

# A General Dispersive Multiconductor Transmission Line Model for Interconnect Simulation in SPICE

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## Abstract

*Although numerous methods have been proposed for interconnect simulation, no single model exists for all kind of transmission line problems. This paper presents a new, single, general dispersive coupled uniform/nonuniform transmission line model which can be used for interconnect simulation in SPICE. The mathematical model is based on the use of Chebyshev polynomials for the representation of the spatial variation of the transmission-line voltages and currents. A simple collocation procedure is used to obtain a matrix representation of the transmission line equations with matrix coefficients that are first polynomials in  $s$ , and in which terminal transmission-line voltages and currents appear explicitly. Thus, the model is compatible with both the SPICE's numerical integration algorithm and the modified nodal analysis formalism.*

## 1 Introduction

Simulation of package and chip interconnects using coupled transmission line models is now routine practice for noise, delay and overall signal degradation prediction in the design of high-speed electronic systems. Over the past fifteen years a variety of models have been proposed for interconnect simulation. In most cases, their development was driven by the desire to effect computationally efficient and accurate interconnect simulation within the framework of SPICE-like circuit simulators [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11].

A comparison of the aforementioned transmission-line models reveals variations in modeling accuracy and computation efficiency among the models. These variations are dependent on one or more of the following factors: a) the physical properties of the interconnects; b) the bandwidth of interest to the specific transient simulation; c) the specific design application. For example, the method of characteristics is ideal for modeling lossless interconnects; however, its extensions for handling lossy lines become computationally inefficient for electrically long lines of significant loss. On the other hand, model-order reduction schemes, such as AWE and its derivatives and enhancements, have been proven highly efficient for lossy transmis-

sion line simulation [5] [9] [11]. However it has been also observed that when the macromodels, obtained using order-reduction techniques, combined with the nonlinear drivers and loads, stability problems may arise. From the above discussion it becomes obvious that a single transmission-line model that can provide highly accurate broadband performance without penalizing efficiency is not available yet. Instead, "hybrid" approaches are being proposed where, with the frequency range of validity and computation efficiency of the various models properly quantified, a selection of the "optimal model" is made based on a predetermined modeling error and desired computation efficiency for the specific application of interest[12].

Even though such a "hybrid" approach is very sensible from an engineering point of view, there are important advantages associated with the availability of the single, general transmission-line model mentioned in the previous paragraph. First and foremost such a general model, equipped with an a-priori estimate of its accuracy, becomes an invaluable tool for evaluation and verification of other models, especially for those cases that the aforementioned "hybrid" approach is used. Second, there is a definitive simplicity associated with the use of a single, general model that is hard to overlook. In essence, a transmission line system becomes simply another "element" in the circuit simulator, handled the same way no matter what its per-unit-length electrical properties might be. Finally, the use of such a model for accurate simulation during the final stages of design for the purpose of design verification is highly desirable, especially if the model is compatible with model-order reduction schemes. The development of such a general transmission line model is the subject of this paper.

## 2 New Transmission Line Model

The general approach to include the multiconductor transmission line (MTL) systems into a circuit simulator is to treat them as linear multiports described by a suitable relationship between terminal voltages and currents:

$$\mathbf{A}(s)\mathbf{V}_t(s) + \mathbf{B}(s)\mathbf{I}_t(s) = 0 \quad (1)$$

where  $\mathbf{V}_t(s)$  and  $\mathbf{I}_t(s)$  are column vectors containing, respectively, the terminal voltages and currents of the

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MTL system. The matrices  $\mathbf{A}(s)$  and  $\mathbf{B}(s)$  are described in terms of line parameters. In order to be able to use numerical integration algorithm to analyze circuits containing transmission lines, the elements of the matrices  $\mathbf{A}(s)$  and  $\mathbf{B}(s)$  need to be first-degree polynomials in  $s$ . The development of such a model using Chebyshev polynomials is presented next.

