Electromagnetic Parasitic Extraction via a Mulipole Method with Hierarchical Refinement†

Michael W. Beattie and Lawrence T. Pileggi
Carnegie Mellon University
Department of Electrical and Computer Engineering
5000 Forbes Ave., Pittsburgh, PA 15213

ABSTRACT

The increasing interconnect density and operating frequencies of system-on-a-chip (SOC) designs necessitates extraction of parasitic electromagnetic couplings beyond the localized confines of functional design blocks. In addition, SOC design styles and gridless variable-width routing make it increasingly difficult to use precharacterized library shapes for parasitic extraction. A comprehensive capacitance and inductance extraction solution requires a hierarchical data representation and fast runtime algorithms. We illustrate through examples that both the multipole method and hierarchical refinement, which are the two most successful approaches for parasitic extraction to date, work efficiently only under certain, limiting conditions. To improve this situation we present an approach which combines the best of both methods into a concurrent multipole refinement representation of the electromagnetic interaction which is efficient for arbitrary interconnect configurations. We use a generalized formulation of electromagnetic interactions to exploit the similarities in capacitance and inductance extraction for greater efficiency.

I. INTRODUCTION

For on-chip circuitry the performance of a signal path can be dominated by the delay due to parasitic capacitive (and recently inductive) coupling between the interconnect wiring. Since modern chip design pushes the performance of a given technology to its limits, it is necessary to find increasingly more accurate predictions of the delay due to the interconnect. The growing complexity of today’s integrated systems, however, makes this computationally very expensive.

For this reason is has been a common approach to simply segment a given circuit into cells which may be dealt with separately for extraction purposes. This method is useful if the capacitive and inductive interaction is strong within and weak among the cells. Due to the steadily increasing density of on-chip structures, and especially as inductive coupling begins to emerge for parasitic extraction, we may have to consider interconnect parasitics for larger portions of a chip and electromagnetic interactions between cells.

Several approaches have been proposed to address the extraction of huge complex systems. The multipole method, developed by Greengard and Rokhlin [1], exploits the fact that with increasing distance the interaction between larger and larger well-separated sets of point sources (such as charges or currents) may be lumped together. This reduces the computational complexity of finding the potential for a given source distribution to linear time in terms of number of point sources, but without exceeding a given error bound. Shell methods [4] assume that the capacitive or inductive coupling from one section (such as a panel or a filament) to another can be neglected beyond a certain mutual distance. The Green’s function of the system is altered to remove the (negligible) coupling to far-away sections and the remaining coupling terms are modified in a manner that guarantees the stability of the resulting sparse potential or inductance matrix. The complexity of this method is \(O(k^2 n)\), where \(k\) depends on the size of the shell surface chosen.

Unfortunately, both shell and multipole methods assume that we are dealing with either point sources (infinitesimal volume) or filament sources (infinitesimal cross-section). Therefore, in order to accurately capture near field effects between adjacent conductors, a very fine subdivision of conductors is necessary, which increases the computation time. Although the complexity still remains \(O(n)\), the pre-factor is much larger.

Hierarchical refinement is another method of dealing with the same class of problems to which parasitic extraction belongs, namely the solution of Fredholm-type integral equations. This approach has been successfully applied to computationally expensive problems in physics and in computer graphics [6]. It has been reported that this method also has \(O(n)\) complexity [5].

Recently, hierarchical refinement has also been applied to capacitance extraction [7], where a set of conductors is assumed given and the appropriate non-uniform sectioning is found by recursively subdividing the surface until the discretization error drops below a given limit. The discretization error is approximated by an error predictor function that suggests that this error depends only on the potential matrix element between the two interacting sections and on their sizes. While hierarchical refinement was described in [7] as superior to the fast multipole method, we will show that refinement does not maintain \(O(N)\) complexity for systems where the average segment size is much smaller than the overall system size, such as for typical on-chip interconnect systems.

In this paper we present an algorithm that combines the best features of the multipole method (with its advantages for large sy-

† This work was supported in part by the Semiconductor Research Corporation, a grant from Intel Corporation and the National Science Foundation.

1. \(k\) is the average number of neighboring sections within the shell of each section. \(n\) is the total number of sections in the system.
2. \(n\) is number of sections in the system (degree of refinement for a given system).
3. \(N\) is the number of original segments in the system (independent of refinement of the system).
tems) and hierarchical refinement (which captures near field contributions well) in a concurrent top-down/bottom-up extraction approach. We propose, to operate with sections which have a finite size, not with point objects as in [1] and to use hierarchical refinement to efficiently evaluate the near-field coupling between the sections.

