Analytical Estimation Of Transition Activity From Word-Level Signal Statistics

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Abstract— **Presented here is an *analytical* methodology to determine the average signal activity, T, from the high-level signal statistics, a statistical signal generation model, and the signal encoding. Simulation results for 16 bit signals generated via $AR(1)$ and $MA(1)$ models indicate an estimation error in T of less than 2%. The application of the proposed method to the estimation of T in DSP hardware is also explained.**

I. INTRODUCTION

Power dissipation has become a critical design concern in recent years driven by the emergence of mobile applications. Reliability concerns and packaging costs have made power optimization relevant even for tethered applications. As system designers strive to integrate multiple-systems on-chip, power dissipation has become an equally important parameter that needs to be optimized along with area and speed. Therefore, extensive research into various aspects of low-power system design is presently being conducted. These include *power reduction* techniques [3–5]; *low-power synthesis* techniques; [7, 19]; *power estimation* [15]; and *fundamental limits* on power dissipation [18]. While the work presented in this paper focuses on estimation of signal transition activity, our eventual objective is to enable low-power synthesis.

At the logic and circuit levels, such techniques as, [8, 10–12, 14, 17, 20] exist for power estimation. However, these techniques are applicable once the design has reached a substantial degree of maturity because a gate or transistor level description of the circuit is required. While a large amount of work has been done at the circuit and logic levels, less has been done for power estimation at the architectural level. Architectural level power estimation tools are of critical importance as they allow the system designer to choose between competing architectures and also permit major design changes when it is easiest to do so. In [9, 16], techniques based upon the concept of *entropy* are presented for estimating the average transition density inside a combinational circuit block.

Architectural level power estimation requires estimation of the transition activity and the capacitance. Our aim in this paper is to derive an analytical method for determining the average signal transition activity, T, for power estimation purposes, from available signal statistical description. The closest approach to our work is described in [6] where a word-level signal is broken up into: 1.) uncorrelated data bits, 2.) correlated data bits, and 3.) sign bits. The uncorrelated data bits are from the least significant bit (LSB) up to a certain break-point BP_0, with a fixed transition activity. The transition activity of the sign bits, which are from the most significant bit (MSB) to another break-point BP_1, are measured by an RTL simulation. A linear model is employed for the switching activity of correlated data bits, which lie between the sign bits and uncorrelated data bits. Empirical equations defining BP_0 and BP_1 in terms of such word-level statistics as mean (μ), variance (σ^2), and autocorrelation (ρ) are also presented.

Our approach, in this paper, is similar to [6] in that we present a method for estimating the average number of transitions in a signal from its word-level statistical description. However, unlike [6], the proposed approach is analytical requiring: 1.) high-level signal statistics, 2.) a statistical signal generation model, and 3.) the signal encoding (or number representation) to estimate the transition activity for that signal. Therefore, the two novel features of the proposed method are: 1.) it is a completely *analytical* approach and 2.) its computational complexity is depends on the *width* of the signal word rather than on its *length* (i.e., number of samples). Both of these features distinguish the proposed approach from most existing techniques to estimate signal transition activity. While [6] also estimates power dissipation by characterizing input capacitances, we focus only on estimation of transition activity.

We first derive a new relation between the transition activity (t_i), probability (p_i) and the autocorrelation (ρ_i) for a single bit signal b_i. Then, we employ word-level signal statistics (namely μ, σ, and ρ), signal generation models (such as auto-regressive (AR), moving-average (MA) and auto-regressive moving-average ($ARMA$) models), along with a certain number representation (such as unsigned, sign-magnitude, one's complement, or two's complement) to estimate the word-level transition activity T of the signal. Proceeding further, we propagate the input statistics through commonly used digital signal processing (DSP) blocks such as adders, multipliers, multiplexers, and delays. The word-level transition activities of all the signals in a system composed of these DSP blocks is determined and accumulated to determine the total transition activity of the filter.

Simulation results for 16 bit signals generated via first order AR model ($AR(1)$) and first order MA model ($MA(1)$) indicate that an error in T of less than 2% can be achieved. Employing AR(1) and MA(10) models for audio and video signals, the proposed method results in errors of less than 10%. Simulation results with $AR(1)$ and $MA(1)$ inputs show that errors less than 4% are achievable in the estimation of the total transition activity in the filters.

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II. PRELIMINARIES

In this section, we will present definitions and review existing results that will be employed in later sections.