Let  $l$  be the length of the transmission line system, i.e. ( $0 \leq x' \leq l$ ). The transformation  $x = 2x'/l - 1$  maps the domain  $[0, l]$  onto the domain  $[-1, 1]$ , which is convenient in dealing with Chebyshev polynomials. For the sake of simplicity, we consider first the simple two-conductor transmission line case. The transformed Telegrapher's equations for a two-conductor transmission line of length  $l$ , with one of the conductors taken as reference, are:

$$\frac{d}{dx}V(x, s) = -\frac{l}{2}(R(x) + sL(x))I(x, s) \quad (2)$$

$$\frac{d}{dx}I(x, s) = -\frac{l}{2}(G(x) + sC(x))V(x, s) \quad (3)$$

where  $V(x, s)$  and  $I(x, s)$  are, respectively, voltage and current distribution along the line;  $R(x)$ ,  $L(x)$ ,  $C(x)$ , and  $G(x)$  are per-unit-length (p.u.l.) line parameters. For the sake of clarity, the model is first developed for the case of lines with frequency-independent p.u.l. parameters. The extension of the model to lines with frequency-dependent p.u.l. parameters is given in Subsection 2.1. In the proposed methodology, the line voltages and currents are approximated by their truncated Chebyshev expansions:

$$V(x, s) = \sum_{m=0}^M a_m(s)T_m(x) \quad (4)$$

$$I(x, s) = \sum_{m=0}^M b_m(s)T_m(x) \quad (5)$$

where  $T_m(x) = \cos(m \cos^{-1} x)$  is the  $m$ th degree Chebyshev polynomial of type 1. The choice of Chebyshev polynomials for the representation of the spatial variation of the line voltage and current is motivated by the exponential rate of convergence of Chebyshev expansions [13]. Because of this property, highly accurate approximations of the voltage and current distributions along the lines can be effected with a small number of polynomials ( $M$ ). More specifically, it can be shown that  $\pi$  expansion functions per wavelength are required for a highly accurate resolution of a sinusoidal variation using a truncated Chebyshev representation [13]. In view of this, the following expression is used for the selection of the number of Chebyshev polynomials for the representation of the spatial voltage and current distributions on a line of length  $l$

$$M = 4 \frac{l}{\lambda_{\min}} + 2 \quad (6)$$

where  $\lambda_{\min}$  is the minimum wavelength of interest.

Let  $V(x_n, s)$ ,  $I(x_n, s)$ ,  $n = 0, 1, 2, \dots, M$ , be the voltage and current values at the points  $x_n$  defined by

$$x_n = \cos(\pi n/M), \quad n = 0, 1, \dots, M. \quad (7)$$

Clearly,  $x_0 = 1$  and  $x_M = -1$  correspond, respectively, to the far- and near-end terminals of the transmission line. It can be shown that the coefficients  $a_m$  and  $b_m$  in (4) and (5) are given by [13]

$$a_m(s) = \frac{2}{M} \frac{1}{c_m} \sum_{n=0}^M \frac{V(x_n, s)T_m(x_n)}{c_n} \quad (8)$$

$$b_m(s) = \frac{2}{M} \frac{1}{c_m} \sum_{n=0}^M \frac{I(x_n, s)T_m(x_n)}{c_n} \quad (9)$$

where  $c_m$  is defined as

$$c_m = \begin{cases} 1, & m \neq 0, M \\ 2, & \text{otherwise} \end{cases} \quad (10)$$