We will further show that the interactions are evaluated in linear time for arbitrary interconnect systems. Our new algorithm is based on the construction algorithm for the spatial subdivision tree for the multipole method and the hierarchical refinement procedure for near-field evaluation. We further derive a discrete representation of the electromagnetic coupling within an interconnect system to exploit similarities between capacitance and inductance extraction for higher efficiency.

II. BACKGROUND

A. Electromagnetic Interaction Matrix

In this section we will derive a discrete representation of the general electromagnetic coupling within an interconnect system. Our intent is to establish the similarities between calculating the electrostatic potential matrix \( P \) and the inductance matrix \( L \) using analogous formulas containing similar integrals.

Maxwell’s equations, which govern all electromagnetic interaction of interest for extraction, can be written as inhomogenous wave equations for the electromagnetic potentials [9][pg. 220]:

\[
(V^2 - \frac{1}{c^2}\partial_t^2)\Phi = -\Gamma
\]

\[
\nabla \cdot \hat{A} = \frac{1}{c^2}\partial_t \Phi
\]

where \( \Phi \) is the vector \( (\phi ; A_x ; A_y ; A_z) \) containing the four potential components and \( \Gamma \) is the source vector \( (\rho / \varepsilon ; \mu_jx ; \mu_jy ; \mu_jz) \) containing the charges and currents generating the potentials. Since the magnetic vector potential \( \hat{A} \) is defined via \( \hat{B} = \nabla \times \hat{A} \), the divergence of \( \hat{A} \) can be fixed arbitrarily much like the constant for the electrostatic potential \( \phi \). The solution of (1) is

\[
\Psi^\alpha(\tau, t) = \frac{1}{4\pi c} \int \frac{\Gamma^\alpha_{ \tau} \left[ \tau' - \frac{|\tau' - \tau|}{c} \right]}{|\tau' - \tau|} d\tau'
\]

which is the retarded potential\(^6\). Eq. (3) is valid for cartesian coordinate systems (which we use for clarity in this paper) since we assume that the components of \( \Gamma^\alpha \) are expressed in terms of a basis constant throughout the integration domain. To solve the integral in (3) we discretize the system into sections and average \( \Psi^\alpha \) over each section. Eq. (3) then takes the form of

\[
\psi^\alpha_i(t) = \frac{1}{W_i} \frac{1}{W_j} \frac{1}{\pi^2} \int \frac{\Gamma^\alpha_{ \tau} \left[ \tau' - \frac{|\tau' - \tau|}{c} \right]}{|\tau' - \tau|} dW_i^\alpha dW_j^\alpha
\]

where \( W^\alpha_i \) is the content (panel area or filament cross-section) of section \( i \) for field type \( \alpha \) and

\[
\psi^\alpha_i(t) = (\psi_i(t) ; A_{i,x}(t) ; A_{i,y}(t) ; A_{i,z}(t))
\]

is the vector of average potentials for section \( i \). To handle the time dependency of \( \Gamma^\alpha \) in the integrand we perform a Laplace transform of (4):

\[
\psi_i^\alpha(s) = \frac{1}{W_i} \frac{1}{W_j} \int \int \frac{e^{-s|\tau' - \tau|}}{|\tau' - \tau|} dW_i^\alpha dW_j^\alpha
\]

which uses the Laplace transform translation property

\[
L\{f(t - \alpha)\} = e^{-\alpha s} F(s)
\]

where \( F(s) \) is the Laplace transform of \( f(t) \).

Using a zeroth order Galerkin approach\(^5\) we are implicitly assuming that the source vector \( \Gamma^\alpha \) is spatially constant within the section \( j \). We define the discrete source vector as

\[
\gamma^\alpha_j = (q_j(s)/\varepsilon ; \mu_jx_x \mu_jy ; j ; \mu_jz(s)) \tag{8}
\]

where \( q_j \) and \( j_{j}(x,y,z) \) are the charge and the three current components for this section. We can extract \( \gamma^\alpha_j \) from the integral in (6) and write

\[
\psi^\alpha_j = \left[ K^\alpha \right] \gamma^\alpha_j
\]

by defining the matrices \( K^\alpha \), which in turn form the four diagonal blocks of the electromagnetic interaction matrix \( K \) in

\[
\Psi(s) = K(s) \gamma(s)
\]

where \( \Psi(s) \) and \( \gamma(s) \) are formed by concatenating the four column vectors \( \psi^\alpha_j(s) \) or \( \gamma^\alpha(s) \). The capacitance matrix can be found by inverting \( K^0 \), while \( K^1, K^2 \) and \( K^3 \) represent the partial inductance matrix in three dimensions.

