SPIE: Sparse Partial Inductance Extraction†

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ABSTRACT

Extracting the inductance of complex interconnect topologies is a formidable task, and simulating the resulting dense partial inductance matrix is even more difficult. Furthermore, it is well known that simply discarding smallest terms to sparsify the inductance matrix can render the partial inductance matrix indefinite and result in an unstable circuit model. In this paper, we describe a methodology for incrementally generating a sparse partial inductance matrix based on using moments about s=0 to determine when a sufficient number of mutual inductances have been captured. The minimally required mutual inductances are extracted for a provably stable model.

1.0 Introduction

Inductance extraction is difficult because mutual inductance depends on the current return path --- which is unknown prior to extracting and simulating a circuit model. Rosa introduced the concept of partial inductances [1][5] to avoid this difficulty by assuming that each segment has a return current at infinity. Ruehli introduced partial inductance to modern ICs and proposed the PEEC (Partial Equivalent Element Circuits) model to handle general three dimensional interconnects[6][7]. Weeks extended PEEC models to include skin effect and proximity effect shortly thereafter[8]. Kamon, et. al. more recently developed algorithms to solve for the effective inductance from the partial inductances with the utmost efficiency[2].

While partial inductance extraction methods are general and can be solved with excellent efficiency, the problem size can be overwhelming. Instead of coupling among all of the loops, there is now coupling among all of the wire segments. This corresponds to an extremely large, dense, partial inductance matrix. Because it is difficult to invert (factor) a large dense matrix, it is often desirable to sparsify the partial inductance matrix, either to serve as an approximate solution, or as a preconditioner for an iterative matrix equation solver.

One would expect that not all of the mutual couplings are important for an accurate simulation of the complete system. However, there are two problems associated with simply discarding small (far away) mutual coupling terms: 1) determining which mutuals are important without generating and simulating the entire inductance matrix, and 2) maintaining a stable circuit model.

The complete partial inductance matrix is positive definite. It has been shown that simply discarding mutual terms can render this matrix indefinite. We will show in this paper that an indefinite L matrix corresponds to a system that may violate conservation of energy. This can result in an equivalent circuit model with positive poles. Recently, a shifted formulation of magnetic potential vector was proposed to generate a provably-stable sparse partial inductance matrix by assuming that the segment currents return at a finite radius instead of infinity[3].

In this paper, the physical significance and modeling consequences of truncating far away mutual inductance are demonstrated. A methodology is described for finding the minimally required mutual inductances to accurately approximate the complete partial inductance matrix over the entire frequency spectrum.

2.0 Partial Inductance

Since inductance is defined only for closed loops, partial inductances can be visualized as the inductance of a conductor segment as it forms a loop with infinity. That is, the return current path for the inductance is assumed to close at infinity, as shown in Fig.1.

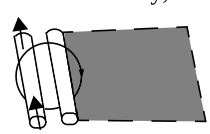

Fig. 1: Visualization of partial mutual inductance for 2 conductor segments. Both segment loops are assumed to close at infinity.

Partial inductances are best analyzed in terms of the normalized magnetic vector potential drop along a conductor segment due to current in that, or another segment. Consider two conductor segments, *i* and *j*, with a current I_j in segment *j*. The partial self inductance L_{jj} along the segment *j* is given by

$$L_{jj} = \frac{1}{I_j a_j}\left[\int_{a_j}\int_{l_j} \boldsymbol{A}_{jj} \cdot d\boldsymbol{l}_j da_j\right] \tag{1}$$

where $\boldsymbol{A}_{jj}$ is the magnetic vector potential along segment *j* due to the current I_j in segment *j*, which has a cross section a_j. The partial mutual inductance M_{ij}, which relates the induced voltage drop along segment *i* due to a change in the current along segment *j*, is given by a similar expression

$$M_{ij} = \frac{1}{I_j a_i}\left[\int_{a_i}\int_{l_i} \boldsymbol{A}_{ij} \cdot d\boldsymbol{l}_i da_i\right] \tag{2}$$

In (2) $\boldsymbol{A}_{ij}$ is the magnetic vector potential along segment *i* due to the current I_j in segment *j*. Segment *i* has a cross section a_i.