Let $x(n)$ be a B-bit signal with *mean* μ, *variance* σ^2, and *lag-i temporal correlation* $\rho(i)$. In this paper, we will be interested only in $\rho(1)$ and therefore we will denote it by ρ. Let $\mathcal{X}$ be the set of values that $x(n)$ can assume. Let p_i be the probability that the i^{th} bit, $b_i(n)$, of $x(n)$ is 1. If $\mathcal{X}_i$ is the set of all elements in $\mathcal{X}$ such that the i^{th} bit is 1, then:

$$p_i = Pr(x(n) \in \mathcal{X}_i) \quad (1)$$
$$= \sum_{\forall j \in \mathcal{X}_i} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(j-\mu)^2}{2\sigma^2}} \text{(assuming normal distr.)} \quad (2)$$

Clearly, the value of p_i is dependent on the statistical distribution of the values in $\mathcal{X}$. While we have provided an example of a normal distribution here, there is no restriction on the distribution itself. Note that, the probability distribution of $x(n)$ can either be estimated or obtained from the knowledge of the parameters of the signal generation models to be discussed in the next subsection. However, without loss of generality, we will assume that the probability distribution of $x(n)$ is known *a priori.*

The *temporal correlation*, ρ_i, of the i^{th} bit is defined as:

$$\rho_i = \frac{E[(b_i(n)-p_i)(b_i(n-1)-p_i)]}{E[(b_i(n)-p_i)^2]} = \frac{E[b_i(n)b_i(n-1)]-p_i^2}{p_i-p_i^2}. \quad (3)$$

If $p_i = 1$ or $p_i = 0$ then ρ_i is defined to be 1.

The *transition activity* (or transition probability [15]), t_i, of the i^{th} bit is defined as

$$t_i = Pr(b_i(n) \neq b_i(n-1)). \quad (4)$$

If the bits $b_i(n)$ and $b_i(n-1)$ are independent then the transition activity is given by [15],

$$t_i = 2p_i(1-p_i). \quad (5)$$

We define the word-level transition activity T as follows

$$T = \sum_{i=0}^{B-1} t_i. \quad (6)$$

We will employ Auto-Regressive Moving-Average ($ARMA$) signal generation models [2] to calculate transition activity. These signal models are commonly employed to represent stationary signals in general and have found widespread application in speech [1] and video coding [13]. Furthermore, signals obtained from such sources as speech, audio, and video can also be modeled employing $ARMA$ models.

An (N, M) order ARMA model ($ARMA(N, M)$) can be represented as

$$x(n) = \sum_{i=0}^{N} b_i\gamma(n-i) + \sum_{i=1}^{M} a_i x(n-i) \quad (7)$$

where the signal $\gamma(n)$ is a uncorrelated noise source with zero mean, and $x(n)$ is the signal being generated. If a given signal source, such as speech, needs to be modeled via (7), then we can choose coefficients a_i and b_i to minimize a certain error measure (such as the mean-squared error) between $x(n)$ and the given source. In that case, we say that $x(n)$ represents the given signal source. As mentioned in the previous subsection, if the a_i's and b_i's in (7) are known, along with the distribution of $\gamma(n)$, then we can obtain the probability distribution of $x(n)$.

It is possible to transform (7) into one that depends only on the inputs, $\gamma(n)$, as shown below:

$$x(n) = \sum_{i=0}^{\infty} h_i\gamma(n-i), \quad (8)$$

where h_i can be computed according to the following recursion,

$$h_k = b_k + \sum_{i=1}^{N} a_i h_{k-i}, \quad (9)$$

where $h_k = 0$ for $k < 0$, and $h_0 = b_0$. Finally, AR and MA models are special cases of $ARMA$. An M^{th} order auto-regressive ($AR(M)$) signal model is identical to an $ARMA(0, M)$ model. Similarly, an N^{th} order moving-average ($MA(N)$) signal model is the same as an $ARMA(N, 0)$ model.

In proving Theorem 1 in the next section, we will also employ the following result from [10],

Lemma 1: $\mathrm{E}[b_i(n)b_i(n-1)] = p_i - \frac{t_i}{2}$

III. WORD-LEVEL SIGNAL TRANSITION ACTIVITY

In this section, we will present techniques for estimating word-level transition activity T of a signal $x(n)$ from its word-level statistics. We will first present a theorem relating bit-level quantities, which are the transition activity t_i, the probability p_i, and temporal correlation, ρ_i. Next, two techniques for estimating ρ_i are presented. The first is referred to as the *exact method*, whereby ρ_i is explicitly determined for the B bits $i = 0, \ldots, B-1$ in $x(n)$. The second method is called the *approximate method* in which break-points BP_0 and BP_1 (as defined in [6]) are determined from an $ARMA$ model of the signal. Simulation results will be provided in support of the theory.