Substituting (8) and (9) into (4) and (5), respectively,

$$V(x, s) = \sum_{m=0}^M V(x_m, s)g_m(x) \quad (11)$$

$$I(x, s) = \sum_{m=0}^M I(x_m, s)g_m(x) \quad (12)$$

where  $g_m(x)$  are given by

$$g_m(x) = \frac{(1-x^2)T'_M(x)(-1)^{m+1}}{c_m M^2(x-x_m)} \quad (13)$$

As expected, the polynomials  $g_m(x)$  have the Lagrange polynomial-type property  $g_m(x_n) = \delta_{mn}$ , where  $\delta_{mn}$  is the Kronecker delta. To obtain an approximation to the differential equations given in (2) and (3), a collocation method is used with collocation points those in (7). Substituting (11) and (12) into (2) and (3),

$$\sum_{m=0}^M V(x_m, s) \frac{d}{dx} g_m(x) = -\frac{l}{2}(R(x) + sL(x)) \times \sum_{m=0}^M I(x_m, s)g_m(x) \quad (14)$$

$$\sum_{m=0}^M I(x_m, s) \frac{d}{dx} g_m(x) = -\frac{l}{2}(G(x) + sC(x)) \times \sum_{m=0}^M V(x_m, s)g_m(x) \quad (15)$$

To perform the collocation, the derivatives of polynomials  $g_m$  at the collocation points are needed. They are found to be

$$\left. \frac{d}{dx} g_m(x) \right|_{x=x_n} = D_{nm} \quad (16)$$

where

$$D_{nm} = \frac{c_n}{c_m} \frac{(-1)^{n+m}}{x_n - x_m} \quad (n \neq m)$$

$$D_{nn} = -\frac{x_n}{2(1-x_n^2)} \quad 1 \leq n \leq M-1$$

$$D_{00} = \frac{2M^2+1}{6} = -D_{M,M}$$

In view of the above results and definitions, the system of linear equations obtained from the collocation process may be cast in the form

$$\sum_{m=0}^M V(x_m, s) D_{nm} = -\frac{l}{2}(R(x_n) + sL(x_n))I(x_n, s)$$

$$\sum_{m=0}^M I(x_m, s) D_{nm} = -\frac{l}{2}(G(x_n) + sC(x_n))V(x_n, s)$$

for  $n = 0, 1, \dots, M$ .

Let  $\mathbf{D}$  be the  $(M+1) \times (M+1)$  square matrix with elements  $D_{nm}$ . Then, the above system of equations may be cast in matrix form as follows:

$$\mathbf{D}\mathbf{V}_s(s) = -\mathbf{Z}(s)\mathbf{I}_s(s) \quad (17)$$

$$\mathbf{D}\mathbf{I}_s(s) = -\mathbf{Y}(s)\mathbf{V}_s(s) \quad (18)$$

where

$$\mathbf{V}_s(s) = [V(x_0, s), V(x_1, s), \dots, V(x_M, s)]^T$$

$$\mathbf{I}_s(s) = [I(x_0, s), I(x_1, s), \dots, I(x_M, s)]^T$$

and,  $\mathbf{Z}(s)$  and  $\mathbf{Y}(s)$  are diagonal matrices

$$\mathbf{Z}(s) = \frac{l}{2} \text{diag}\{R(x_0) + sL(x_0), \dots, R(x_M) + sL(x_M)\}$$

$$\mathbf{Y}(s) = \frac{l}{2} \text{diag}\{G(x_0) + sC(x_0), \dots, G(x_M) + sC(x_M)\}$$

The next step is to express (17) and (18) in terms of the terminal voltages and currents of the transmission line. For this purpose, we use the following colon notation to select specific rows and columns of a matrix. Let submatrix of  $A$  that is between the  $i$ th and  $j$ th rows, and  $m$ th and  $n$ th columns of  $\mathbf{A}$ . Similarly,  $\mathbf{A}^{(i,j,m)}$  is a column vector of length  $(j-i+1)$  having as elements the elements of the  $m$ th column of the matrix  $\mathbf{A}$  between (and including) rows  $i$  and  $j$ . Recognizing that (17) and (18) constitute the approximation of a two-point boundary value problem, two boundary conditions (involving the values of terminal voltages or the values of the terminal currents or impedance relationships between the terminal voltages and currents) need be specified for the problem to be well posed. This implies that two of the equations in (17) and (18) associated with the terminal quantities need be eliminated in favor of the aforementioned boundary conditions. Without loss of generality, the first and last of the equations in (17) are the ones eliminated. Consequently, using the aforementioned colon notation (17) and (18) are cast in the form