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The magnetic vector potential $\boldsymbol{A}_{ij}$ is defined as

$$\boldsymbol{A}_{ij} = \frac{\mu_0}{4\pi a_j}\left[\int_{a_j}\int_{l_j}\frac{I_j}{r_{ij}}\, d\boldsymbol{l}_j\, da_j\right] \qquad (3)$$

In this expression, r_{ij} is the geometric distance between two points in segment i and segment j.

3.0 Potential Instability

3.1 Eigen-Energy

The instability caused by discarding partial mutual inductance terms is most easily explained in terms of the eigenvalues and eigenvectors of the partial inductance matrix. Given a set of n conductor segments, the energy stored in the system's magnetic field is

$$E = \boldsymbol{i}^{\boldsymbol{T}} \cdot \boldsymbol{L} \cdot \boldsymbol{i} \qquad (4)$$

where $\boldsymbol{i}$ is the vector of currents flowing through the conductor segments and $\boldsymbol{L}$ is the partial inductance matrix, assuming no sources are included. According to conservation of energy, we know that

$$E \geq 0 \qquad (5)$$

From (4), it is apparent that in terms of the eigenvectors of the partial inductance matrix $\boldsymbol{L}$, the current vector $\boldsymbol{i}$ can be represented as

$$\boldsymbol{i} = \sum_{j=1}^{n} a_j \cdot \boldsymbol{v}_j \qquad (6)$$

where $\boldsymbol{v}_j$ is the normalized j th eigenvector of $\boldsymbol{L}$, and a_j is the coefficient of the linear expansion. From (6), the energy stored in the conductors can be expressed as *eigen-energy*

$$E = \sum a_j^2 \cdot \lambda_j \qquad (7)$$

In summary, physically speaking, an eigenvalue of the partial inductance matrix $\boldsymbol{L}$ represents some eigen-energy (stored in the magnetic field) of the interconnect system for a particular current assembly. Since the energy must always be non-negative, the eigenvalues are always positive real.

It is well known that the return current always takes the tightest closed return path at high frequency. This corresponds to the system achieving its lowest energy state, or smallest inductance path. From the point view of eigen-energy, the tightest closed return path is related to the eigen-direction for the smallest possible eigen-energy (assembly energy). So, the high frequency behavior is determined by the smallest eigenvalues of the partial inductance matrix.

It is only at low frequency that the return currents may assume large assembly energy instead of the smallest assembly energy, which means large eigen-energy is a significant portion of the assembly energy. So the large eigenvalues of the partial inductance matrix is a good monitor of low frequency behavior. From the pole and zero theory of the linear circuit, it is known that the smallest poles determine the low frequency behavior of the circuit. As a result, we can conclude that the largest eigenvalues of the partial inductance matrix correspond to the smallest poles of the circuit. This observation serves as a good theoretical basis for our methodology of extracting the minimum required mutuals in Section 5.0.

3.2 Potential Instability

In this subsection, we give an example to demonstrate the instability due to such a truncation procedure. Fig.2 describes a 3-D four-conductor system. The conductors are parallel to each other with circular cross section and physical dimensions as shown.

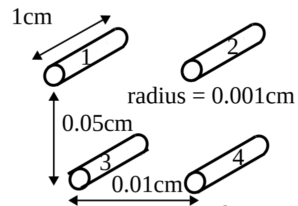

Fig. 2: An example of four-conductor system

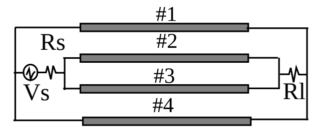

Fig. 3: Circuitry for the four-conductor system.

We calculated the partial inductance matrix and the eigenvalues of the full inductance matrix are all positive. However, discarding the smallest mutual terms in the matrix, we obtain the truncation only partial inductance matrix, which has a negative eigenvalue -0.27e-9. The corresponding eigenvector is $[1\ -1\ -1\ 1]^{\mathrm{T}}$

So, by truncating the smallest terms in the partial inductance matrix, we create a **negative** eigenvalue, which corresponds to a potential **negative** eigen-energy for approximating a **physically stable** system. If we now connect the four-conductor system with exterior circuitry as in Fig.3 (Rs and Rl are $1\,\Omega$ and $10\ \Omega$ respectively.) such that the eigen-direction corresponding to the negative eigenvalue is stimulated, a positive pole 4.3656e10 appears.