A. Transition Activity For Single-bit Signals

For single-bit signals, we have an expression given by (5) for independent bits $b_i(n)$ and $b_i(n-1)$. We now present a general result which is also applicable when the temporal correlation between $b_i(n)$ and $b_i(n-1)$ (i.e., ρ_i) is not zero.

Theorem 1:

$$t_i = 2p_i(1-p_i)(1-\rho_i) \quad (10)$$

Proof: Substitute for $E[b_i(n)b_i(n-1)]$ from Lemma 1 into (3) and solve for t_i.

Note that, substitution of $\rho_i = 0$ (corresponding to the case of uncorrelated bits) in (10) reduces it to (5). In subsequent sections, we present two methods (the exact and approximate methods) for calculating ρ_i from word-level statistics. These will then be substituted in (10) to obtain t_i.

B. Calculation of ρ_i: The Exact Method

From (3), we see that it is necessary to compute p_i and $E[b_i(n)b_i(n-1)]$ in order to estimate ρ_i. As p_i can be obtained from the probability distribution function of $x(n)$, we will now focus upon $E[b_i(n)b_i(n-1)]$, which is given by:

$$E[b_i(n)b_i(n-1)] = Pr((b_i(n)=1) \wedge (b_i(n-1)=1)) = Pr(x(n) \in \mathcal{X}_i \wedge x(n-1) \in \mathcal{X}_i) \quad (11)$$

In particular, we will employ $AR(1)$ and $MA(N)$ signal models to estimate $E[b_i(n)b_i(n-1)]$. First, we present the following result for an $AR(1)$ model.

Theorem 2: *For an $AR(1)$ signal,*

$$E[b_i(n)b_i(n-1)] = \sum_{\forall j \in \mathcal{X}_i} Pr(x(n-1)=j) \sum_{\forall k \in \mathcal{X}_i} Pr(\gamma(n)=k-a_1 j) \quad (12)$$

Proof: From the definition of $E[b_i(n)b_i(n-1)]$ in (11), we have

$$E[b_i(n)b_i(n-1)] = \sum_{(\forall j \in \mathcal{X}_i)} \sum_{(\forall k \in \mathcal{X}_i)} Pr(x(n)=k \wedge x(n-1)=j). \quad (13)$$

Substituting the expression for an $AR(1)$ model (obtained by substituting $N=0$, $M=1$ and $b_0=1$) in (7)) into (13), we obtain

$$E[b_i(n)b_i(n-1)] = \sum_{(\forall j \in \mathcal{X}_i)} \sum_{(\forall k \in \mathcal{X}_i)} Pr(\gamma(n)+a_1 j = k \wedge x(n-1)=j)$$

$$= \sum_{(\forall j \in \mathcal{X}_i)} \sum_{(\forall k \in \mathcal{X}_i)} Pr(\gamma(n)+a_1 j = k) Pr(x(n-1)=j) \quad (14)$$

where the last step is justified because $\gamma(n)$ and $x(n-1)$ are independent. Note that, (12) can now be obtained by a simple rewriting of (14). Furthermore, each of the summations in (12) can be evaluated via the knowledge of the probability distribution function. □

In order to verify Theorem 2, we compared the measured values of t_i and ρ_i for the data generated by an $AR(1)$ signal, SIG2, in Table I, with the estimated values predicted by the theorem. Figure 1 indicates that the measured and theoretical values match well. The error in T was less than 1%. The mean absolute error in t_i for SIG1 was 1.03%.

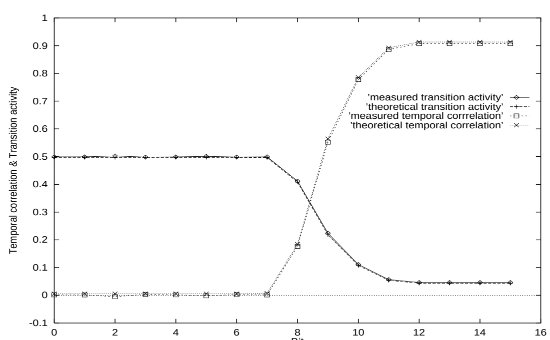

Fig. 1. Measured & Theoretical t_i & ρ_i for SIG2

We now consider an $MA(1)$ process and present the following result.