$$\begin{bmatrix} \mathbf{D}_a & \mathbf{D}_b \\ \mathbf{Y}_a(s) & \mathbf{Y}_b(s) \end{bmatrix} \begin{bmatrix} V_{near}(s) \\ V_{far}(s) \end{bmatrix} +$$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{Z}_a(s) & \mathbf{D}_c \\ \mathbf{D}_d & \mathbf{D}_e & \mathbf{D}_f & \mathbf{Y}_c(s) \end{bmatrix} \begin{bmatrix} I_{near}(s) \\ I_{far}(s) \\ \mathbf{I}_s(s) \\ \hat{\mathbf{V}}_s(s) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (19)$$

where the following abbreviated array notation has been used:  $\mathbf{D}_a = \mathbf{D}_{(2:M, M+1)}$ ,  $\mathbf{D}_b = \mathbf{D}_{(2:M, 1)}$ ,  $\mathbf{D}_c = \mathbf{D}_{(2:M, 2:M)}$ ,  $\mathbf{D}_d = \mathbf{D}_{(1:M+1, M+1)}$ ,  $\mathbf{D}_e = -\mathbf{D}_{(1:M+1, 1)}$ ,  $\mathbf{D}_f = \mathbf{D}_{(1:M+1, 2:M)}$ ,  $\mathbf{Y}_a(s) = \mathbf{Y}_{(1:M+1, M+1)}(s)$ ,  $\mathbf{Y}_b(s) = \mathbf{Y}_{(1:M+1, 1)}(s)$ ,  $\mathbf{Y}_c(s) = \mathbf{Y}_{(1:M+1, 2:M)}(s)$ ,  $\mathbf{Z}_a(s) = \mathbf{Z}_{(2:M, 2:M)}(s)$ . Also,  $V_{near}(s) = V(x_M, s)$ ,  $V_{far}(s) = V(x_0, s)$ ,  $I_{near}(s) = I(x_M, s)$ ,  $I_{far}(s) = -I(x_0, s)$ , while

$$\hat{\mathbf{V}}_s(s) = [V(x_1, s), V(x_2, s), \dots, V(x_{M-1}, s)]^T$$

$$\hat{\mathbf{I}}_s(s) = [I(x_1, s), I(x_2, s), \dots, I(x_{M-1}, s)]^T$$

Equation (19) can be written as,

$$(\mathbf{A}^R + s\mathbf{A}^I)\mathbf{V}_t(s) + (\mathbf{B}^R + s\mathbf{B}^I)\mathbf{J}(s) = \mathbf{0} \quad (20)$$

where

$$\mathbf{J}(s) = \begin{bmatrix} \mathbf{I}_t(s) \\ \mathbf{V}(s) \\ \hat{\mathbf{I}}(s) \end{bmatrix},$$

and  $\mathbf{V}_t = [V_{near}, V_{far}]^T$ ,  $\mathbf{I}_t = [I_{near}, I_{far}]^T$  are, respectively, the vectors of terminal voltages and currents of the line. In the time domain, the above equation becomes

$$\mathbf{A}^R \mathbf{v}_t(t) + \mathbf{A}^I \frac{d}{dt} \mathbf{v}_t(t) + \mathbf{B}^R \mathbf{j}(t) + \mathbf{B}^I \frac{d}{dt} \mathbf{j}(t) = \mathbf{0} \quad (21)$$

which is compatible with the numerical integration algorithm used in SPICE.