Note that truncation-only does not always cause positive poles -- more on this in Section 6.0.

4.0 Sparse Partial Inductance formulation with spherical return currents

In [3], a redefinition of the magnetic vector potential was proposed in order to sparsify the inductance matrix while maintaining its positive semi-definite property. Unlike normal partial inductance that assumes that conductor (partial) segment currents return at infinity (Fig.1), all incremental currents are assumed to return at a finite and constant radius r_0 from their origin. This is a reasonable approximation for realistic circuits, since currents can not really return at infinity.

The definition in (3), which describes the magnetic vector potential along conductor segment i due to an isolated current in conductor segment j, is replaced by

$$\mathbf{A}_{ij} = \frac{\mu_0 \boldsymbol{I}_j}{4\pi}\int_{l_j} f\left(r_{ij}, r_0\right) dl_j \qquad (8)$$

where

$$\begin{aligned} f\left(r_{ij}, r_0\right) &= \left(\frac{1}{r_{ij}} - \frac{1}{r_0}\right), \qquad & r_{ij} \leq r_0 \\ &= \quad 0 \quad , & r_{ij} > r_0 . \end{aligned} \qquad (9)$$

As demonstrated in [3], the spherical model of partial inductance can achieve significant sparsity. However, one problem remains for determining the required radial return distance r_0, without having to generate the entire inductance matrix. In the next section, we present an algorithm to find r_0.

5.0 Selecting the Return Current Radius

Consider the circuit behavior for a full partial inductance matrix which corresponds to the return current shell at infinity. When we decrease r_0, the possible return current is constrained within the radius of r_0. At high frequency, when the inductance dominates the impedance, the current will always take the tightest return path corresponding to the least inductance path and the restriction of the return path within r_0 will not affect the circuit simulation results.

At low frequency, resistance dominates the total impedance and the current will take the least resistance return path. If the least resistance return path is outside the constrained return current shell and the mutuals outside r_0 are significant, the behavior of the approximated circuit model will deviate dramatically from that of the full partial inductance model. Therefore, matching the low frequency behavior is the challenging problem for any sparse approximation over the frequency spectrum of interest.

The low frequency behavior of linear circuits is governed by the dominant poles i.e. the smallest poles, of circuit. Moments of the impulse response for an expansion about s=0 are known to be excellent indicators of the dominant pole value. Namely, the ratio of two successive moments, (m_j / m_{j+1}), is known to quickly converge to the value of dominant pole with increasing j[4]. Moreover, we know that the dominant poles of the circuit will stop changing with increasing r_0 as the added mutual inductances no longer affect the lowest frequency poles. So we will use the moment ratio as a convergence criterion for the sparse approximation. Fortunately, moments about s=0 are trivial to obtain for this problem.

Usually for inductance problems, mesh formulation is preferred because it requires a smaller number of unknowns compared to other types of formulation. However, for simplicity, we will explain moment computation using MNA formulation. For a linear circuit containing resistors and inductors, the Laplace-domain MNA formulation is

$$(\boldsymbol{G} + s\boldsymbol{C}(r_0)) \cdot \boldsymbol{x} = \boldsymbol{b} \tag{10}$$

where $\boldsymbol{x}$ contains the nodal voltages and the inductor currents, and $\boldsymbol{b}$ represents the sources. The matrix $\boldsymbol{G}$ contains the conductances and the connectivity information, and the matrix $\boldsymbol{C}(r_0)$ includes the partial inductance matrix as a block submatrix that is r_0 dependent. The first moment vector is given by

$$\boldsymbol{G} \cdot \boldsymbol{x}_0 = \boldsymbol{b} \tag{11}$$

To compute the first moment vector, the matrix G is LU factored. The matrix $\boldsymbol{G}$ is very sparse and can be factored very efficiently(it is nearly tri-diagonal for most packaging problems.). The higher order moment vectors are computed recursively as

$$\boldsymbol{G} \cdot \boldsymbol{x}_i = -\boldsymbol{C}(r_0) \cdot \boldsymbol{x}_{i-1}, \qquad i>0 \tag{12}$$

