is the maximal specified number of harmonics, G_{mn} and C_{mn} are defined as follows.

$$G_{mn} = \sqrt{\sum_{h=0}^{K} \left|g_{mn}^{(h)}\right|^2} \tag{10}$$

$$C_{mn} = \sqrt{\sum_{h=0}^{K} \left|c_{mn}^{(h)}\right|^2} \tag{11}$$

By such a way the conductance $g_{mn}^{(h)}$ or capacitance $c_{mn}^{(h)}$ is rejected for $h > htg_{mn}$ and $h > htc_{mn}$ respectively. As a result rejecting unimportant Jacobian entries is performed.

The parameter μ impacts on the threshold value. The limiting cases $\mu = 1$ and $\mu = 0$ correspond to the BD preconditioner and full harmonic Jacobian respectively.

Numerical experiments show that it is expedient to choose the value of μ from the interval $[10^{-5}, 10^{-3}]$.

B. Adaptively Estimating the Nonlinearity Degree (AP2)

It is easy to show that the substitution

$$y' = b - y \tag{12}$$

transfer the (6) into the following equation

$$JD^{-1}y' = JD^{-1}b - b \tag{13}$$

This equation is useful for estimating harmonics of variables because components of rhs vector (13) corresponding to linear variables will be zero, and the solution of system (13) contains zeroes for all harmonics of linear variables. It is also expected that for weakly nonlinear variables rhs vector (13) will contain smaller number of essential harmonics in comparison with vector *b*.

Now apply a preconditioner *P* to the linear system (13). We obtain

$$JD^{-1}P^{-1}Py' = JD^{-1}b - b \tag{14}$$

or

$$JD^{-1}P^{-1}z = b' \tag{15}$$

where

$$z = Py' \tag{16}$$

$$b' = JD^{-1}b - b \tag{17}$$

Therefore we would like to solve by iterative solver the linear problem (15) with respect to the vector *z* and with the preconditioner *P* applied.

We will construct the preconditioner *P* as an approach to the product JD^{-1}, i. e.

$$P \approx JD^{-1} \tag{18}$$

In order to obtain approximation we first estimate the number of harmonics of variables and then take into account only those members of product (18) which satisfy the predefined condition.

To determine the number of harmonics of variables we find the minimal total number of harmonics $\sum_{i=1}^{N} K_i$ (K_i, number of harmonics for node *i*) that satisfies to the following inequality [7]:

$$\left\|b - \tilde{b}\right\| \le \|b\| \cdot tol, \tag{19}$$

where *tol* is the given relative tolerance and $\tilde{b}$ is the reduced

RHS vector. The procedure of determination sequentially finds and drops the harmonic with minimal contribution to RHS vector until condition (19) is true. Finally the procedure gives numbers K_i, $i=1,..,N$ where N number of variables.

Then when we form the block of preconditioner defined by numbers k, l we put into P only those entries (i,j) of corresponding block of product (18) for which :

$k < K_i$

$l < K_j$

Therefore this technique allows to construct the preconditioner in the form of approximation (18) presented as block matrix with varying sparseness. Since the RHS (14) gives the estimate of nonlinearity of a circuit [7] then the sparseness of a preconditioner depends on nonlinearity of a circuit. Note that parameter *tol* will control the sparseness of preconditioner: the case *tol=0* corresponds to the dense matrix and the case *tol=1* corresponds to the most sparse matrix.

IV. Experimental Results

The computational efficiency of the first proposed adaptive preconditioner AP1 is demonstrated in comparison with the block-diagonal (BD) preconditioner. The typical dependencies of CPU time (sec) on input signal amplitudes (Volt/Ampere) are shown for Fifth pole filter (Fig.1), rectifier circuit (Fig.2) and amplifier class “C” (Fig.3).

The presented cost curves are obtained by HB analysis using two types of preconditioners: BD and proposed AP1 with $\mu = 0.0001$. The significant difference of slopes can be seen from these figures. If for weakly nonlinear regimes the cost curves are close (or for weakly nonlinear example in Fig.3 AP1-curve is even higher than BD-curve for small amplitudes) then for large input signal the efficiency of AP1 is essentially increased and its advantages become clear.

The advantages of the second proposed adaptive preconditioner AP2 are demonstrated on two examples of strongly nonlinear circuits: rectifier circuit and OpAmp ua741

We characterize the computational efforts of iterative solver by the number of linear iterations of iterative algorithm. The cost of factoring the preconditioner is described by the average number of nonzero elements of preconditioner.