The development of the model for the case of MTL's will not be presented here. It is very similar to the one for the two-conductor line. This is easily seen by noting that for an  $(N+1)$ -conductor MTL (with the  $(N+1)$ st conductor taken as reference) the collocation procedure that led to (17) and (18) produces now the following system of equations

$$\begin{bmatrix} \mathbf{D} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}^1(s) \\ \vdots \\ \mathbf{V}^N(s) \end{bmatrix} =$$

$$- \begin{bmatrix} \mathbf{Z}^{11}(s) & \dots & \mathbf{Z}^{1N}(s) \\ \vdots & \ddots & \vdots \\ \mathbf{Z}^{N1}(s) & \dots & \mathbf{Z}^{NN}(s) \end{bmatrix} \begin{bmatrix} \mathbf{I}^1(s) \\ \vdots \\ \mathbf{I}^N(s) \end{bmatrix}, \quad (22)$$

$$\begin{bmatrix} \mathbf{D} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{I}^1(s) \\ \vdots \\ \mathbf{I}^N(s) \end{bmatrix} =$$

$$- \begin{bmatrix} \mathbf{Y}^{11}(s) & \dots & \mathbf{Y}^{1N}(s) \\ \vdots & \ddots & \vdots \\ \mathbf{Y}^{N1}(s) & \dots & \mathbf{Y}^{NN}(s) \end{bmatrix} \begin{bmatrix} \mathbf{V}^1(s) \\ \vdots \\ \mathbf{V}^N(s) \end{bmatrix} \quad (23)$$

where  $\mathbf{V}^i(s)$  is the vector of voltage samples along the  $i$ th conductor,

$$\mathbf{V}^i(s) = [ V^i(x_0, s) \quad V^i(x_1, s) \quad \cdots \quad V^i(x_M, s) ]$$

and

$$\mathbf{Z}^{ij} = \frac{l}{2} \text{diag}\{R^{ij}(x_0) + sL^{ij}(x_0), \dots, R^{ij}(x_M) + sL^{ij}(x_M)\}$$

$\mathbf{I}^i(s)$  and  $\mathbf{Y}^{ij}(s)$  are defined similarly.

## 2.1 Transmission lines with frequency-dependent line parameters

It is well-known that high-frequency wave propagation in transmission lines in inhomogeneous media is characterized by geometric dispersion. Furthermore, lossy substrates and the finite conductivity of the conductors combine with the aforementioned geometric dispersion and result in frequency dependent p.u.l. line parameters. The frequency dependence needs to be taken into account for the accurate simulation of high-speed pulse propagation in interconnects. In the following, we extend the methodology of the previous section to the case with frequency-dependent line parameters.

Even though the methodology is also applicable to nonuniform lines, we choose to develop the model for the case of a uniform line in order to keep the notation as simple as possible. For a uniform line with frequency-dependent line parameters the transformed Telegrapher's equations become

$$\frac{d}{dx}V(x, s) = -\frac{l}{2}Z(s)I(x, s) \quad (24)$$

$$\frac{d}{dx}I(x, s) = -\frac{l}{2}Y(s)V(x, s) \quad (25)$$

where p.u.l. impedance and admittance functions,  $Z(s)$  and  $Y(s)$ , are related to the p.u.l. line parameters through the expressions

$$Z(s = j\omega) = R(\omega) + j\omega L(\omega) \quad (26)$$

$$Y(s = j\omega) = G(\omega) + j\omega C(\omega) \quad (27)$$

It will be assumed that the p.u.l. line parameters are either calculated over the frequency range of interest using a full-wave analysis method, or are determined from measurements. In the proposed method, we approximate the p.u.l. impedance and admittance functions with rational functions of  $s$ ,

$$Z(s) = \frac{b_Z(s)}{a_Z(s)} = \frac{b_{Z0} + \cdots + b_{Zq}s^q}{a_{Z0} + \cdots + a_{Zq}s^q} \quad (28)$$

$$Y(s) = \frac{b_Y(s)}{a_Y(s)} = \frac{b_{Y0} + \cdots + b_{Yq}s^q}{a_{Y0} + \cdots + a_{Yq}s^q} \quad (29)$$

where  $q$  is the maximum of the orders of the four polynomials. The coefficients of the rational functions are forced to be real and obtained from the available  $R, L, C, G$  data using least squares fitting. More discussion about this issue can be found in [11].

