Blending Symbolic Matrix and Dimensional Numerical Simulation Methodology for Mechatronics Systems

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Abstract

The methodology for the integration of design domains towards the purpose of controlling dynamic mechatronics systems is the current challenge of the modern engineer. Scaling issues for both the mechanical and electrical parameters are critical to the successful design and implementation of a mechatronic system. In approaching the scaling design methodology for future submicron fabrication, new disciplines of symbolic matrix techniques and dimensional analysis must be developed and applied in the design of these mechatronics systems. This paper presents both an overview of the techniques and insight using computer aided design packages for the blending of symbolic matrix techniques using the admittance matrix created by SPICE and dimensional analysis using Buckingham's Π parameters.

1. Introduction

The boundaries of traditional engineering disciplines have been blended by the new age of integrated circuit fabrication techniques, which have even imposed a new concept of sensors and actuators, known as Microelectromechanical Devices (*MEMs*). Looking at one of today's products, the *MEMs* accelerometer which is used in automotive airbags, represents the integration of interdependent electrical and mechanical components. Likewise, the term, "*Mechatronics*", has been coined for the field of study involving the analysis, design, synthesis and selection of *MEM* systems, which combine both electronic and mechanical components with modern controls and embedded microprocessors. An important aspect of the mechatronic design methodology and philosophy considers the applicability of dimensional analysis on both the electrical and mechanical components to achieve performance stabilization. The mechatronics system matrix form, can be solved by direct application of determinant theory and tensor methods; further reduction of the mechatronics system matrix can be obtained through matrix algebra.

2. Dimensional Analysis (*Buckingham's Theorem)*

Dimensional analysis[1,3] provides the key to insight for telling us how the numerical value of a quantity changes when the basic units of measurement are subjected to prescribed changes. It has its beginnings with Maxwell (1871), the Scottish physicist who used symbols of the type [F], [M], [L], [T], [ϕ] to denote force, mass, length, time, and temperature, respectively. The dimensions of these physical quantities can be manipulated algebraically and the results can be interpreted to provide information about the physical processes involved in the scaling process. The fundamental theorem of dimensional analysis is: *If an equation is dimensionally homogenous it can be reduced to a relationship among a complete set of dimensionless products of the system variables, or also know as Buckingham's Theorem.* The dimensionless products are called Π terms. These terms do not depend on the fundamental units of measurement. The set of dimensionless products is complete when each product is independent and any other dimensionless product that can be formed from the variables is a product of the pi terms in the set. This proposes the following questions of: How many dimensionless products form a complete set? How are the dimensionless products formed? These questions will be answered by looking at the following two examples (Sections 2.1 & 2.2 & Appendix A).

2.1 Microfluidic Scaling Example[3,7]

To give some insight to dimensionless products, lets consider a microfluidic example[7]. The first step is to list all the variables that are involved in the phenomenon. Suppose the drag force, F, on a smooth body, in a stream of incompressible fluid with a relative velocity, v, body diameter, D, mass density of fluid, ρ, and viscosity of fluid, ν, is to be found. So, we have the variables F, v, D, ρ, ν. The dimensionless term will have the form:

$$\pi = F^a v^b D^c p^d \nu^e \qquad (1)$$

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where the literal constants a to e must be determined. Since π is dimensionless, then:

$$a + d + e = 0$$
$$a + b + c - 3d - e = 0 - 2a - b - e = 0 \quad (2)$$

This set of equations must be solved. Any solution of these equations will result in a dimensionless Π term. From matrix algebra, the number of independent solutions of a set of simultaneous equations equals the number of variables for the equation set minus the rank of the coefficient matrix. The coefficient matrix becomes: (It is the array of numbers which multiply the variables a, b, c, d and e.)

$$[c] = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -3 & -1 \\ -2 & -1 & 0 & 0 & -1 \end{bmatrix} \quad (3)$$

The rank of a matrix is defined as the order of the largest non-zero determinant that can be constructed from the rows and columns of the matrix. For the dimension matrix of this example, one of the 10 possible 3x3 determinants is

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & -1 \end{bmatrix} \quad (4)$$

Therefore, the rank of the matrix is *3*. Applying the rule stated above: The number of independent solutions equals the number of system variables, *5*, minus the rank of the dimension matrix, *3*. This gives two Π terms. The other three are expressed in terms of the other two, which are called excess variables.

