Power Sensitivity - A New Method to Estimate Power Dissipation Considering Uncertain Specifications of Primary Inputs

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Abstract
Power dissipation in CMOS circuits heavily depends on the signal properties of the primary inputs. Due to uncertainties in specification of such properties, the average power should be specified between a maximum and a minimum possible value. Due to the complex nature of the problem, it is practically impossible to use traditional power estimation techniques to determine such bounds. In this paper, we present a novel approach to accurately estimate the maximum and minimum bounds for average power using a technique which calculates the sensitivities of average power dissipation to primary input signal properties. The sensitivities are calculated using a novel statistical technique and can be obtained as a by-product of average power estimation using a Monte Carlo based approach. The signal properties are specified in terms of signal probability (probability of a signal being logic ONE) and signal activity (probability of signal switching). Results show that the maximum and minimum average power dissipation can vary widely if the primary input probabilities and activities are not specified accurately.

1 Introduction
The increasing use of portable computing and communication systems makes power dissipation a critical parameter to be minimized in circuit design. Therefore, power estimation tools are badly needed.

In order to accurately estimate power, traditional power estimation techniques [3, 4, 5, 6] require exact signal properties of primary inputs. However, accurate signal property values for primary inputs may not often be available. Since power dissipation strongly depends on the input signal properties, uncertainties in specifications of input signal properties make the estimation process difficult. In fact, average power dissipation of a circuit should be represented as a range given by [Power_min, Power_max]. Traditional power estimation techniques cannot deal with the complexity of the problem since it is practically impossible to try all ranges of signal properties to estimate the minimum and maximum average power dissipation, when the number of primary inputs is large. In this paper we present a novel approach to accurately estimate such bounds for average power dissipation using a technique which calculates the sensitivities of average power to primary input signal properties. The signal properties are specified in terms of signal probability and signal activity. The sensitivities are calculated using an efficient statistical technique and can be obtained as a by-product of average power estimation using a Monte Carlo based approach, the details of which are given in section 3.

2 Preliminaries
2.1 Signal Probability and Activity
The primary inputs of a circuit are modeled to be mutually independent strict-sense-stationary (SSS) mean-ergodic 1-0 processes [5]. Under this assumption, the probability of the primary input node \( x_i \), to assume logic ONE, \( P(x_i|t) \), becomes constant and independent of time and is denoted by \( P(x_i) \), the equilibrium signal probability of node \( x_i \). \( P(x_i) \) is the average fraction of clock cycles in which the equilibrium value of node \( x_i \) is logic ONE.

The activity \( A(x_i) \) is defined as the average number of switching events per unit time. If we assume that all primary inputs to the circuits under consideration switch only at the leading edge of the clock and that the circuits are delay-free, we can define normalized activity denoted by \( a(x_i) \), as \( A(x_i)/f \), where \( f \) is the clock frequency. Therefore, \( a(x_i) = P(x_i(t-T)|x_i(t) + \tau_i(t-T|x_i(t)) = P(x_i(t-T|x_i(t)) + P(\tau_i(t-T|x_i(t)) \). Since \( x_i \) is SSS, \( P(x_i(t-T)) = P(x_i(t)) \). We also have \( P(x_i|t) = P(x_i(t-T|x_i(t)) + P(\tau_i(t-T|x_i(t)) \) and \( P(x_i(t-T)) = P(x_i(t)|x_i(t)) + P(\tau_i(t-T|x_i(t)) \). Therefore, we can derive

\[
P(x_i(t-T)|x_i(t)) = P(\tau_i(t-T|x_i(t)) = a(x_i)/2 \quad (1)
\]

and

\[
P(x_i(t-T|x_i(t)) = P(x_i(t)) - a(x_i)/2 \quad (2)
\]

\[
P(\tau_i(t-T|x_i(t)) = 1 - P(x_i(t)) - a(x_i)/2 \quad (3)
\]
2.2 Power Dissipation in CMOS Logic Circuits