Theorem 3: *Let $j,k,l \in \mathcal{X}_i$, where $j+b_1k \in \mathcal{X}_i$ and $k+b_1l \in \mathcal{X}_i$. Then, for an $MA(1)$ signal $x(n)=\gamma(n)+b_1\gamma(n-1)$,*

$$E[b_i(n)b_i(n-1)] = \sum_j \sum_k \sum_l Pr(\gamma(n)=j)Pr(\gamma(n-1)=k) Pr(\gamma(n-2)=l)$$

Proof: The proof is similar to the proof of Theorem 2 and can be obtained by substituting the expression for an $MA(1)$ signal into (11). The expression for an MA(1) signal can be obtained by substituting $N=1$, $M=0$, and $b_0=0$ into (7). □

In Figure 2, we show the simulation results in support of Theorem 3. Again, we compared the measured values for t_i and ρ_i in data generated by the $MA(1)$ signal, SIG3, in Table I with the values predicted by the theorem. In this case, we found that the errors between the measured and predicted values of T were less than 2%.

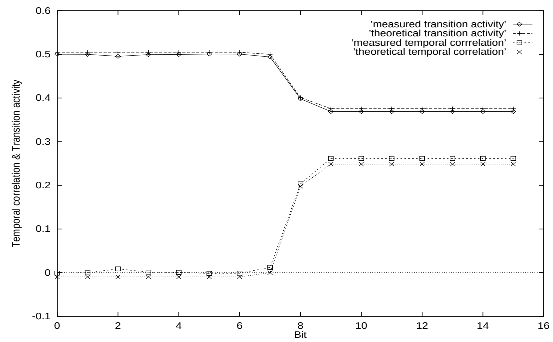

Fig. 2. Measured & Theoretical t_i & ρ_i for SIG3

Finally, we consider the computation of $E[b_i(n)b_i(n-1)]$ for an $MA(2)$ signal and show that Theorem 3 can also be extended to calculate $E[b_i(n)b_i(n-1)]$ for an $MA(N)$ signal. For an $MA(2)$ signal $x(n)=\gamma(n)+b_1\gamma(n-1)+b_2\gamma(n-2)$, the quantity $E[b_i(n)b_i(n-1)]$ is given by:

$$E[b_i(n)b_i(n-1)] = \sum_{j,k,l,m} Pr(\gamma(n)=j)Pr(\gamma(n-1)=k) Pr(\gamma(n-2)=l)Pr(\gamma(n-3)=m),$$

where $j,k,l,m : j+b_1k+b_2l \in \mathcal{X}_i$ and $k+b_1l+b_2m \in \mathcal{X}_i$. It can be checked easily that $E[b_i(n)b_i(n-1)]$ for $AR(M)$ and $ARMA(N,M)$ signals is difficult to calculate for $M>1$. However, we can estimate $E[b_i(n)b_i(n-1)]$ for an $AR(M)$ or an $ARMA(N,M)$ signal by approximating the signal with an $MA(N')$ signal, where N' is sufficiently large, or by approximating with an AR(1) signal.

C. Estimation of ρ_i: The Approximate Method

In the previous subsection, an exact method for computing ρ_i was presented. For large values of B, this computation can become expensive. In order to alleviate this problem, we will present a computationally efficient method (referred to as the *approximate method*) to estimate ρ_i from word-level statistics using a model similar to that described in [6].

In Figure 3, we plot ρ_i for the signals in Table IV. It can be seen that ρ_i is approximately zero for the LSBs and close to the word-level temporal correlation ρ for the MSBs. Furthermore, there is a region in between the LSBs and MSBs where ρ_i increases approximately linearly. As proposed in [6], we divide the bits in the word into three regions of contiguous bits referred to as the LSB, *linear*, and MSB regions. The break-points BP_0 and BP_1 separate the LSB from the linear region and the linear from the MSB region, respectively. Furthermore, the graph of ρ_i for the LSB, linear, and MSB regions has slopes of zero, non-zero, and zero respectively.

In spite of this similarity with [6], the proposed approach differs from [6] in: 1.) the way BP_0 and BP_1 are computed, and 2.) our use of (10) (to compute t_i) and (6) to compute T *analytically*. In particular, we do not employ simulations to estimate transition activity of the MSBs.