B. Fast Multipole Method

This extraction method is described in detail in [1] and is founded on the observation that with increasing distance from a group of sources, the potential functions \( \Psi^\alpha(\tau, t) \) can be approximated very well by multipole expansions. The description

\[
4. \text{The field type index } \alpha \text{ is zero for the electrostatic component and 1,2,3 for the (x,y,z) inductive components.}
\]

\[
5. \text{Higher order approximations of the source distribution within a section provide more accuracy, but have no fundamental impact on the main topic of this paper.}
\]
in [1] assumes point sources, which implies that the given conductors have been subdivided into sections such that the discretization error is below a given threshold. Error bounds presented in [1] refer to the error introduced by the truncation of the multipole expansion. The error due to the discretization is not considered.

Starting from the entire system of point sources, the bounding box of the system is repeatedly spatially subdivided using a quad–tree structure in 2–D (oct–tree in 3–D) until each childless box corresponding to a leaf in the spatial subdivision tree contains no more than $s$ source points. Each of these localized collections of source points creates a potential, and the far field is approximated by a multipole expansion. The spatial subdivision tree provides the means of defining a neighborhood for each childless spatial box $B$, comprising all other childless boxes which are not well–separated$^6$ from the source points in $B$. The extent of this neighborhood is fixed for each childless box and the multipole approximation accuracy is set by choosing the appropriate order of multipole terms$^7$. Electromagnetic coupling is evaluated directly for points within the neighboring boxes, and via multipole expansions for boxes further away. A hierarchy of increasingly coarser multipole approximations is constructed by translating the child box expansions to the center of the parent box then adding them.

To summarize, it is important to note that the multipole technique starts with the finest possible representation of the system sources and from it builds a coarser and coarser representation of the source distribution. This is done by recursive translation and superposition of the multipole expansions of the children of a spatial subdivision box into the next higher level. The finest necessary subdivision of the system is independently determined prior to the multipole potential evaluation phase.

C. Hierarchical Refinement

The method of hierarchical refinement has been used in many different scientific disciplines (see [6][7]). The starting point is generally the entire (not subdivided) domain within which interaction occurs or, as in [7], each conductor into which the domain is already subdivided.

For each pair of conductors and for each conductor individually, an estimate is calculated of how much the interaction $K_{ij}^\alpha$ would change if the conductor were subdivided. If this error is above a given error threshold, the larger of both conductors is subdivided and the same estimation technique is applied to each of the subdivisions recursively until the accuracy condition is satisfied. In [7], the starting point is a set of $N$ conductors which are to be subdivided. Recursive refinement must be applied to each pair of conductors, leading to $O(N^2)$ time complexity for the data structure construction. In most cases, if the accuracy requirements are chosen reasonably, this will not lead to any refinement, since the majority of conductor pairs are far enough apart. In many cases, however, the error estimate will be so far below the threshold that a grouping of conductors (hierarchical encou-}

ment) seems warranted in order to trade unnecessary accuracy for faster computation, since the interaction evaluation will also have an $O(N^2)$ time complexity in this case. Unfortunately, this would make it necessary to find nearest neighbors to conductors or groups of conductors which would amount to a Delaunay triangulation with $O(N\log N)$ time complexity for a general non–regular system.

We note, however, that this grouping of sources in order to find less accurate but faster computable approximations of the group is a central feature of the multipole technique described in Section B. We observe that the spatial subdivision tree defined neighborhoods for each source point which would be of great use for hierarchical refinement. We also observe that hierarchical refinement assumes the coarsest given representation of the system and builds from there a finer and finer picture of the interactions via hierarchical subdivision of the entire domain. Hierarchical refinement also finds the finest required subdivision of the system according to a given accuracy threshold; this is a prerequisite for the multipole technique.

Our conclusion is that a technique which melds multipole method and hierarchical refinement together to create a concurrent top–down/bottom–up approach to exploit the best of both approaches will lead to a better overall performance for arbitrary interconnect systems.