Then the MNA stamp given in (19) becomes

$$\left[ \begin{array}{c|c} \mathbf{D}_a & \mathbf{D}_b \\ \hline \mathbf{0} & \frac{b_Y(s)}{a_Y(s)} \\ \frac{b_Y(s)}{a_Y(s)} & \mathbf{0} \end{array} \right] \begin{bmatrix} V_{near}(s) \\ V_{far}(s) \end{bmatrix} +$$

$$\left[ \begin{array}{c|c|c|c} \mathbf{0} & \mathbf{0} & \frac{b_Z(s)}{a_Z(s)}\mathbf{1} & \mathbf{D}_c \\ \hline \mathbf{D}_d & \mathbf{D}_e & \mathbf{D}_f & \frac{b_Y(s)}{a_Y(s)}\mathbf{1} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0}^T & \mathbf{0}^T \end{array} \right] \begin{bmatrix} I_{near}(s) \\ I_{far}(s) \\ \mathbf{I}_s(s) \\ \mathbf{V}_s(s) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

where  $\mathbf{1}$ 's and  $\mathbf{0}$ 's are identity matrices and column vectors of zeros, respectively, with suitable sizes. Multiplying the first  $M-1$  rows of the above matrices with  $a_Z(s)$  and the remaining  $M+1$  rows with  $a_Y(s)$ , we obtain

$$(\mathbf{A}_0 + \cdots + \mathbf{A}_q s^q)\mathbf{V}_t + (\mathbf{B}_0 + \cdots + \mathbf{B}_q s^q)\mathbf{J} = \mathbf{0} \quad (30)$$

where,

$$\mathbf{A}_i = \left[ \begin{array}{c|c} a_{Zi} \cdot \mathbf{D}_a & a_{Zi} \cdot \mathbf{D}_b \\ \hline \mathbf{0} & b_{Yi} \\ b_{Yi} & \mathbf{0} \end{array} \right]$$

and

$$\mathbf{B}_i = \left[ \begin{array}{c|c|c|c} \mathbf{0} & \mathbf{0} & b_{Zi} \cdot \mathbf{1} & a_{Zi} \cdot \mathbf{D}_c \\ \hline a_{Yi} \cdot \mathbf{D}_d & a_{Yi} \cdot \mathbf{D}_e & a_{Yi} \cdot \mathbf{D}_f & b_{Yi} \cdot \mathbf{1} \\ \hline \mathbf{0}^T & \mathbf{0}^T & \mathbf{0}^T & \mathbf{0}^T \end{array} \right]$$

Next, we introduce a set of new vectors,

$$\begin{aligned} s\mathbf{V}_t &= \mathbf{V}_{t1} \\ &\vdots \\ s\mathbf{V}_{t(q-1)} &= \mathbf{V}_{tq} \end{aligned} \quad (31)$$

and

$$\begin{aligned} s\mathbf{J} &= \mathbf{J}_1 \\ &\vdots \\ s\mathbf{J}_{q-1} &= \mathbf{J}_q \end{aligned} \quad (32)$$

Then, (30), (31), and (32) may be cast in the form

$$\begin{bmatrix} \mathbf{A}_0 \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ s\mathbf{1} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \mathbf{V}_T + \begin{bmatrix} \mathbf{B}_0 & \mathbf{B}_1 & \cdots & \mathbf{B}_q & \mathbf{A}_1 & \cdots & \mathbf{A}_q \\ s\mathbf{1} & -\mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & s\mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{1} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & s\mathbf{1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{1} \end{bmatrix}$$

$$\times \begin{bmatrix} \mathbf{J} \\ \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_q \\ \mathbf{V}_{T1} \\ \mathbf{V}_{T2} \\ \vdots \\ \mathbf{V}_{Tq} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \quad (33)$$

where  $\mathbf{0}$ 's are either column vectors or matrices of zeros with suitable sizes. Equation (33) can be written as

$$(\mathbf{A}^R + s\mathbf{A}^I)\mathbf{V}_i(s) + (\mathbf{B}^R + s\mathbf{B}^I)\mathbf{J}(s) = \mathbf{0}. \quad (34)$$

If the line has frequency-independent parameters, (20) is stenciled to the MNA matrix instead of (34).