For now, we will assume that 2's complement representation is employed. By definition, $\rho_i = 0$ for $i < BP_0$ and $\rho_i = \rho_{BP_1}$ for $i \geq BP_1 - 1$. Hence, we can make the following

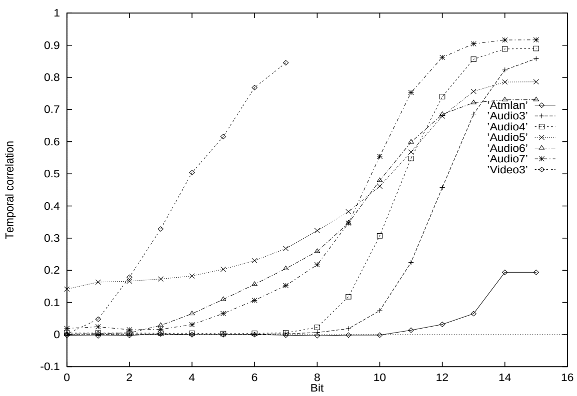

Fig. 3. Temporal correlation versus bit

approximation:

$$\rho_i = \begin{cases} 0 & (i < BP_0) \\ \frac{(i - BP_0 + 1)\rho_{BP_1}}{BP_1 - BP_0} & (BP_0 \leq i < BP_1 - 1) \\ \rho_{BP_1} & (i \geq BP_1 - 1) \end{cases} \quad (15)$$

We now examine the relation between BP_0, BP_1, and ρ_{BP_1} and word-level statistics.

C.1 Calculation of BP_0

For an uncorrelated signal, $\gamma(n)$, a reasonable estimate of BP_0 is given by $\log_2 \sigma_\gamma$, where σ_γ^2 is the variance of $\gamma(n)$ [6]. If the signal $x(n)$ has non-zero correlation then it can be modeled using a signal model, which can then be used to calculate BP_0. For instance, if $x(n)$ is modeled using an $ARMA$ model then it can be expressed using (8). Since the signals $h_i\gamma(n-i)$ are uncorrelated, BP_0 for each of the signals can be estimated as $\log_2 |h_i|\sigma_\gamma$. The break-point BP_0 for a signal $x(n) = \sum_i h_i\gamma(n-i)$ can now be estimated as the maximum of the BP_0's of the signals $h_i\gamma(n-i)$. Hence,

$$BP_0 \quad = \quad [\log_2 h_{max}\sigma_\gamma], \quad (16)$$

where $h_{max} = \max(|h_i|)$ and $[.]$ returns the nearest integer. We verified (16) by comparing the measured and estimated values of BP_0 obtained from data generated with the five signals shown in Table I. The results are shown in Table II, where it can be seen that the measured and estimated values of BP_0 match quite well.

C.2 Calculation of BP_1

Let the values of $x(n)$ lie between the values x_{min} and x_{max}. In a normal distribution, $x_{min} = \mu - 3\sigma$ and $x_{max} = \mu + 3\sigma$. We define BP_1 such that for $i \geq BP_1 - 1$, ρ_i is approximately constant. Since the dynamic range of $x(n)$ is x_{max}-x_{min}, the least significant $\log_2(x_{max}$-$x_{min})$ bits are required to cover this range. Hence, we have

$$\begin{aligned} BP_1 &= [\log_2(x_{max}\text{-}x_{min})], \\ &= [\log_2 6\sigma] \text{ (for a normal distribution)}, \quad (17) \end{aligned}$$

TABLE I

Signal details

Signal	$x(n)$	σ_γ	σ	ρ	μ
SIG1	$\gamma(n) - 0.5x(n-1)$	866	1000	-0.50	0
SIG2	$\gamma(n) + 0.99x(n-1)$	141	1000	0.99	0
SIG3	$\gamma(n) + 0.5\gamma(n-1)$	100	111	0.40	0
SIG4	$\gamma(n) + 0.99x(n-1)$	141	1000	0.99	16384
SIG5	$\gamma(n) + 0.4\gamma(n-1) + 0.2\gamma(n-2) +$ $.07\gamma(n-3) + .5x(n-1) + .3x(n-2) +$ $0.1x(n-3) + 0.05x(n-4) - .2x(n-5)$	1000	2309	0.89	0

TABLE II

Measured and Estimated BP_0 and BP_1

Signal	BP_0		BP_1	
	Measured	Estimated	Measured	Estimated
SIG1	11	10	13	13
SIG2	8	7	13	13
SIG3	8	7	10	9
SIG4	8	7	13	13
SIG5	11	10	14	14

where σ^2 is the variance of $x(n)$. The estimate for BP_1 in (17) is different from that in [6], which is also given in (18) below for comparison purposes:

$$BP_1 \quad = \quad [\log_2(|\mu| + 3\sigma)]. \quad (18)$$

When $|\mu| \leq 3\sigma$, both (17) and (18) are approximately equal with the maximum difference of 1 occuring at $\mu = 0$. However, in the case where $|\mu| \gg 3\sigma$ the difference between (17) and (18) is non-trivial. This is because for $|\mu| > 3\sigma$ there are 3 regions in which ρ_i is a constant. The first region consists of the bit positions i such that $i < BP_0$. The second region has bit positions i lying between BP_1 and another break-point BP_2, which will be defined later. The third region consists of bits with positions beyond BP_2 where the bits do not have any transitions. BP_2 can be calculated by computing the common MSBs in the binary representations of the numbers x_{max} and x_{min}.