$$a = -d - e$$
$$b = -2a - e = 2d + 2e - e = 2d + e \quad (5)$$
$$c = 3d + e - a - b = 2d + e$$

Values can be chosen for d and a to solve for a, b and c. It makes it simple to choose $d=1$ and $e=0$, resulting in $a=-1$, $b=2$, and $c=2$; then choose $d=0$ and $e=1$, $a=1$, $b=1$, and $c=1$. These two sets of values can now be substituted back, resulting in two independent Π terms.

$$\pi_1 = F^{-1} v^2 D^2 p^{1}$$
$$\pi_2 = F^{-1} v^1 D^1 p^1 \quad (6)$$

It is possible to obtain valid Π terms from algebraic manipulation of Eqn. 6, as shown in Eqn. 7.

$$\pi_3 = \frac{\pi_1}{\pi_2} = \frac{vDp}{v} \quad (7)$$

Eqn. 7 is actually the familiar Reynolds number. Any two of these three Π terms is a complete set. π_1 could be selected for calculation of the drag force as a function of the object's diameter, velocity, and liquid density. π_3 could determine the conditions at which nonlinear flow past the object would exist.

2.2 Electrical Circuit Scaling Example[3,7]

It is necessary to determine how the instantaneous current of a series LC circuit is influenced by the voltage, inductance, and capacitance. The coefficient matrix is:

Variable	*C*	*L*	*E*	*i*	*t*
kg	-1	1	1	0	0
m	-2	2	2	0	0
s	4	-2	-3	0	1
A	2	-2	-1	1	0

Table 1: Coefficient Matrix for LC Circuit

Columns 3, 4 and 5 of rows 1,3 and 4 form a non-zero determinant. The rank of the matrix is three and the number of Π terms is two. Choosing a and b as the excess variables, then c, d, and e become:

$$c = +a - b$$
$$d = -a + b \quad (8)$$
$$e = -a - b$$

The solution matrix becomes:

Variable	*C*	*L*	*E*	*i*	*t*
π_1	1	0	-1	-1	-1
π_2	0	1	-1	1	-1

Table 2: Solution Matrix for LC Circuit

Then $\pi_1 = \frac{CE}{it}$, $\pi_2 = \frac{Li}{Et}$, and likewise $\pi_3 = \sqrt{\pi_1 / \pi_2} = \sqrt{\frac{C}{L}}(\frac{E}{i})$. This gives the maximum current I, as a scalable, and is similar to the results obtained form the conservation-of-energy principle, where I=v $\sqrt{C/L}$. Or, another term is generated if $\pi_3 = 1 / \sqrt{\pi_1 \cdot \pi_2} = \sqrt{\frac{1}{LC}} \cdot t$, where classically, the angular frequency ω, is known to be $\sqrt{\frac{1}{LC}}$. Identical to the results obtained by classical network analysis[5].

2.3 Nonlinear Polynomial Device -Tunnel Diode

The tunnel diode is a transconductance two-terminal, *pn* junction device that has a negative resistance region in its current versus voltage characteristic. We have seen that dimensional analysis allows us to express mathematical equations in the form of dimensionless parameters, but what happens in the case where a physical phenomena is not described by exact mathematical equations? Take for instance, the characterization of the tunnel diode, which for a SPICE simulation *(GTD POLY(1))* is described by the following polynomial expression:

```
I(V)= -3.95510115972848E-17
+1.80727308405845E-01*C*V
-2.93646217292003E+00*C*V^2
+4.12669748472374E+01*V^3
-6.09649516869413E+02*V^4
+6.08207899870511E+03*V^5
-3.73459336478768E+04*V^6
+1.44146702315112E+05*V^7        (9)
-3.53021176453665E+05*V^8
+5.34093436084762E+05*V^9
-4.56234076434067E+05*V^10
+1.68527934888894E+05*V^11
```

Experimentally, the tunnel diode polynomial expression may be adjusted, by the use of MAPLE, with the insertion of the parameter C. As shown in Fig. 1, just the variation of C from .99 to 1.01, results in significant variations in the transconductance model of the tunnel diode.