III. COMBINING THE MULTIPOLe METHOD WITH HIERARCHICAL REFINEMENT

If the average size of a conductor is much smaller than the size of the system, then the application of hierarchical refinement influences only the computation of the potential caused on a certain section by the conductors nearby. The potential caused by all the other conductors further away still has to be evaluated one by one, so that the runtime increases as $O(m^3)$, when the amount of conductors, $m$, in the system is increased. The algorithm still has a $O(n)$ complexity for any given system when the number of subdivisions in the system, $n$, is increased; however, the combined complexity $O(nm^2)$ varies in a quadratic fashion, making hierarchical refinement alone look worse and worse as the system size (and with it $m$) increases. Therefore, we expect (and observe) excellent performance for hierarchical refinement compared with multipole for small, compact examples. For real–world applications, however, the situation is quite different. To construct an algorithm where performance does not deteriorate as that of hierarchical refinement for increasing system size, but which still keeps its advantages, we need to integrate the multipole technique for efficient far field evaluation.

In [1], an algorithm for the multipole technique is outlined which sorts a given set of $N$ point sources into a spatial subdivision tree so that in the resulting structure each childless box contains not more than $s$ source points. We are proposing, however, to operate with sections which have a finite size, not with point objects. The algorithm we present in the following paragraphs starts with $N$ sections and sorts them into the hierarchy of boxes of the spatial subdivision tree. This tree is created concurrently in such a manner that eventually each box of the spatial subdivision

---

6. Defined in [1]; A point is well–separated from collection of sources or their bounding sphere if the distance between the bounding sphere and the point is larger than the diameter of the bounding sphere.

7. The opposite approach, that is fixing the expansion order and increasing the direct coupling distance as necessary for accuracy is valid, too and applied in the hierarchical refinement method in the next section.
tree is associated only with sections of similar size (or smaller for childless boxes). That is, each set of sections associated with a node in the spatial subdivision tree forms a compact system of conductors. This system is now (together with the sections in the neighborhood of the current box) subjected to the algorithm suited best for this constellation, namely hierarchical refinement, to prepare the structure for the near field evaluation (which is found by direct calculation in [1]). We also construct refinement structures between the sections in the current box and all sections within the neighborhoods of the ancestors of the current box, because the conductors in the current box are in turn within the descendants of the neighborhood of all these conductors and therefore not well separated.

Figure 1: Top view of a 2d system after Step 1. The sections (rectangles) and the bounding box have been created by a different process.

Figure 2: Top view of system after Step 2 with \( s = 4 \). Root box sections (light grey); level 1 box (medium grey); level 2 box (dark grey).

The time complexity of the near field evaluation is now only linear (as opposed to quadratic for the direct method used in [1]). The multipole expansion of each box, on the other hand, captures the far field interactions between well-separated groups of conductors. Its potential evaluation step has a linear complexity in terms of the number of sections in the interconnect system. Therefore, the complexity of the new algorithm will be linear.

A. Construction of the Spatial Subdivision Tree

Our algorithm begins with construction of the data structure which contains the coupling coefficients for the electromagnetic interaction in a hierarchical manner and allows for the evaluation of the coupling in linear time.

Step 1: All root sections in the system are constructed. Segments are read either from a description file or extracted from layout and then decomposed into simpler sections. For our examples these are either filament boxes or rectangular surface panels. The bounding box of the system is determined in parallel and its dimensions define the root box of the spatial subdivision tree (root box of space tree) (see Fig. 1).

Step 2: Each originally created root section is associated by the \texttt{insert()} algorithm in Fig. 3 with \texttt{(linked to, put within)} one box of similar size or to one childless box (leaf box) which is larger than that section. Since sections can intersect neighboring boxes, we assign each section to that box which overlaps with most of the content of that section.

The parameter \( s \) in Fig. 3 controls the number of root sections in each box. This parameter has little influence on the insertion procedure if the average section size is comparable or larger than the average distance, since most conductors are then linked permanently to some box of similar size. The smaller the average size becomes with respect to the average distance (increasingly point-like sources), the more influence \( s \) has since most conductors are stored only temporarily until a leaf box is subdivided. This subdivision is deferred until \( s \) conductors have accumulated in a leaf box. Therefore, \( s \) limits the depth growth of the space tree when our new algorithm starts to behave like the multipole method described in [1], but does not influence the result if our algorithm is used on compact systems where the refinement component dominates. The larger \( s \) is, the slower is the depth growth of the space tree. As a result, increasing the value of \( s \) diminishes the impact of the multipole component on the performance of the new algorithm; for \( s = \infty \) the multipole evaluation is practically disabled and the entire system is represented by the hierarchical refinement structure.