Among the three sources of power dissipation — switching current, short-circuit current, and leakage current, the switching power is the most dominant in current day technology. Thus the average power for a CMOS circuit can be approximated by $\text{Power}_{\text{avg}} = \frac{1}{2} V_{dd} \sum_j \sum_{\text{all nodes}} C_j A_j$, where $V_{dd}$ is the supply voltage, $C_j$ is the node capacitance, $A_j$ is the activity at node $j$. Since $A_j$ is proportional to the normalized activity $a_j$ and $C_j$ is approximately proportional to the fanout at node $j$, we can define the normalized power dissipation measure $\Phi$ as: $\Phi = \sum_j f_{\text{fanout}(j)} a_j$, where $f_{\text{fanout}(j)}$ is the fanout number at node $j$.

2.3 Power Sensitivity

To measure the effect of primary input uncertainties on power dissipation, we define power sensitivity to primary input activity $S_{i}(x_i)$ and power sensitivity to primary input probability $S_{P}(x_i)$ as follows:

$$S_{a}(x_i) = \lim_{\Delta a_i \to 0} \frac{\Delta \text{Power}_{\text{avg}}}{\Delta a_i} = \frac{\partial \text{Power}_{\text{avg}}}{\partial a_i}(x_i)$$

(4)

$$S_{P}(x_i) = \lim_{\Delta P_i \to 0} \frac{\Delta \text{Power}_{\text{avg}}}{\Delta P_i} = \frac{\partial \text{Power}_{\text{avg}}}{\partial P_i}(x_i)$$

(5)

where $a_i$ and $P_i$ are the activity and probability of primary input $x_i$, respectively.

Power$_{\text{avg}}$ is proportional to $\Phi$. Therefore, we can define normalized power sensitivity to primary input activity $\zeta_{a}(x_i)$ and normalized power sensitivity to primary input probability $\zeta_{P}(x_i)$ in terms of $\Phi$ as follows:

$$\zeta_{a}(x_i) = \frac{\partial \Phi}{\partial a_i} = \sum_{j \in \text{all nodes}} f_{\text{fanout}(j)} \frac{\partial a_j}{\partial a_i}(x_i)$$

(6)

$$\zeta_{P}(x_i) = \frac{\partial \Phi}{\partial P_i} = \sum_{j \in \text{all nodes}} f_{\text{fanout}(j)} \frac{\partial \Phi}{\partial P_i}(x_i)$$

(7)

where $a_j$ is the activity of node $j$.

3 Efficient Statistical Technique to Estimate Power Sensitivity (STEPS)

A naive approach to estimate power sensitivity would be to simulate a circuit to obtain the average power dissipation based on nominal values of primary input signal probabilities and activities. Then assign a small variation to only one primary input and re-simulate the circuit. After all the primary inputs have been exhausted, power sensitivity can be obtained using $\Delta \text{Power}_i/\Delta \theta_i$, where $\theta_i$ can be $P(x_i)$ or $a_i(x_i)$. This naive method can be easily implemented. However, it involves $n+1$ times of power estimation. If the number of primary inputs is large, this method can be computationally expensive. Therefore, the naive simulation method is impractical for large circuits with large number of primary inputs. A practical symbolic method was proposed in [2]. However, this approach requires circuit partitioning for large circuits, which can introduce error. In this section we present an efficient technique (STEPS) to estimate power sensitivities as a by-product of statistical power estimation using a Monte Carlo based approach.

The basic idea of Monte-Carlo based statistical method to estimate power dissipation is to simulate a circuit with random patterns applied to primary inputs. Stopping criterion is used to determine when node activities have converged to its correct value [1]. Let us formulate how to estimate power sensitivity using Monte Carlo technique.