We have verified (17) by comparing it with the measured values of BP_1 obtained from data generated by various signals in Table I. The results are shown in Table II where it can be seen that the measured and estimated values match closely. In Figure 4, we plot ρ_i and t_i for signals SIG2 and SIG4. Note that from Table I, SIG2 and SIG4 are identical except for their mean μ. It can be seen from Figure 4, that the value of BP_1 (13) is independent of μ, which is also indicated by (17). For SIG4 BP_2 is 15 because the binary representations of x_{max} (19384) and x_{min} (13384) have only 1 common MSB.

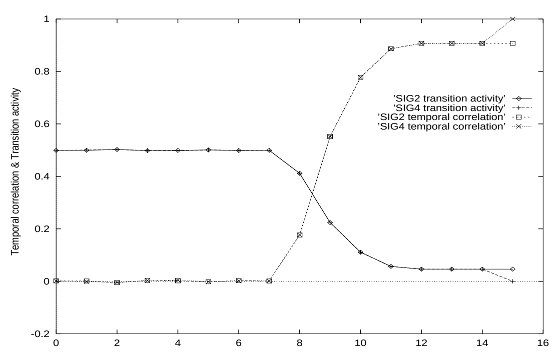

Fig. 4. ρ_i and t_i vs. i for SIG2 and SIG4

All that now remains in the approximate method is to estimate the value for ρ_{BP_1}. If the model for $x(n)$ is known then we can use the exact method to calculate ρ_{BP_1}. On the other hand, if the model for $x(n)$ is not available, then we assume that $\rho_{BP_1} = \rho$. This is especially valid for audio and video signals (see Figure 3).

D. Calculation of T

Employing (6), Theorem 1, (16), (17), we computed T for the signals in Table I for 2's complement. The measured and estimated T for all the signals using our method and the DBT method in in [6] are shown in Table III. The errors for our method are less than 2% for 2's complement representation.

E. Effect of Signal Encoding/Number Representation

The results presented so far in this section (Theorems 2 and 3) have implicitly included the effect of the signal encoding. This is because the elements of the sets $\mathcal{X}$ and $\mathcal{X}_i$ will depend upon the signal encoding. In this subsection, we examine explicitly the effect of number representation on transition activity.

In the previous subsections, we have considered 2's complement number representation. The unsigned representation will have the same transition activity as 2's complement because the MSBs of the former behave identical to the sign bits of the latter.

The 1's complement representation is identical to the 2's complement for positive numbers. For negative numbers, we can generate the 2's complement representation from that of the 1's complement by adding a 1 to the LSB, which will usually affect only the LSBs. In the approximate method, since we assume that LSBs are uncorrelated, the activity of the LSBs in the 1's complement will be close to that of the 2's complement. The remaining bits will have the same temporal correlation as in the 2's complement representation. Therefore, ρ_i for 1's complement representation will be the same as that for 2's complement. The measured and estimated T for various signals employing 1's complement are shown in the second set of the three columns in Table III. The measured T was obtained by generating data using the signal model and measuring T in that data. The error in T is the same as the error for the 2's complement representation, except for one signal, SIG5, where it is slightly higher. The error in T is less than 2% for 1's complement representation.

In the sign magnitude representation there is only one sign bit; namely, the MSB, $b_{B-1}(n)$. This bit will have the same temporal correlation as the sign bits in 2's complement representation because the temporal correlation of the sign bit depends on the sign transitions. The bits $b_i(n)$ for $i < BP_0$ are uncorrelated as in the case of 2's complement. We again assume a linear model for ρ_i for $BP_0 \le i < BP_1 - 1$. The resulting expression for ρ_i is as follows:

$$\rho_i = \begin{cases} 0 & (i < BP_0) \\ \frac{(i - BP_0 + 1)\rho_{BP_1}}{BP_1 - BP_0} & (BP_0 \le i < BP_1 - 1) \\ 1 & (BP_1 - 1 \le i < B - 1) \\ \rho_{BP_1} & (i = B - 1) \end{cases} \tag{19}$$

The measured and estimated T for the signals are shown in the last three columns of Table III. As before, the measured T was obtained by generating data using the signal model and measuring T in that data. It can be seen that the error in T is less than 2% for all the signals except for SIG4 where the error is less than 5%.