Since the matrix $\boldsymbol{G}$ has already been factored, for the higher order moment vectors, only forward and back substitutions are required. $\boldsymbol{C}(r_0)$ does not have to be factored. For each higher order moment, we multiply $\boldsymbol{C}(r_0)$ with a vector to obtain the right hand side vector in (12). Now, suppose that the output of interest is a linear combination of $\boldsymbol{x}$,

$$H(s) = \boldsymbol{l}^{\boldsymbol{T}} \cdot \boldsymbol{x} \tag{13}$$

Then, the moments of output are obtained as,

$$m_i = \boldsymbol{l}^T \cdot \boldsymbol{x}_i \tag{14}$$

The order of moments used here depends on level of accuracy. Usually, the higher order moments we utilized, the higher accuracy we obtained. However, for indirect coupling problems, the lower order moments are zero, while the higher order moments are non-zero. In such cases, we have to use higher order moment ratio to monitor low frequency behavior. Note that $\boldsymbol{G}$ is only factored once. So we continue increasing the value of r_0 until the moment ratio converges, which means sufficient mutuals have been captured. A brief description of our extraction algorithm is given as follows:

SPIE
{
read in the input file and the initial guess of $r_0^{\,0}$;
compute sparse formulation of partial inductance matrix $\boldsymbol{L}^0$;
use MNA to build the circuit equations in matrix form and calculate $\boldsymbol{G}^{-1}$;
calculate the moments to the order specified or such that $m_{j+1}^{\;0}$ is not zero;
$ratio^0 = m_j^{\;0} / m_{j+1}^{\;0}$;
$metric^0 = 1$;
$k=0$;
while($metric^k > error$)
{
increase $r_0^{\,k}$ to $r_0^{\,k+1}$ and compute the partial inductance matrix $\boldsymbol{L}^{k+1}$;
update the $\boldsymbol{C}(r_0)$ matrix and calculate $m_j^{\;k+1}$ and $m_{j+1}^{\;k+1}$ using $\boldsymbol{G}^{-1}$ and $\boldsymbol{C}(r_0)$, then $ratio^{k+1}$;
$metric^{k+1} = (ratio^k - ratio^{k+1}) / ratio^{k+1}$;
k++;
}
output the required information;
}

To demonstrate the efficacy of this approach, we consider the 2cm by 2 cm ground plane circuit in Fig.4.The plane is meshed into 20

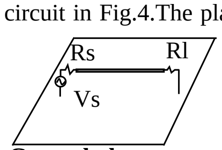

Fig. 4: Ground plane example.

segments both in the x and y directions of the plane. One 1.8cm long signal line runs 0.03mm above the plane and returns along the ground plane.

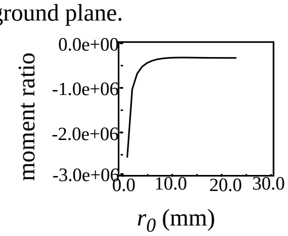

Fig. 5: The convergence of the moment ratio with respect to r_0 for the ground plane example.

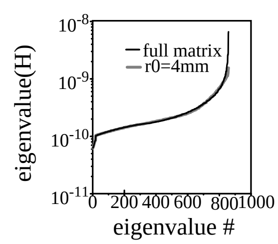

Fig. 6: Eigenvalues for the partial inductance matrix of the ground plane.

With the external connectivity and the partial inductance matrix for the inductances, the moment ratio for the current flowing through the signal line is easily calculated and the results are plotted in Fig.5. If we assume that the moment ratio reaches convergence (meaning the dominant pole no longer is changing as we add more mutuals) at r_0=4mm, we have a matrix sparsity of 95.02%. Because of the large size of the full dense partial inductance matrix, we are unable to compare the approximation result with the exact solution. However, we will assert correctness of our result in terms of eigen-value pattern of the partial inductance matrix in Fig.6.