The experimental dependence of number of nonzero elements in preconditioner on parameter *tol* for the rectifier circuit is shown in the Fig. 4. This dependence is monotone and the number of nonzero decreases from 168100 to 4100. The experimental dependence of number of linear iterations is shown in the Fig. 5. For example calculation with *tol=0.01* takes 674 linear iterations and number of nonzero is 77731 or less than 50% of dense matrix. Note that block-diagonal preconditioner (*tol=1*) takes 4390 linear iterations and the number of nonzero is 4100.

The experimental dependencies of number of nonzero elements in preconditioner and number of linear iterations on parameter *tol* for the OpAmp ua741 circuit are shown in the Fig. 6 and Fig. 7 respectively. The calculation with *tol=0.01* takes 311 linear iterations and number of nonzero is 341750 or less than 11% of dense matrix. Note that block-diagonal preconditioner (*tol=1*) takes 1616 linear iterations and number of nonzero is 51301. Therefore the calculation with *tol=0.01* demonstrates five times reduction of number of linear iterations.

The numerical efficiency is obtained for strong nonlinear circuits where the cost of linear iterations is essential due to large number of iterations. The speed up 2.2 and 1.51 is obtained for rectifier circuit and OpAmp ua741 respectively.

V. Conclusion

The opportunity to redistribute computational efforts between preconditioner forming and linear iterations opens additional flexibility to reduce overall simulation time. In many cases little additional complication of the preconditioner allows substantial reduction of the number of linear iterations.

The proposed approach generates an adaptive preconditioner whose density is determined by the degree of nonlinearity of a circuit. In particular the best numerical efficiency is obtained if the large circuit contains a few extremely nonlinear components.

The proposed adaptive preconditioners allow iterative methods to reestablish good convergence properties for such strong nonlinear problems where previously direct methods had advantages compared with iterative.

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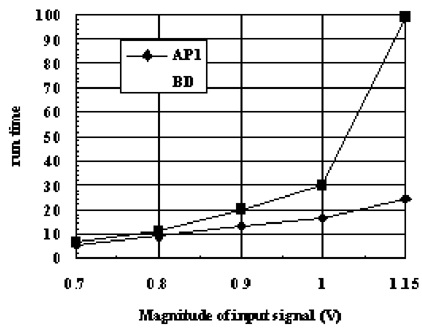

Fig.1 The dependence of run time on input signal (filter)

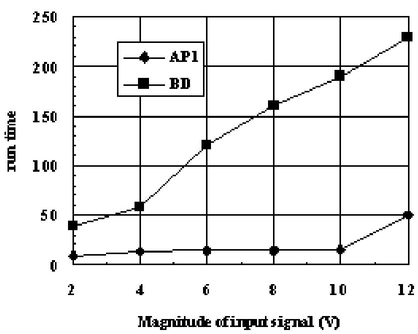

Fig.2 The dependence of run time on input signal (rectifier)

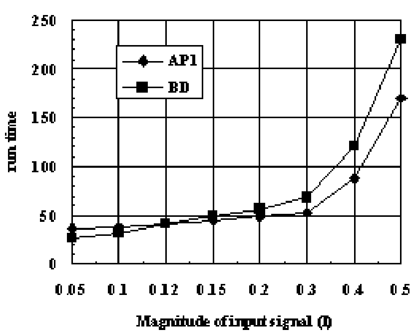

Fig.3 The dependence of run time on input signal (amplifier)

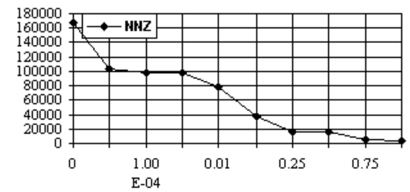

Fig.4 The dependence of number of nonzero elements on parameter *tol*

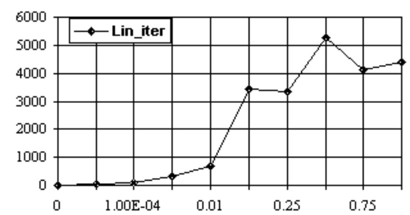

Fig.5 The dependence of number of linear iterations on parameter *tol*

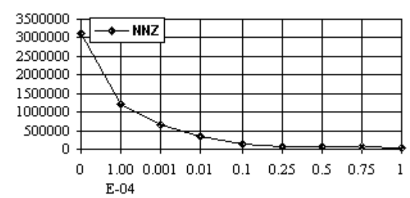

Fig.6 The dependence of number of nonzero elements on parameter *tol*

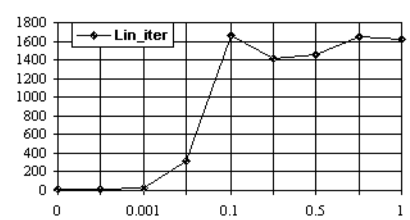

Fig.7 The dependence of number of linear iterations on parameter *tol*