In the course of this algorithm the root box is subdivided as necessary until all boxes contain only root sections of similar size and the leaf boxes contain at most \( s \) sections which are significantly smaller than the leaf box they are linked to (see Fig. 2). Within each box there is now a compact system of conductors (average size comparable or larger than average distance) so we will reduce the near field evaluation complexity to linear from quadratic when we use the hierarchical refinement algorithm instead of the brute force point-to-point potential evaluation between sources within a neighborhood as in [1][Ch. 3].

Step 3: We use hierarchical refinement to construct an efficient representation of the near field coupling between root sections in a box \( B \) and all root sections which are not well separated from the root sections in \( B \). For every box \( B \) in the space tree we refine every root section linked to \( B \) with every other root section which is linked to a box in the neighborhood of \( B \) or the neighborhood of an ancestor of \( B \) (see Fig. 4). Couplings to root sections outside the neighborhoods of \( B \) and its ancestors will be

8. The neighborhood of a box \( B \) is \( B \) itself, all boxes adjacent to \( B \) and of same size and all leaf boxes adjacent to \( B \) of larger size than \( B \).

9. To be within a box means to be linked to it; to be within a neighborhood of a box means to be within a box in that neighborhood. A section \( E \) linked to a descendant box \( D \) of a box \( B \) is not within \( B \), but within \( D \), although \( E \) is physically inside the spatial confines of both boxes \( B \) and \( D \).
B. Multipole Potential Evaluation with Refinement

We next provide the procedure to evaluate the potential in each electromagnetic interaction as described in the previous section. This algorithm refines the given root sections and the spatial subdivision tree. Let \( B' \) be a box to which one such smaller section is linked. The current box \( B \) is within the neighborhood of an ancestor of \( B' \) and hierarchical refinement will be applied as described above once \( B' \) is the current box.

The refinement proceeds recursively until the error condition is satisfied. Alternatively, the recursion can also be stopped once the two current sections are far enough apart to have the multipole expansion handle their coupling as far field component during the evaluation phase.

\[
\text{insert}(\text{section } E, \text{ box } B, \text{ int } s) \{ \\
\quad \text{if (size}(E) \text{ similar to size}(B))} \\
\quad \quad \text{link}(E \text{ to } B \text{ ; permanently}) \\
\quad \text{else /* } E \text{ smaller than } B \text{ */} \\
\quad \quad \text{if (children of } B \text{ exist)} \\
\quad \quad \quad \text{ } C = \text{ child of } B \text{ which contains most of } E \\
\quad \quad \quad \text{insert}(E, C, s) \\
\quad \quad \text{else /* } B \text{ is leaf box */} \\
\quad \quad \quad \text{if (s or more sections in } B) \\
\quad \quad \quad \quad \text{create children of } B \\
\quad \quad \quad \quad \text{for (each section } T \text{ temporarily linked to } B) \\
\quad \quad \quad \quad \quad \text{ } C = \text{ child of } B \text{ which contains most of } T \\
\quad \quad \quad \quad \quad \text{insert}(T, C, s) \\
\quad \quad \quad \quad \text{unlink}(T \text{ from } B) \\
\quad \quad \text{endif} \\
\quad \quad \text{C = child of } B \text{ which contains most of } E \\
\quad \quad \text{insert}(E, C, s) \\
\quad \text{else /* less than } s \text{ sections in } B \text{ */} \\
\quad \quad \text{link}(E \text{ to } B \text{ ; temporarily}) \\
\quad \text{endif} \\
\text{endif} \\
\}
\]

Figure 3: Algorithm for \text{insert()} which recursively locates all sections into boxes which are of similar size as the section or into the leaf boxes.

Result: This algorithm refines the given root sections and the spatial subdivision tree as necessary. Ultimately, a structure is created which represents near field couplings between root sections by an efficient hierarchical refinement tree while allowing the evaluation of far field couplings via the multipole representation. The direct interaction is evaluated through hierarchical refinement for all pairs of conductors which are not well-separated, indicated by the cone-like area in the spatial subdivision tree in Fig. 5. The vertical axis shows the level within the space tree to which the sections are linked. Since for typical on-chip interconnect most root sections will be linked to the lower levels of the space tree (section size much smaller than system size), most couplings will be covered by the multipole component of the algorithm.