TABLE III

Word-level transition activity for different number representations

Signal	Unsigned, Two's complement					One's complement			Sign magnitude		
	Meas.	Est.	% Err.	DBT	% Err.	Meas.	Est.	% Err.	Meas.	Est.	% Err.
SIG1	8.79	8.82	0.34	8.93	1.59	8.79	8.82	0.34	6.07	6.16	1.48
SIG2	4.99	5.03	0.80	5.05	1.20	4.99	5.03	0.80	4.65	4.74	1.94
SIG3	6.97	6.94	0.43	6.82	2.15	6.97	6.94	0.43	4.20	4.15	1.19
SIG4	4.99	5.03	0.80	5.25	5.21	4.99	5.03	0.80	4.65	4.86	4.52
SIG5	6.54	6.42	1.83	6.24	4.59	6.55	6.42	1.98	5.91	5.89	0.34

TABLE IV

Description of data-sets

Signal	Description	μ	σ	ρ
Audio3	2.88MB, 16 bit PCM music	1.43	7349.20	0.96
Audio4	2.88MB, 16 bit PCM music	-17.63	4040.40	0.97
Audio5	0.37MB, 16 bit PCM speech	59.46	2661.75	0.90
Audio6	0.61MB, 16 bit PCM speech	23.62	2328.79	0.96
Audio7	2.88MB, 16 bit PCM music	-39.35	3086.30	0.99
ATM	0.80MB, 16 bit comm. channel	0.49	5581.60	0.30
Video3	9.70MB (380 QCIF frames), 8 bit	99.71	55.57	0.92

From the expressions for ρ_i (15), (19) we see that the temporal correlation and hence transition activity for unsigned, 1's complement, and 2's complement representations are nearly equal. Also, the transition activity for sign magnitude is less than or equal to 2's complement because the number of sign bits in sign magnitude representation (one) is less than or equal to that in 2's complement representation. These conclusions are supported by the results in Table III.

IV. RESULTS WITH REALISTIC BENCHMARK SIGNALS

We have so far presented results using the synthetic signals in Table I. In this section, we will present simulation results for the signals in Table IV. First, we apply the approximate method (see section III(C)) to compare the measured and estimated T for these signals. Then, we process these signals through direct form FIR and IIR, transpose FIR and IIR filters to compute the total T in these structures.

A. Realistic benchmark signals

For the signals described in Table IV, the approximate method was employed to compare the measured and estimated T. The results are shown in Table VI where the measured T was calculated directly from the data. We assumed $\rho_{BP_1} = \rho$. To estimate BP_0 we assumed AR(1) models for all data sets except Audio5 and Video3. The measured and estimated BP_0 and BP_1 are shown in Table V. We used MA(10) models for Video3 and Audio5 because AR(1) models resulted in higher errors. In general, the model order should be chosen such that the mean square modeling error is below a certain level. In Table VI we see that for unsigned, 2's complement, and 1's complement representations, the estimation error in T is less than 10%. For sign magnitude, the error in T is less than 18%. The mean absolute error in t_i for Audio3 was 11%.

B. Total word-level transition activity, T, for FIR and IIR filters

In this subsection, we present the measured and estimated transition activity with signals in Table I and Table IV for a direct form filter and its transpose (see Table VII), and an IIR filter and its transpose (see Table VIII). The word-level statistics of the input are propagated through the fil-

TABLE V

MEASURED AND ESTIMATED BP_0 AND BP_1

Signal	BP_0		BP_1	
	Measured	Estimated	Measured	Estimated
Audio3	10	11	16	15
Audio4	9	10	15	15
Audio5	0	3	15	14
Audio6	4	9	14	14
Audio7	5	9	14	14
ATM	12	12	15	15
Video3	1	1	8	8

TABLE VI

WORD-LEVEL TRANSITION ACTIVITY

Signal	Uns., 2's comp.			One's complement			Sign magnitude		
	Meas.	Est.	% Err.	Meas.	Est.	% Err.	Meas.	Est.	% Err.
Audio3	6.42	6.32	1.56	6.43	6.32	1.71	6.17	6.24	1.13
Audio4	5.80	6.06	4.46	5.80	6.06	4.46	5.55	5.89	6.13
Audio5	4.78	4.40	7.95	4.79	4.40	8.14	4.22	4.23	0.24
Audio6	5.38	5.59	3.90	5.38	5.59	3.90	4.62	5.43	17.53
Audio7	5.05	5.52	9.31	5.05	5.52	9.31	4.78	5.44	13.81
ATM	7.76	7.56	2.58	7.76	7.56	2.58	7.09	6.94	2.12
Video3	2.31	2.15	6.93	2.31	2.15	6.93	2.16	2.15	0.15