A logic circuit can be described by a set of completely specified Boolean functions. Each Boolean function maps primary input vector to an internal or primary output signal. The statistics of internal signals and primary output signals are completely determined by the logic transition at primary inputs. Therefore, the instantaneous power dissipation of a circuit is completely determined by two consecutive input vectors $V^0$ and $V^T$, where the superscripts denote time and $T$ is the clock cycle. The expected value of average power can be expressed as follows:

$$E[\text{Power}] = \sum_{V^0, V^T} \text{Power}(V^0 V^T) P(V^0 V^T)$$

(8)

where $V^0 = (i_0^0, i_1^0, \ldots, i_n^0)$ and $V^T = (i_0^T, i_1^T, \ldots, i_n^T)$ are primary input vectors, and $I_i^0 = x_i^0$ or $\overline{x_i^0}$, and $I_i^T = x_i^T$ or $\overline{x_i^T}$. The power consumption in every clock cycle is a random variable and is denoted as $\text{Power}$. $\text{Power}(V^0 V^T)$ represents the power consumption due to the pair of input vectors $V^0$ and $V^T$. For a particular pair of consecutive vectors, $\text{Power}(V^0 V^T)$ is independent of the probability and activity values of primary inputs. The probability of having consecutive input vectors $V^0$ followed by $V^T$ is represented by $P(V^0 V^T)$. Therefore, power sensitivity can be expressed as follows:

$$\zeta_{\theta_i} = \frac{\partial \Phi}{\partial \theta_i} = \frac{\partial E[\text{Power}]}{\partial \theta_i} = \sum_{V^0, V^T} \left( \text{Power}(V^0 V^T) \frac{\partial P(V^0 V^T)}{\partial \theta_i} \right)$$

(9)

All the primary inputs are assumed to be spatially independent. Therefore, we have:

$$\frac{\partial P(V^0 V^T)}{\partial \theta_i} = P(I_i^0 I_i^T) \ldots \frac{\partial P(I_i^0 I_i^T)}{\partial \theta_i} \ldots P(I_n^0 I_n^T)$$

(10)

From basic calculus, we have

$$\frac{\partial P(I_i^0 I_i^T)}{\partial \theta_i} = P(I_i^0 I_i^T) \frac{\partial \ln P(I_i^0 I_i^T)}{\partial \theta_i}$$

(11)
Substituting the above two equations into equation (9), we get,

\[ \zeta_i = \sum_{V^T \in V_T} [\text{Power}(V^T V^T) P(V^T V^T)] \]

\[ = E[\text{Power}(P(I_i^T I_i^*)^T)] \]

Multiplying \( \text{Power} \) by a factor \( \frac{\partial \ln(P(I_i^T I_i^*)^T)}{\partial \theta_i} \), we obtain a sample of power sensitivity. Therefore, power sensitivity can be estimated simultaneously with average power. The only expression left to be evaluated is \( \frac{\partial \ln(P(I_i^T I_i^*)^T)}{\partial \theta_i} \). Since \( I_i^T \) is either \( x_i^0 \) or \( x_i^1 \) and \( I_i^T \) is either \( x_i^T \) or \( x_i^0 \), we have four combinations for \( P(I_i^T I_i^*)^T) \): \( P(x_i^0 x_i^0) \), \( P(x_i^1 x_i^1) \), \( P(x_i^0 x_i^1) \), and \( P(x_i^1 x_i^0) \). Each of the expressions can be expressed in terms of probability and activity \( (P(x_i) \) and \( a(x_i) \) by equations (1), (2), and (3) (note that \( x_i(t-T) \) and \( x_i(t) \) are replaced by \( x_i^0 \) and \( x_i^1 \), respectively). Therefore, \( \frac{\partial \ln(P(I_i^T I_i^*)^T)}{\partial a(x_i)} \) can be calculated as follows:

\[ \frac{\partial \ln(P(x_i^0 x_i^0)^T)}{\partial a(x_i)} = \frac{\partial \ln(P(x_i) - \frac{1}{2}a(x_i))}{\partial a(x_i)} \]

\[ = -\frac{1}{P(x_i) - \frac{1}{2}a(x_i)} \]