B. Multipole Potential Evaluation with Refinement

After the construction of the hierarchical representation of the electromagnetic interaction as described in the previous section, we next provide the procedure to evaluate the potential in each section from a given source distribution throughout the system. As suggested in the sections above, this procedure combines the characteristics of the evaluation for the multipole method and hierarchical refinement. We used the (3d generalization of the) adaptive multipole algorithm as described in [1][Ch. 2.5] as the starting point for the new algorithm. The use of hierarchical refinement enabled us to simplify the evaluation somewhat (the coupling contributions from lists \( W_b \) and \( X_b \) are now evaluated via the refinement part of the tree).

Figure 4: Evaluation of the interaction for the white section \( B \) in the lower right corner. Via far field multipole expansions (light grey); refinement with sections within neighborhood of \( B \) (black); refinement with sections in neighborhood of parent of \( B \) (medium grey); refinement with sections in neighborhood of grandparent of \( B \) (= root box) (dark grey).

Step 1: We start with the multipole evaluation part of the algorithm to find the contribution from the far field interaction. This is similar to Step 2 in [1][Ch. 2.5] (Step 1 has been modified and described in Section A). The multipole expansion for an arbitrary box can be found recursively by adding the translated multipole expansions from the child boxes as in [1] plus the multipole expansions of the root sections linked to that box. The shifting procedure for multipole expansions is given in [1][Ch. 3]. To create the multipole expansion of a root section, we consider the leaf sections of its refinement tree to be point-like. This is reasonable, since we have (through our choice for the refinement accuracy) indirectly determined a upper bound on the resolution for the source density. We can rearrange the leaf section source without changing the potential significantly. The multipole construction proceeds from the leaf boxes to the root box. Note that the multipole expansion of a box defined in this manner contains only contributions from root sections which are linked to that box or its descendants, but not from root sections which are linked to higher level boxes and which happen to intersect this box.

Step 2: Once the multipole expansions for all space boxes are found, the steps 4 and 7 can be processed as given in [1][Ch. 2.5] to find the local expansions of the far fields for all boxes in the space tree. The list \( V_b \) of boxes which contribute to the local expansion of a box \( b \) is defined here as in [1]: all children of the colleagues of \( b \)'s parent which are well-separated from \( b \).

The local expansions we calculate are, however, different from those in [1] since the multipole expansions of the boxes \( V_b \) do not contain the potential of all conductors intersecting the \( V_b \), but only the potential of the root sections linked to the \( V_b \) and their descendants. These are all root sections which are mainly located within the boxes of \( V_b \), and are not larger than those boxes. Adding these reduced potential contributions to the local
expansion of \( b \) we find that the local expansion of any box \( b \) contains the contributions of all conductors which are well-separated from \( b \). Since the hierarchical refinement tree contains all couplings between the root sections linked to \( b \) and those conductors which are not well-separated from \( b \) (see Fig. 5), this definition of the local expansion complements the hierarchical refinement and makes the potential computation complete.

**Step 3:** For this reason we need to evaluate the local potential expansions for each root section in a box \( b \). For each box \( b \) we find all leaf sections of all root sections linked to \( b \) and evaluate the local expansion of \( b \) at the center of each leaf section. The result is added to the total potential of the leaf section.

**Step 4:** Steps 3, 5 and 6 in the algorithm in [1][Ch. 2.5] are now replaced by the evaluation of the hierarchical refinement tree which is similar to the procedure presented in [7]. All sections which are not well-separated from the box \( b \) are contained in the lists \( U_b, W_b, X_b \). The source values are given for the leaf sections of all root sections within each box \( b \). Traversing the hierarchical refinement tree upwards for each root section in \( b \) we calculate the source for a section by accumulating the source from each of its children. During the subsequent downward traversal of the refinement tree for each root section we evaluate the near field potential to this section. Then we add the potential of each section to all of its children, where the leaf sections contain initially the far field potential calculated in the previous step and all other sections are initially zero. In this manner we evaluate the direct coupling via the efficient hierarchical refinement tree rather than by the direct evaluation technique used in [1]. This procedure simplifies the near field evaluation with respect to the pure multipole technique, since all near field contributions are now computed through the same refinement tree rather than three separate techniques for the lists \( U_b, W_b, X_b \), which are also much more difficult to construct than the simple neighborhood box set (see Footnote 8).

