Extending Moment Computation to 2-Port Circuit Representations

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Abstract

In this paper, we present an extension of moment computation to 2-port circuits. Our formulas are applicable to both transfer function moments and driving-point admittance moments. Given the input admittances, output admittances, and transfer functions of two 2-ports, our formulas compute the input admittance, output admittance, and transfer function when these 2-ports are combined either in parallel or in series. A nice conclusion of our work is the discovery our formulas form an elegant framework integrating the results from two classical papers, Rubinstein et al. & O'Brien and Savarino, for computing the Elmore delay and driving-point admittance moments in RC trees.

1 Introduction and Motivations

In recent years, timing analysis methods based on moments have become increasingly popular [1] [4]. These methods are typically faster than SPICE by two orders of magnitude for digital and/or interconnect circuits, while retain a high degree of accuracy. The efficient computation of moments is crucial for these timing analysis methods, since this step can be very time-consuming, particularly for large circuits.

A majority of large circuits are composed of subcircuits, and it would be beneficial if moment computation can take advantage of the hierarchies formed by these subcircuits to shorten computation time. For example, if the moments of subcircuits are already computed, it would be desirable to reuse these moments in computing the moments of the overall circuit, instead of having to calculate the moments of the overall circuit from the ground up. What is needed is a way to calculate moments *hierarchically*.

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2 Applications of Our Formulas

Figure 1 illustrates the applications of our formulas. In Figure 1, 2-ports P_1 and P_2 are in parallel, with P_3 in series with both of them. *Driver* and *Output* are the driver and output stage, respectively. **Please note there is no loop in this circuit: signals can only propagate "down" the circuit, not "up".**

Given the transfer function, input admittance, and output admittance of every 2-port in this circuit, our formulas compute the input and output admittances of the entire circuit, as well as the transfer function between arbitrary nodes on either *Path*1 or *Path*2, and when *Path*1 and *Path*2 are combined. Thus with our formulas, we can compute the transfer function, input admittance, and output admittance of the overall circuit *hierarchically* from those of the individual constituent 2-ports.

3 Notations

In this section we introduce the notations used in the rest of this paper. For an illustration of these notations, please refer to Figure 3.

- $\overline{H}_{a,b}$, $\overline{H}_{c,d}$: transfer functions from a to b and from c to d of 2-ports P_1 and P_2, respectively. $\overline{H}_{a,b} = \frac{V_b}{V_a}\Big|_{I_b=0} = \sum_{i=0}^{\infty} \overline{h}_{a,b}^{(i)} s^i$; $\overline{H}_{c,d} = \frac{V_d}{V_c}\Big|_{I_d=0} = \sum_{i=0}^{\infty} \overline{h}_{c,d}^{(i)} s^i$. $\overline{h}_{a,b}^{(i)}$ and $\overline{h}_{c,d}^{(i)}$ are the ith moments of $\overline{H}_{a,b}$ and $\overline{H}_{c,d}$, respectively.

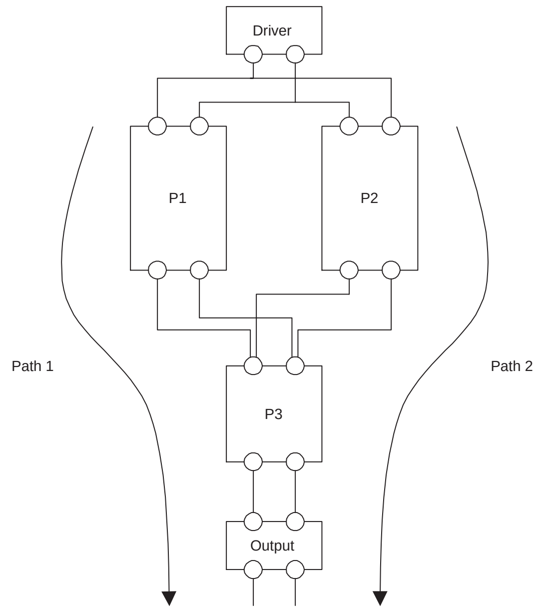

Figure 1: Illustration of the applications of our formulas.

- $H_{a,b}$: transfer function from a to b of 2-ports P_1 and P_2 *in series*. $H_{a,b} = \sum_{i=0}^{\infty} h_{a,b}^{(i)} s^i$, where $h_{a,b}^{(i)}$ is the ith moment of $H_{a,b}$.

- $\overline{Y}_a$, $\overline{Y}_c$: input admittances of 2-ports P_1 and P_2, respectively. $\overline{Y}_a = \frac{I_a}{V_a}\Big|_{I_b=0} = \sum_{i=0}^{\infty} \overline{y}_a^{(i)} s^i$; $\overline{Y}_c = \frac{I_c}{V_c}\Big|_{I_d=0} = \sum_{i=0}^{\infty} \overline{y}_c^{(i)} s^i$. $\overline{y}_a^{(i)}$ and $\overline{y}_c^{(i)}$ are the ith moments of $\overline{Y}_a$ and $\overline{Y}_c$, respectively.