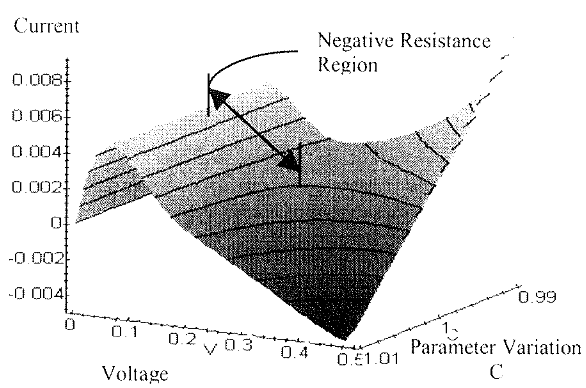

Figure 1: Tunnel Diode V-I Characteristics

2.4 Two-Port Device: Admittance Matrix

The majority of active devices, such as CMOS, BJT & BiCMOS amplifiers, may be classified as two-port networks. This means that the functions of the network are expressed in terms of an input voltage and current. The matrix equation, in terms of admittances ($Y_{11}, Y_{12}, Y_{21}, Y_{22}$) [4], is shown in Eqn. 10.

$$\begin{bmatrix} I_i \\ I_o \end{bmatrix} = \begin{bmatrix} Y_{11} Y_{12} \\ Y_{21} Y_{22} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} \qquad (10)$$

2.4.1 TOTAL-SPICE CAD Package [2,8]

The TOTAL-SPICE CAD package allows matrix input, block diagram manipulation and state-space analysis, digital and continuous time response, root locus, digital and continuous frequency response, polynomial operations, matrix operations, digital and continuous transformation, and SPICE simulation. The TOTAL-SPICE CAD package can easily create the admittance matrix, as is shown by analyzing the circuit in Fig. 2.

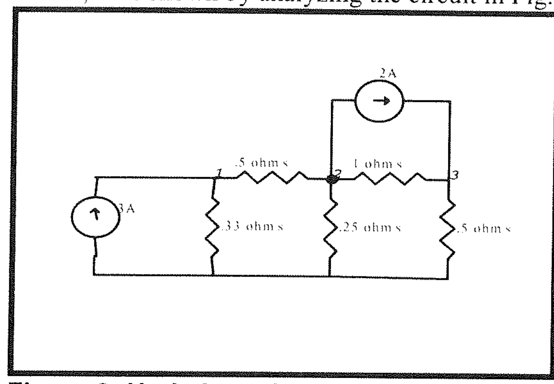

Figure 2: Node Locations for TOTAL-SPICE Simulation

The node equations for Fig. 2 are the following:

$$\begin{aligned} 5v_1 - 2v_2 &= 3 \\ -2v_1 + 7v_2 - v_3 &= -2 \qquad (11) \\ -v_2 + 3v_3 &= 2 \end{aligned}$$

The results of inputting Fig. 2 into TOTAL-SPICE are shown in Fig. 3.

```
***TOTAL-SPICE V1.3 AUG  98***
.OPTION GNUPLT MATRIX
I1 0 1 DC 3A
R1 1 0 .333OHM
R2 1 2 .5OHM
R3 2 0 .25OHM
R4 2 3 1OHM
I2 2 3 DC 2A
R5 3 0 .5OHM
.PRINT DC V(1)
.END
 NODE    V     NODE    V        NODE    V
 (1)  0.5905  (2)  -0.0228   (3) 0.6591
TOTAL POWER DISSIPATION   3.14E+00  WATTS
```

Figure 3: TOTAL-SPICE Results

TOTAL-SPICE generates the following equations, which can be directly loaded (by the *read* command) into MAPLE [5] as shown in Table 3.

```
NODE1 := 5.0030 V1 - 2.0000 V2 = 3.0000
NODE2 := -2.0000 V1 + 7.0000 V2 - 1.0000 V3 = -2.0000
NODE3 := -1.0000 V2 + 3.0000 V3 = 2.0000

{V2 = -.02284805814, V1 = .5905064729, V3 =
.6590506473}
```