(11)

\[ \frac{\partial \ln(P(x_i^1 x_i^1)^T)}{\partial a(x_i)} = \frac{\ln(1 - P(x_i) + \frac{1}{2}a(x_i))}{\partial a(x_i)} \]

\[ = -\frac{1}{1 - P(x_i) + \frac{1}{2}a(x_i)} \]

(12)

\[ \frac{\partial \ln(P(x_i^0 x_i^1)^T)}{\partial a(x_i)} = \frac{\ln(P(x_i^0 x_i^1))}{\partial a(x_i)} \]

\[ = \frac{\ln(a(x_i))}{\partial a(x_i)} \]

\[ = \frac{1}{a(x_i)} \]

(13)

and \( \frac{\partial \ln(P(I_i^T I_i^*)^T)}{\partial P(x_i)} \) has the following four combinations:

\[ \frac{\partial \ln(P(x_i^0 x_i^0)^T)}{\partial P(x_i)} = \frac{\partial \ln(P(x_i) - \frac{1}{2}a(x_i))}{\partial P(x_i)} \]

\[ = \frac{1}{P(x_i) - \frac{1}{2}a(x_i)} \]

(14)

\[ \frac{\partial \ln(P(x_i^1 x_i^1)^T)}{\partial P(x_i)} = \frac{\ln(1 - P(x_i) + \frac{1}{2}a(x_i))}{\partial P(x_i)} \]

\[ = -\frac{1}{1 - P(x_i) + \frac{1}{2}a(x_i)} \]

(15)

\[ \frac{\partial \ln(P(x_i^0 x_i^1)^T)}{\partial P(x_i)} = \frac{\ln(P(x_i^0 x_i^1))}{\partial P(x_i)} \]

\[ = \frac{\ln(a(x_i))}{\partial P(x_i)} \]

\[ = \frac{1}{a(x_i)} \]

(16)

\[ \frac{\partial \ln(P(I_i^T I_i^*)^T)}{\partial P(x_i)} \]

\[ = 0 \]

4 Power Sensitivity Method

Traditional power estimation techniques require exact specification of primary input signal distribution. However, in general, accurate primary input properties may not be available. Since power dissipation heavily depends on the input signal specifications, uncertain primary input specifications in turn result in uncertain average power dissipation. Therefore, average power should be represented by a range given by the maximum and minimum values of average power. However, traditional power estimation methods cannot handle the complexity of such estimation. It is practically impossible to exhaust all the 2^n (n is the primary input number) combinations of primary input specifications to obtain such bounds. Our method which calculates power sensitivities to primary input specifications, on the other hand, can deal with this complexity efficiently.

Section 3 shows that STEPS can estimate power sensitivity as a by-product of average power estimation with nominal values of signal probability and activity. Hence, the minimum and maximum average power of a circuit can easily be computed as follows:

\[ \Phi_{\text{min}} = \Phi_{\text{avg}} - \sum_{i \in \text{all } P^T} \zeta_{a(x_i)} \Delta a(x_i) \]

(17)

\[ \Phi_{\text{max}} = \Phi_{\text{avg}} + \sum_{i \in \text{all } P^T} \zeta_{a(x_i)} \Delta a(x_i) \]

(18)

\( \Phi_{\text{avg}} \) is the average normalized power dissipation measure. It can be estimated during the average power estimation process using STEPS based on nominal values of primary input signal properties. \( \zeta_{a(x_i)} \) is the power sensitivity to activity \( a(x_i) \) of primary input \( x_i \). \( \Delta a(x_i) \) is the activity variation.

5 Experimental Results

We have implemented the power sensitivity method to estimate minimum and maximum average power dissipation considering uncertain primary input specifications. All the primary inputs are assumed to have probability and activity values (nominal) of 0.5 and 0.26, respectively.