This result for the two-conductor line can be extended easily to the case of MTL systems. For such systems, the entries of the p.u.l. impedance and admittance matrices are approximated as

$$Z^{ij} = \frac{b_Z^{ij}(s)}{a_Z^i(s)}, \quad Y^{ij} = \frac{b_Y^{ij}(s)}{a_Y^i(s)}, \quad i, j = 1, \dots, N$$

The reason for choosing the same denominator polynomial for all entries in a row of the p.u.l. impedance and admittance matrices can be explained as follows. In the MNA stamp of a MTL system, any row contains only one row of either the p.u.l. impedance or admittance matrix. Therefore, when we multiply that row with  $a_Z^i(s)$  or  $a_Y^i(s)$ , we obtain an equation set which has polynomials in  $s$  as coefficients and the maximum order of these polynomials is kept as small as possible.

### 3 Numerical Experiments

The new transmission line model has been implemented in SPICE3f4 and several example circuits have been tested. We present three examples in this section.

*Example 1:* Consider the stripline circuit shown in Fig. 1. The stripline structure was analyzed using the method given in [14] and the p.u.l. line inductance and resistance parameters were calculated at a set of frequency points. Then, a third order rational approximation was obtained for the impedance function,  $Z(s)$ . Fig. 2 depicts the comparison of the output voltage obtained a) using the frequency-dependent  $R$  and  $L$  parameters and b) using frequency-independent values for  $R$  and  $L$ , namely, the (infinite-frequency) external inductance value for  $L$  and the dc value for  $R$ .

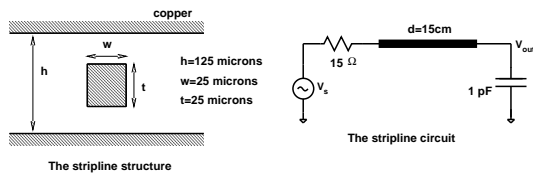


Figure 1: The circuit for example 1.

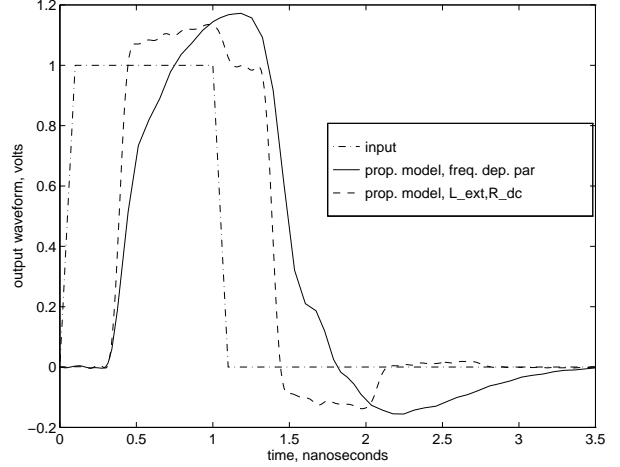


Figure 2: The response of the circuit of example 1.

Clearly, the pulse suffers significant dispersion which is not accurately modeled using simply the dc value of the p.u.l. resistance. The CPU time for dispersive case was 0.84 seconds.

*Example 2:* Consider the interconnect circuit shown in Fig. 3. The input to the driver is a pulse of 0.1 ns rise and fall times and 5 ns duration. The output response is shown in Fig. 4. In the same figure the SPICE output is also given. The SPICE output was found using SPICE's lossy line model. For the proposed model the CPU time was 26.73 seconds, while the lossy line model requires 87.85 seconds.