- $\overline{Y}_b$, $\overline{Y}_d$: output admittances of 2-ports P_1 and P_2, respectively. $\overline{Y}_b = \frac{I_b}{V_b}\Big|_{V_a=0} = \sum_{i=0}^{\infty} \overline{y}_b^{(i)} s^i$; $\overline{Y}_d = \frac{I_d}{V_d}\Big|_{V_c=0} = \sum_{i=0}^{\infty} \overline{y}_d^{(i)} s^i$. $\overline{y}_b^{(i)}$ and $\overline{y}_d^{(i)}$ are the ith moments of $\overline{Y}_b$ and $\overline{Y}_d$, respectively

- Y_a, Y_d: input and output admittances, respectively, of 2-ports P_1 and P_2 *in series*. $Y_a = \sum_{i=0}^{\infty} y_a^{(i)} s^i$; $Y_d = \sum_{i=0}^{\infty} y_d^{(i)} s^i$. $y_a^{(i)}$ and $y_d^{(i)}$ are the ith moments of Y_a and Y_d, respectively.

4 Formula Derivations

4.1 Parallel Case

For 2-ports P_1 and P_2 in parallel as shown in Figure 2, the input admittance, Y, is the sum of the input admittances of P_1 and P_2, and we have $Y = \overline{Y}_a + \overline{Y}_c$. Similarly, the output admittance Y' is the sum of the output admittances of P_1 and P_2, with $Y' = \overline{Y}_b + \overline{Y}_d$.

Theorem 4.1 gives the formula for the new transfer function $H_{ac,bd}$ when P_1 and P_2 are in parallel:

Theorem 4.1 *When two 2-ports P_1 and P_2 are in parallel, the new transfer function $H_{ac,bd}$ is*

$$H_{ac,bd} = \frac{V_b}{V_a} = \frac{V_d}{V_c} = \frac{\overline{Y}_b \overline{H}_{a,b} + \overline{Y}_d \overline{H}_{c,d}}{Y_b + Y_d} \tag{1}$$

Expanding into moments, we have $(q \geq 1)$

$$h_{ac,bd}^{(q)} = \frac{1}{\overline{y}_b^{(1)} + \overline{y}_d^{(1)}} \sum_{l=1}^{q} \left(\overline{h}_{a,b}^{(l)} \overline{y}_b^{(q+1-l)} + \overline{h}_{c,d}^{(l)} \overline{y}_d^{(q+1-l)} \right) - \frac{1}{\overline{y}_b^{(1)} + \overline{y}_d^{(1)}} \sum_{l=1}^{q-1} h_{ac,bd}^{(l)} \left(\overline{y}_b^{(q+1-l)} + \overline{y}_d^{(q+1-l)} \right) \tag{2}$$

where $h_{ac,bd}^{(q)}$ $(q \geq 1)$ is the qth moment of $H_{ac,bd}$, with $h_{ac,bd}^{(0)} = 1$. Since all quantities on the right-hand side of (1) are known, $H_{ac,bd}$ can be computed. Please refer to [2] for the proof of Theorem 4.1.

4.2 Series Case

Theorem 4.2 below computes $H_{a,b}$, Y_a, and Y_d, when P_1 and P_2 are in series:

Theorem 4.2 *For 2-ports P_1 and P_2 in series as shown in Figure 3, we have*

$$H_{a,b} = \frac{V_b}{V_a} = \frac{\overline{H}_{a,b}}{1 + Y_c/Y_b} \tag{3}$$

$$Y_a = \frac{I_a}{V_a} = \overline{Y}_a + \frac{\overline{Y}_b \overline{Y}_c \overline{H}_{a,b}^2}{Y_b + Y_c} \tag{4}$$

$$Y_d = \frac{I_d}{V_d} = \frac{\overline{Y}_d \left(\overline{Y}_b + \overline{Y}_c\right)}{\overline{Y}_d \overline{H}_{c,d}^2 + \overline{Y}_b + \overline{Y}_c} \tag{5}$$

Expanding into moments, we have $(q \geq 0)$

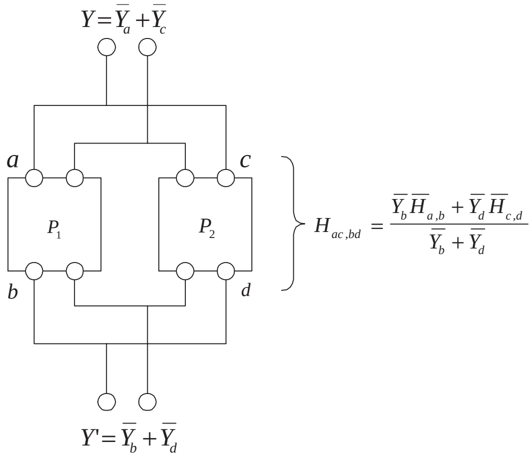

Figure 2: P_1 in parallel with P_2, illustrating the overall input (Y) and output (Y') admittances, as well as the overall transfer function ($H_{ac,bd}$).