Fig.6 gives the eigenvalue distributions for the full matrix and the sparsified matrix. One interesting observation is that the eigenvalues of the sparsified matrix agree very well with those of the full matrix for all but the few largest eigenvalues. Noting that these large eigenvalues of $\boldsymbol{L}$ correspond to the low frequency current assemblies, and since we are accurately approximating the dominant pole, we can assume that these eigenvalues correspond to impossible or negligible current assemblies given the connectivity of this circuit. We would further postulate that by matching the eigenvalues corresponding to the high frequency energies, we are providing an accurate approximation over all frequencies of interest.

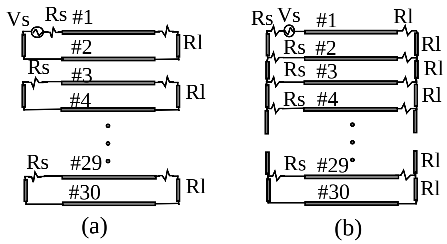

Fig. 7: Two circuits with identical partial inductance matrices (in the horizontal direction), but with different connectivity among the elements in circuits (a) and (b).

Next, we consider two 30-conductor packaging examples in Fig.7. The length of the lines is 0.5cm and the spacing between the lines is 0.2cm. Both circuits have the same horizontal topology but different connectivity. In order to consider the edge effect, we also model the vertical inductance effect in our extracted circuits. Referring to circuit Fig.7 (a), each signal line has its return path nearby. The r_0 for this circuit when moment ratio reaches convergence is 0.6cm, which translates to 84% sparsity for horizontal partial inductance matrix, and 90% sparsity for vertical partial inductance matrix. However, in circuit Fig.7 (b), all of the signal lines share one return path and the r_0 at moment ratio convergence is 1.2cm. The sparsity for horizontal partial inductance matrix and vertical partial inductance matrix are 50% and 65% respectively. Our methodology agrees with our understanding of good packaging design practice, namely, the circuit in Fig.7 (a) is a superior design.

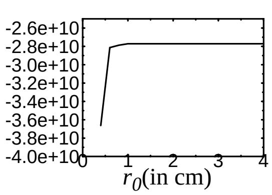

Fig. 8: Moment ratio convergence for circuit in Fig.7 (a).

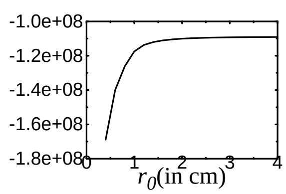

Fig. 9: Moment ratio convergence for circuit in Fig.7 (b).

6.0 Discussion and Alternative Potential Functions

For the original partial inductance concept, the return current is assumed at infinity. However, this introduces a full matrix, which is a formidable task for simulation. A simple solution is to truncate the smallest terms in the partial inductance matrix to achieve sparsity, but this can cause an unstable circuit model, as shown in Section 3.0. However, if the block diagonality is carefully preserved, that is, no indirect coupling between two uncoupled lines through a third line exists, the partial inductance matrix for truncation-only is still positive definite. The shift and truncation formulation of partial inductance assumes the return current is within a certain radius and is provably stable in any situation. For short lines, it works fine. But for long straight lines, we have to increase r_0 large enough to capture the forward coupling, or else the total loop inductance will be underestimated. A better model for long straight lines would be to use a tubular shell as plotted in Fig.10. The shell is infinitely long in the horizontal direction. The tubular shell may lead to unsymmetrical partial inductance matrix when the wires are routed in arbitrary direction. However, when wires are restricted in orthogonal directions, the partial inductance matrix remains symmetric.

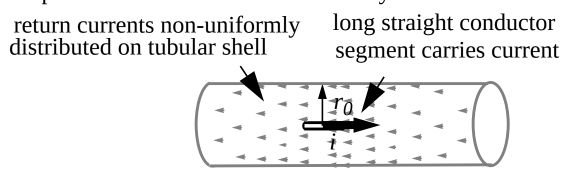

Fig. 10: Long straight conductor segment with current *i* and a cut away view of the tubular shell of return current *-i* at a radius r_0.

7.0 Conclusions

In this paper, we studied the physical significance and modeling consequences of discarding far away mutual terms in the partial inductance matrix. A moment ratio convergence criterion was proposed to create a methodology for determining the minimally required mutual inductance. For more complex interconnect structure where capacitance effect is considered, we expect to use a slightly different criterion. We also introduce alternative potential functions that may be useful for addressing the on-chip inductance problem.

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