**Result:** Each leaf section contains now the initial source value and the resulting potential value. This procedure calculates \( \Psi(s) = K(s) \hat{T}(s) \) where \( K(s) \) contains both the electrostatic potential matrix \( P \) and the inductance matrix \( L \). The construction of the neighborhood list and \( V_b \) is less costly than the previously needed lists \( U_b, V_b, W_b \) and \( X_b \). The local potential expansion complements the refinement data structure and completes the evaluation of the potential.

![Figure 5](image.png)

**Figure 5:** The sections from the 2D example in Fig. 1 are shown with the greyscale code from Fig. 4 indicating the potential evaluation method (refinement, multipole). The level in the picture corresponds to the level in the spatial subdivision tree to which the sections are linked (root box is uppermost level). The arrows illustrate the cone–like expansion of the part of the subdivision tree for which the coupling is evaluated by using hierarchical refinement.

### IV. Computational Results

To illustrate the efficacy of our new hierarchical potential evaluation algorithm we implemented the fast multipole method (with point–to–point near field evaluation) and the refinement method for comparison with our approach. We applied these methods to three examples to demonstrate the performance during construction of the hierarchical data structure and during capacitance and inductance extraction under different conditions. We used an IBM AIX 4.1.5 workstation with a 233MHz PowerPC 604e and with a memory limit of 128 MByte for all examples.

**A. Crossing Bus Example**

The example structures which have been given in [1] and [7] to show the efficiency of the pure multipole and pure refinement methods differ greatly in compactness. In [1] all conductors given to the multipole evaluation routine are assumed to be much smaller than the average minimal distance between all conductors in the system (point sources). In [7] the conductors are approximately of the size of the entire system. The average minimal distance is here much smaller the objects given to the refinement procedure.

For comparison, we have computed the capacitance matrix for the example with 4x4 crossing bus lines from [2][7] (Width, Height and Spacing 1 µm, Length 9 µm). In Table 1 the lines were not segmented before extraction. For Table 2 the lines were initially segmented into 50 units each in length direction. \( T_{Setup} \) is the time necessary to create the hierarchical data structure; \( T_{Eval} \) is the time necessary to compute \( Pq; T_{Invert} \) is the iterative inversion time.
In Table 1 the hierarchical refinement method works optimally since the conductors are much larger than their average minimal distance. The amount of coupling terms stored in the refinement tree structure only depends linearly on the size of the system. The multipole technique, however, has in this case a comparatively much larger runtime. Our new approach has only a minimal overhead compared to the efficient refinement procedure.

For Table 2 the segment size is significantly smaller than the system size. This situation is frequently encountered in on-chip circuitry, since single nets are composed of many segments which are much smaller than the entire net they belong to. In this case, most calls to the recursive refinement procedure return immediately, since most segment pairs are already far apart with respect to their size. As a result, the number of nodes in the refinement tree which are directly coupled to one another increases quadratically with the number of segments. This causes the large runtime and memory consumption seen in Table 2 for refinement. Multipole evaluation is much more efficient for the segmented system since the far couplings are computed in linear time using the hierarchical multipole representation. Our new approach is much faster than even the pure multipole method for segmented lines, since it requires less spatial subdivision layers and uses the more efficient refinement method for near field modeling rather than direct potential evaluation.

Table 3 contains a comparison of the partial inductance results for the same crossing bus structure which were calculated by FastHenry [3] and with our new hierarchical approach which also yields inductance between orthogonal lines due to eddy currents. The error in Table 3 is for inductances between parallel wires only.

![Figure 6](image-url)

**Figure 6:** Time needed to set up hierarchical structure.

In our example we fixed the segment size to roughly 1.7 µm to investigate the memory and time demands of each of the three evaluation methods while the system size is growing, but not the minimal feature size. For short bus lengths the system is similar to the compact examples from [7] while for very long bus structures the system begins to look like the examples from [1]. The number of segments and panels in the system increases linearly with the bus length (see Fig. 6).