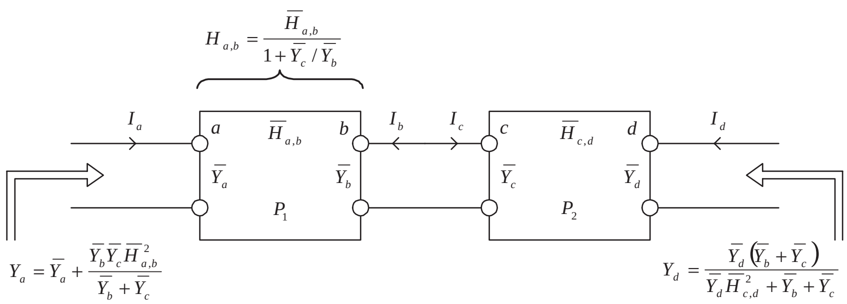

Figure 3: P_1 in series with P_2, illustrating the input ($\bar{Y}_a$, $\bar{Y}_c$) and output ($\bar{Y}_b$, $\bar{Y}_d$) admittances, as well as the transfer functions ($\bar{H}_{a,b}$, $\bar{H}_{c,d}$) of P_1 and P_2.

$$
\begin{aligned}
h_{a,b}^{(q)} &= \frac{1}{\bar{y}_b^{(0)} + \bar{y}_c^{(0)}} \left(\sum_{l=0}^{q} \bar{y}_b^{(l)} \bar{h}_{a,b}^{(q-l)} - \sum_{l=1}^{q} \left(\bar{y}_b^{(l)} + \bar{y}_c^{(l)} \right) h_{a,b}^{(q-l)} \right) \\
y_a^{(q)} &= \frac{1}{\bar{y}_b^{(0)} + \bar{y}_c^{(0)}} \left(\sum_{l=0}^{q} \left(\bar{y}_b^{(l)} + \bar{y}_c^{(l)} \right) \bar{y}_a^{(q-l)} - \sum_{l=1}^{q} \left(\bar{y}_b^{(l)} + \bar{y}_c^{(l)} \right) y_a^{(q-l)} + c_{q+1} \right) \\
y_d^{(q)} &= \frac{1}{d_{1,1}} \left(\sum_{l=0}^{q} \left(\bar{y}_b^{(l)} + \bar{y}_c^{(l)} \right) \bar{y}_d^{(q-l)} - \sum_{l=2}^{q+1} d_{l,1} y_d^{(q+1-l)} \right)
\end{aligned}
$$

where

$$
\begin{aligned}
c_{q+1} &= \sum_{m+n+p+r=q} \bar{y}_b^{(m)} \bar{y}_c^{(n)} \bar{h}_{a,b}^{(p)} \bar{h}_{a,b}^{(r)} \\
d_{1,1} &= \bar{y}_d^{(0)} \left(\bar{h}_{c,d}^{(0)} \right)^2 + \bar{y}_b^{(0)} + \bar{y}_c^{(0)} \\
d_{l,1} &= \sum_{n+p+r=l-1} \bar{y}_d^{(n)} \bar{h}_{c,d}^{(p)} \bar{h}_{c,d}^{(r)} + \bar{y}_b^{(l-1)} + \bar{y}_c^{(l-1)}
\end{aligned}
$$

Please refer to [2] for the proof of Theorem 4.2. Note all quantities on the right-hand side of (3), (4), and (5) are known. Thus, $H_{a,b}$, Y_a. and Y_d can be computed.

Expanding (3), (4), and (5) into moments and re-arranging terms, we get the polynomial expressions for $h_{a,b}^{(q)}$, $y_a^{(q)}$, and $y_d^{(q)}$ as stated in Theorem 4.2.

5 Equivalence with Rubinstein et al. & O'Brien and Savarino

In this section, we demonstrate that our formulas are a natural extension of Rubinstein et al. [5] and O'Brien and Savarino [3].

Theorem 5.1 below states the equivalence of our formulas with [5] and [3]. Theorem 5.1 can be obtained by properly simplifying the formulas in Theorem 4.2 when the circuit is a RC tree.