*Example 3:* The last example is the packaging interconnection system shown in Fig. 5. The coupled lines are identical and their length is  $l = 5$  cm. The p.u.l. line parameters are : ( $L_{11} = L_{22} = L_{33} = 4.976, L_{12} = 0.765, L_{13} = 0.152$ )nH/cm, ( $C_{11} = C_{33} = 1.082, C_{22} = 1.124, C_{12} = -0.197, C_{13} = -0.006$ )pF/cm, and  $R_{11} = R_{22} = R_{33} = 3.448\Omega/cm$ . The output waveform is shown in Fig. 6. The CPU time was 18.92 seconds. The same circuit can be also analyzed using SPICE's lossy MTL model. However, it works under some assumptions: a-) restricts coupling to between adjacent wires only, b-)the lines must be identical and equally spaced [7]. In this case the simulation of the above circuit takes 10.35 seconds.

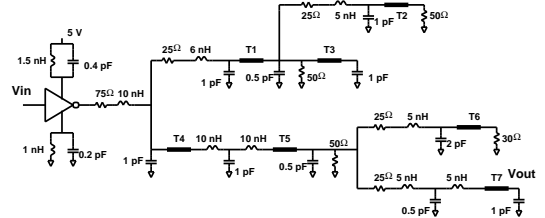


Figure 3: The interconnect circuit.

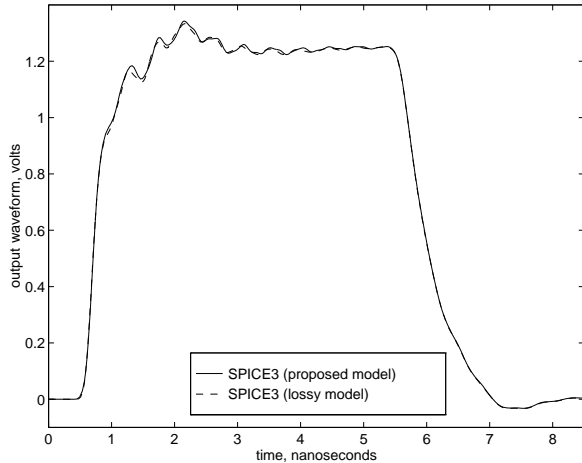


Figure 4: The response of the interconnect circuit.

## 4 Conclusions

We have introduced a new mathematical model that allows the simulation of dispersive, multiconductor, uniform/nonuniform transmission lines in SPICE. The new model is general, robust and does not require any pre-processing before the numerical integration. We have implemented the new model in SPICE3f4 and demonstrated the validity and generality of the proposed method by testing several example circuits.

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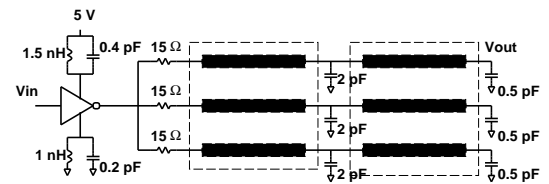


Figure 5: The packaging interconnection circuit.

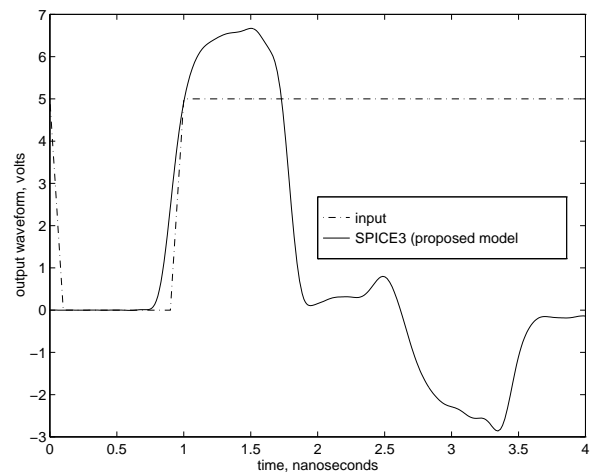


Figure 6: The response of the circuit of example 3.