TABLE VIII

TOTAL TRANSITION ACTIVITY FOR IIR FILTERS

Signal	Direct form			Transpose		
	Meas.	Est.	% Err.	Meas.	Est.	% Err.
SIG1	35.22	35.52	0.85	35.68	35.97	0.81
SIG2	18.36	18.21	0.82	16.82	16.38	2.62
SIG3	26.86	26.92	0.22	26.33	27.25	3.49
SIG4	17.77	17.86	0.51	16.11	15.92	1.18
SIG5	24.88	24.38	2.01	23.66	23.26	1.69
Audio3	24.36	24.26	0.41	22.92	22.56	1.57
Audio4	21.82	22.49	3.07	20.34	20.78	2.16
Audio5	18.06	17.15	5.04	16.99	15.55	8.48
Audio6	20.29	20.85	2.76	19.02	19.39	1.95
Audio7	18.87	20.33	7.74	17.38	18.59	6.96
ATM	30.59	29.25	4.38	30.17	28.68	4.94
Video3	7.69	7.74	0.65	6.22	6.93	11.41

TABLE IX

RUN TIMES IN SECONDS FOR DIRECT FORM FILTER

Signal	Simulation	Approx. method	Fast Method	DBT
Audio3	42.30	2.25	0.06	6.38
Audio4	40.28	2.41	0.13	6.40
Audio5	5.00	3.10	1.23	0.85
Audio6	8.60	2.58	0.18	1.46
Audio7	39.16	2.58	0.21	6.65
ATM	13.05	2.21	0.05	1.86
Video3	138.91	0.01	0.01	37.95

ter and then used to estimate T for each signal in the filter. These are then summed up to obtain T for the filter. The transfer function of the FIR filters is given by $H(z^{-1}) = 0.09765625 + 0.1953125z^{-1} + 0.39453125z^{-2} + 0.1953125z^{-3} + 0.09765625z^{-4}$. The transfer function of the IIR filters is given by $H(z^{-1}) = \frac{1}{1-0.1z^{-1}}$. The errors in T for all the filters are less than 12%. The estimated transition activity using the DBT method in [6] is also presented for the direct form FIR filter. Table IX compares the run times for simulation, the DBT method in [6], and the run time for our approximate method on a 85 MHz SparcStation 5. In most cases the run time for our approximate method is an order of magnitude less than that for simulation. The simulation time depends on the length of the input sequence whereas the time for the approximate method depends on the bit width of the signals (8 for video3 and 16 for the rest). This is because in our method, the computational complexity is determined by the calculation of p_i using (2) where the summation is over 2^B elements where B is the bit width. We can make the computation time of p_i essentially independent of bit width by calculating the sum over points spaced $2^{\frac{BP_0}{2}}$ apart with basically no change in the accuracy of the sum. The running time for the fast approximate method is also shown in Table IX.

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TABLE VII

TOTAL TRANSITION ACTIVITY FOR FIR FILTERS

Signal	Direct form					Transpose		
	Meas.	Est.	% Err.	DBT	% Err.	Meas.	Est.	% Err.
SIG1	148.3	148.9	0.1	150.1	1.2	145.5	146.0	0.4
SIG2	76.6	76.4	0.3	77.6	1.2	72.8	72.3	0.8
SIG3	113.2	113.6	0.4	110.3	2.5	109.0	109.4	0.4
SIG4	74.6	74.8	0.4	82.5	10.7	70.3	70.5	0.4
SIG5	104.6	102.1	2.4	99.3	5.1	101.4	98.6	2.8
Audio3	102.2	100.8	1.4	101.5	0.7	99.1	98.0	1.1
Audio4	91.4	94.4	3.3	96.7	5.8	88.4	90.6	2.5
Audio5	75.8	68.4	9.7	98.4	29.9	73.2	66.6	9.1
Audio6	84.9	86.6	2.0	96.9	14.1	82.0	83.1	1.3
Audio7	78.8	85.7	8.7	89.4	13.5	76.1	82.4	8.3
ATM	129.4	124.8	3.6	113.1	12.6	127.9	122.2	4.5
Video3	31.6	33.0	4.6	43.6	38.2	28.3	31.6	11.8