Theorem 5.1 *For the RC tree in Figure 4, we have (*$i \geq 1$*)*

$$
\begin{aligned}
h_{a,b}^{(i)} &= -R y_a^{(i)} && (6) \\
y_a^{(i)} &= \sum_{l=0}^{i-1} h_{a,b}^{(l)} \hat{y}_c^{(i-l)} && (7) \\
\hat{y}_c^{(i)} &= \bar{y}_c^{(i)} \quad (i \neq 1) && (8) \\
\hat{y}_c^{(i)} &= \bar{y}_c^{(i)} + C \quad (i = 1) && (9)
\end{aligned}
$$

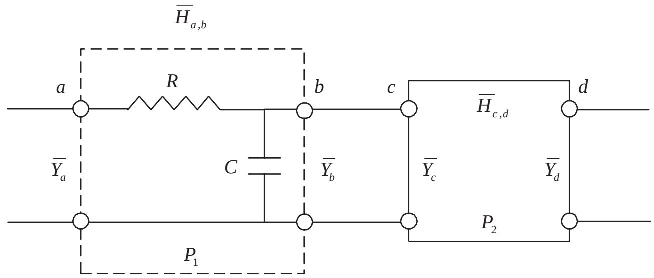

Figure 4: RC tree demonstrating the equivalence of our formulas with Rubinstein et al. [5] & O'Brien and Savarino [3]. P_1 is a single RC segment, while P_2 is an arbitrary RC tree.

- **Equivalence with Rubinstein et al. [5]:** (6) relates $h_{a,b}^{(i)}$, the ith ($i \geq 1$) moment of $H_{a,b}$, to $y_a^{(i)}$, the ith ($i \geq 1$) moment of Y_a. When $i = 1$, (6) becomes

$$h_{a,b}^{(1)} = -Ry_a^{(1)} \tag{10}$$

  Since $y_a^{(1)}$ is the rooted capacitance at node a (see Figure 4), the absolute value of (10) is the Elmore delay for the segment from a to b.

  To compute the Elmore delay of a path consisting of more than one segment, Theorem 5.2 below must be used in conjunction with (10). But first, we state our formulas' equivalence with O'Brien and Savarino [3].

- **Equivalence with O'Brien and Savarino [3]:** (6) and (7) form a recursive relation that can compute the input admittance moment of an arbitrary order, while [3] gives the formulas for only the first three input admittance moments. $\hat{y}_c^{(i)}$ is defined to simplify notation. Due to space limitations, we do not list the case-by-case equivalence with [3]. For details, please refer to [2].

Theorem 5.2 below is used with (6) to compute the Elmore delay of a multiple-segment path:

Theorem 5.2 *Suppose we have, for the RC tree in Figure 4:*

$$\begin{aligned} H_{a,b} &= 1 + h_{a,b}^{(1)}s + h_{a,b}^{(2)}s^2 + \cdots = \sum_{l=0}^{\infty} h_{a,b}^{(l)}s^l \\ \overline{H}_{c,d} &= 1 + \overline{h}_{c,d}^{(1)}s + \overline{h}_{c,d}^{(2)}s^2 + \cdots = \sum_{l=0}^{\infty} \overline{h}_{c,d}^{(l)}s^l \\ H_{a,d} &= 1 + h_{a,d}^{(1)}s + h_{a,d}^{(2)}s^2 + \cdots = \sum_{l=0}^{\infty} h_{a,d}^{(l)}s^l \end{aligned}$$

where $H_{a,b}$, $\overline{H}_{c,d}$, and $H_{a,d}$ are the transfer functions from a to b (with P_2 loaded on P_1), from c to d, and from a to d, respectively; $h_{a,b}^{(l)}$, $\overline{h}_{c,d}^{(l)}$, and $h_{a,d}^{(l)}$ are the lth moments of $H_{a,b}$, $\overline{H}_{c,d}$, and $H_{a,d}$, respectively.

Then we have ($l \geq 1$)

$$h_{a,d}^{(l)} = \sum_{m=0}^{l} h_{a,b}^{(m)} \overline{h}_{c,d}^{(l-m)} \tag{11}$$

When $l = 1$ in (11), we have

$$h_{a,d}^{(1)} = h_{a,b}^{(1)} + \overline{h}_{c,d}^{(1)} \tag{12}$$

with which the Elmore delay from a to d, $h_{a,d}^{(1)}$, can be calculated in terms of $h_{a,b}^{(1)}$, the Elmore delay from a to b, and $\overline{h}_{c,d}^{(1)}$, the Elmore delay from c to d.

Please refer to [2] for the proof of Theorem 5.2.

6 Conclusions

In this paper, we have presented a method to hierarchically compute moments for 2-port circuits. In addition, we demonstrated that our formulas form an elegant framework integrating results from two classical papers: Rubinstein et al. & O'Brien and Savarino.

References

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