Table 3: Maple Results for Electric Circuit

2.4.2 CMOS Op Amp

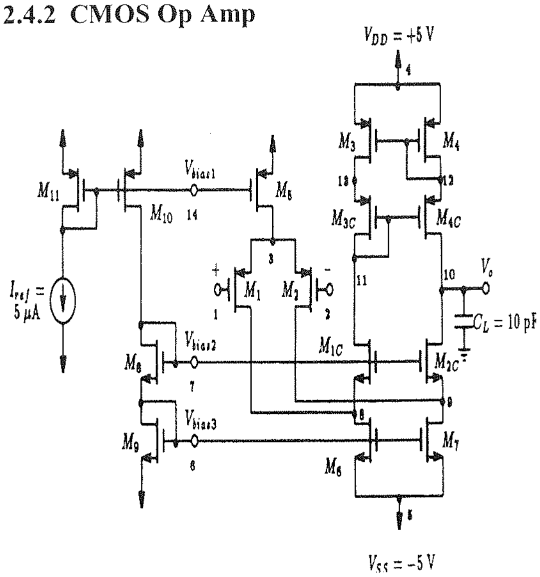

Figure 4: Folded-Cascoded CMOS Op Amp[6]

Fig. 4 shows the folded-cascoded CMOS op-amp used as an example for the generation of the DC Admittance matrix. Table 4 shows the solutions to the node equations as resolved by Maple.

```
>read `C:/TOTAL/MAPLE/MSPICE`;
Node 1 thru Node 3
NODE4 := .66667e-1*V2+1.0000*V4-.13866e-11*V8-.13866e-11*V9-
.13333e-1*V10-.13866e-11*V11-.13333e-1*V12-.13866e-11*V13-
.13866e-11*V14-.13333e-1*V15-.13866e-11*V17-.13866e-11*V26-
.13333e-1*V27-.13866e-11*V28-.13866e-11*V30-.13866e-11*V31-
.13866e-11*V32-.13333e-1*V33-.13866e-11*V34-.13866e-11*V39 =
.78088e-28
Node 5 thru Node 46
NODE47 := -.66667e-1*V23-.66667e-1*V24+.13333*V47 = -
.85710e-3
>Solve({NODE1,NODE2 ... NODE47},{V1,V2 ... V47});

{V27 = -.4778260375e-2, V34 = -507.4483342, V1 = 0, V2 = 0, V3 =
0, V6 = 0, V20 = -1.450452740, V43 = -42401.88653, V44 =
2485.406139, V18 = -.7157805469e-1, V46 = -873.4345368, V5 = -
.1025309055, V22 = -38.15711826, V13 = 4040.875976, V14 = -
2651.483586, .....}
```

Table 4: Maple Results for CMOS Op Amp

3. Conclusion

Dimensional Analysis was applied to both an electrical circuit and a microfluidic device. The dimensionless products were determined, and identified to classical similar results, indicating the effectiveness of Buckingham parameters in the scaling of mechatronic devices. In submicron fabrication of mechatronic MEM devices, the material parameters depend on the electric and magnetic fields, where the Maxwell's equations become nonlinear, and dimensional analysis may become a valuable tool. This is similar to the situation in the nonlinear fluid dynamics example, where the Reynolds number plays a significant part, and is an actual Π parameter. For more complex circuits, the admittance matrix from the TOTAL-SPICE simulator essentially reduces the circuit to a set of equations which are then read into Maple, in symbolic form. The symbolic matrix form, can be solved by direct application of determinant theory and tensor methods, or further reduced by matrix algebra. Since the system topology is viewed as a form of transfer functions, parametric algebraic solutions within the frequency domain, can be generated in the form of Buckingham Π parameters. Finally, if solutions in the time domain, such as transients, are desired, the matrix approach based upon the use of state variables can be employed.

4. References

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[6] A.S. Sedra and G.W. Smith, "SPICE for Microelectronic Circuits", Saunders College Publishing, 3rd Edition, 1992.

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Appendix A : Symbols, Dimensional Formula and Units [3]

Quantity, Symbol	*Dimensional Formula*
Kinematic Quantities	
Time Interval, t	**$[T]$**
Velocity, v	**$[LT^{-1}]$**
Quantities in Mechanics	
Mass, m	**$[M]$**
Force, F	**$[MLT^{-2}]$**
Density, ρ	**$[ML^{-3}]$**
Viscosity, ν	**$[L^2T^{-1}]$**
Electrical Quantities	
Charge, Q	**$[Q]$**
Current, I	**$[QT^{-1}]$**
Voltage, V	**$[ML^2T^{-2}Q^{-1}]$**
Electric Field, E	**$[MLT^{-2}Q^{-1}]$**
Resistance, R	**$[ML^2T^{-1}Q^{-2}]$**
Capacitance, C	**$[M^{-1}L^{-2}T^2Q^2]$**
Inductance, L	**$[ML^2Q^{-2}]$**