The long run simulation method (naive technique described in section 4) is used as a figure of merit for STEPS. Sample number used in this experiment was 3000 while an activity variation of 0.05 was assumed for all the inputs. The comparison is shown in Table 1. The percentage difference is obtained using the expression \( \frac{\sum_{i \in \text{all } P^T} \zeta_{a(x_i)} \text{(STEPS)} - \zeta_{a(x_i)} \text{(SIM)}}{\sum_{i \in \text{all } P^T} \zeta_{a(x_i)} \text{(SIM)}} \), where \( i \) varies from all primary inputs, \( \zeta_{a(x_i)} \text{(STEPS)} \) is the power sensitivity obtained by STEPS and \( \zeta_{a(x_i)} \text{(SIM)} \) is the power sensitivity obtained by simulation. The comparison is shown in Table 1. The CPU time is also shown for a SPARC 5 workstation. Since long run simulation method repeats the estimation procedure \( n + 1 \) times (\( n \) is the number of primary inputs), execution time may be unacceptably
Table 1: Comparison of two methods

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<th>Diff %</th>
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long for large n. Let us consider circuit C7552. It takes approximately 4093 seconds of CPU time to complete one simulation run. The circuit has 207 primary inputs. It would take approximately $8.5 \times 10^5$ seconds (9.8 days) of CPU time to obtain power sensitivities. Circuits with prohibitively long execution time are identified by dashes in the “SIM” and “Diff %” columns of table 1. “Diff %” stands for percent difference between the results obtained by simulation and STEPS. STEPS can estimate power sensitivities simultaneously with average power, and hence, it is much faster than the naive simulation based approach.

Results for power sensitivities indicate that for some circuits power dissipation is much more sensitive to some primary inputs than others. A small activity variation of such highly sensitive primary inputs will result in a dramatic change of the average power. Consider circuit C6. The power sensitivities to activities of the 1st, 2nd, 60th, and 124th primary input (corresponding to primary input number 0, 1, 59, and 123 respectively in Figures 1 and 4) are 237, 276, 133, and 30 respectively. The power sensitivity to the activity of each of the other primary inputs is less than 4. If the activities of the 1st and 2nd primary input have a variation of ±0.05, the power dissipation may change by 30%. Therefore, for power conscious designs, those sensitive primary inputs have to be accurately specified for an accurate estimation of average power.

After obtaining power sensitivities, we use equations (17) and (18) to compute the minimum and maximum average power for each simulated circuit. For simplicity, all primary inputs are assumed to have the same activity variation of ±0.05. However, our method is not limited to such assumption. Results of the minimum and maximum average power for ISCAS and MCNC benchmark circuits are shown in Figures 2 and 3. Results indicate that for some circuits minimum and maximum average power can vary widely if uncertain specifications of primary inputs exist. Consider circuit i2: maximum average power dissipation is 79 units, which is about 46% greater than minimum average power which is 53 units. It should be noted that we do not assume any delay models in deriving equation (10). Therefore, STEPS can handle different delay models for logic gates. Figures 5 and 6 give such bounds for average power based on unit delay model.

6 Conclusions

In this paper we have considered an accurate technique to estimate sensitivities of power dissipation to uncertainties in specification of signal properties of primary inputs. The sensitivities can be obtained as a by-product of the statistical power estimation technique, and hence, different delay models can be easily incorporated. Based on the sensitivity values, minimum and maximum average power can be easily estimated. Our results on ISCAS and MCNC benchmark circuits indicate that for some circuits power dissipation can be very sensitive to some primary inputs. A small activity variation of such sensitive inputs can cause power dissipation to change drastically. Results on minimum and maximum average power show that such bounds can vary widely if the primary input probabilities and activities are not specified accurately.

References

Figure 1: Power sensitivity $\zeta_a$, obtained by simulation for circuit i6

Figure 4: Power sensitivity $\zeta_a$, obtained by \textit{STEPS} for circuit i6

Figure 2: Average power obtained using zero delay model for ISCAS benchmark circuits

Figure 5: Average power obtained using unit delay model for ISCAS benchmark circuits

Figure 3: Average power obtained using zero delay model for MCNC benchmark circuits

Figure 6: Average power obtained using unit delay model for MCNC benchmark circuits