Fig. 6 shows the time spent for each of the methods in the setup phase where the hierarchical representation of the interconnect coupling is generated. For the multipole method the setup time reflects only the time spent to sort the point charges (leaf sections in the tree) into the box tree structure as described in [1], since the actual panel generation is not considered to be a part of the multipole method itself. We used the refinement method to generate the point charges used for the multipole technique.

The refinement graph in Fig. 6 shows a quadratic increase of the setup time with the bus length, which is expected since the refinement procedure needs to be called for each segment pair. For the new approach the setup time is similar to that of refinement for small systems, but as the system grows, the far field evaluation is handled by the multipole component of the new approach. For certain threshold bus lengths new hierarchy levels in the spatial subdivision structure for the new approach are created, slowing the increase of the setup time. This causes the changes in slope at lengths of 30 and 120 microns. These reductions of the slope due to far field component of the new approach keeps the increase of the setup time below a linear bound.

Similar characteristics can be found in Fig. 7 and 8 for the potential evaluation time and the memory usage. For the refine-
ment method, we find again a quadratic dependence of the evaluation time on the bus length (see Fig. 7). The effort of computing the potential becomes more and more equivalent to direct evaluation which has a quadratic time complexity (see previous section).

The evaluation time for the multipole method increases roughly linearly, but the pre-factor is so large that we did not compute examples for bus lengths larger than 40 µm. Since \((p+1)^2\) coefficients are necessary to represent an expansion of order \(p\), the multipole-to-local maps (MTLs) have \((p+1)^4\) nonzero elements. The expansion order necessary for a potential accuracy of \(\varepsilon\) is \(p(\varepsilon, R)+1 = -\log_2(\varepsilon) \) with \(c = 2R/\sqrt[3]{3} - 1\), where \(R\) is the radius of the neighborhood around each box for which MTLs are computed (see [1]). To achieve an accuracy of 10 %, \(p\) must be at least 9 for \(R = 2\) (in units of current box size), so each expansion for each box has 100 real coefficients. Then each MTL has 10,000 nonzero elements. For varying \(\varepsilon\) and \(R\), we need for all MTLs of one box \([7(4\pi R^3/3)] \times (p(\varepsilon, R)+1)^4\) operations. This is the (approximate) number of boxes coupled to the current box via MTLs times the MTL size. The operation count is minimized for \(R_{opt} = 5.164\) for \(\varepsilon < 20\%\) independently of \(\varepsilon\). Therefore, the optimal neighborhood radius is five boxes, not two as assumed in [1].

For the new approach the runtime contribution of the multipole component also increases linearly. However, since the amount of boxes in the spatial subdivision tree depends here only on the number of segments rather than panels, the number of boxes is reduced by the factor \((\# \text{ of segments} / \# \text{ of panels})\).

The increase in memory usage for the pure refinement and the new approach follows the runtime pattern. The memory required for pure refinement increases quadratically and reaches our memory limit of 128MByte at a bus length of about 100 µm. The new approach can handle systems of more than twice that size.

Figure 7: Time needed to evaluate \(P_q=0\) using the previously constructed hierarchical structure. For pure multipole the near field potential is evaluated directly (point-to-point computation).

For the multipole case Fig. 8 shows the memory consumption to be similar to that of the other two methods. The memory usage increases linearly with the bus length, but not as fast as for the evaluation time, since we have exploited the fact that the multipole-to-local maps can be reused for each box in the system (these maps only depend on the translation vector between the two boxes involved). By storing computed matrices in a hash table for reuse we avoid recomputation (contributing to the low setup time for the pure multipole approach shown in Fig. 6) and additional storage.

V. CONCLUSIONS

We have identified similarities and weaknesses of the two most common approaches to the extraction of interconnect electromagnetic interaction; namely, the fast multipole method and hierarchical refinement. By analyzing the conditions under which each algorithm performs optimally, we were able to integrate both approaches into a new, concurrent top-down / bottom-up approach to the hierarchical representation of electromagnetic interaction within on-chip interconnect. We presented new algorithms for constructing the appropriate data structures and efficiently evaluating the potentials given a source distribution. In addition, we formulated the entire procedure in terms of a general electromagnetic approach to exploit the similarities between inductance and capacitance extraction for increased efficiency and generality. Numerical examples demonstrate the efficacy of our comprehensive approach.

REFERENCES


Figure 8: Memory used by our implementation of the three different potential evaluation approaches. The pure multipole graph ends at 40 µm due to excessive runtime and lies between the two other graphs.