An Optimal ILP Formulation for Minimizing the Number of Feedthrough Cells in Standard Cell Placement

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Abstract

Standard cell design style has been widely applied for the design automation of VLSI circuits because of the easy implementation of the layout design. Since the aim of most of standard cell design systems is to minimize the utilization of chip area, the number of feedthrough cells in a standard cell layout will be further minimized to reduce the layout size. In this paper, first, we model a row assignment problem to minimize the number of feedthrough cells in a standard cell placement. Furthermore, an integer linear programming (ILP) optimal approach is proposed to minimize the number of feedthrough cells for the row assignment in a standard cell placement. Finally, two standard cell benchmarks, Primary1 and Primary2, have been tested on the proposed ILP approach for the assignment of different number of rows, and the experimental results show that the ILP approach is efficient for the assignment of cell rows.

1. Introduction

Recently, the partitioning-based approach[1-7] has been widely applied to obtain a standard cell placement. In Fig. 1, a standard cell netlist, the result of the placement phase, and the results of global routing after the assignment of net pins are shown. Basically, the approach is divided into the netlist partitioning, the row assignment (the inter-row placement) and the multi-row linear placement (the intra-row placement). Due to the row-based structure of a standard cell placement, the partitioning techniques have been applied to the row netlist-clustering in a standard cell placement. According to the aspect ratio and the area estimation in a stand cell layout, the number of rows K in a desirous placement will be further obtained. In the netlist partitioning, the standard cells in a cell netlist are clustered into at most K groups of standard cells by a hierarchical or multi-way partitioning operation. First, Dunlop and Kernighan[1] applied Kernighan&Lin-based two-way min-cut partitioning to hierarchically partition a cell netlist into smaller netlists for a standard cell placement. Furthermore, Sauris and Kedem[2] proposed quadrisecion partitioning for a standard cell placement. Hamada, Cheng and Chau[5] proposed two-way minimum ratio-cut partitioning to partition a cell netlist into smaller netlists for a standard cell placement. However, these partitioning-based approaches combine the netlist partitioning with the row assignment or the multi-row linear placement in a standard cell placement into one single integrated phase. Hence, the minimization of the number of feedthrough cells will not be clearly considered in such an integrated phase for a standard cell placement.

For the consideration of feedthrough cells, the minimization of the number of feedthrough cells in a standard cell placement is an NP-complete problem[9]. Cho and Kyung[3] first proposed a heuristic algorithm based on a constrained multi-stage graph to minimize the number of feedthrough cells in a standard cell placement. Furthermore, Bose and Saha[4] also proposed a heuristic algorithm for the row assignment in a standard cell placement. However, these two approaches only partition the cell netlist into two cell subnetslists by a Kernighan&Lin-based two-way min-cut partitioning algorithm. The further netlist partitioning of standard cells will be combined with the row assignment of standard cells, and the number of feedthrough cells will seriously depend on the selection of seed nets in the two-way min-cut partitioning. Hence, the minimization of the number of feedthrough cells in such an integrated row assignment is more difficult than that in a pure row assignment for a standard cell placement. Hence, based on the partitioning-based approach[6-7], we further propose a novel partitioning-based approach for a standard cell placement. Basically, the approach is divided into three phases: the multi-way netlist...
partitioning, the row assignment (the inter-row placement) and the multi-row linear placement (the intra-row placement). In the multi-way netlist partitioning phase, based on the number of rows \( K \) in a desirable placement and the constraint of area in a row, the standard cells will be partitioned into \( K \) cell subnets lists by a multi-way area-ratio-constrained partitioning[8]. Furthermore, the \( K \) subnets lists will be linearized to minimize the number of feedthrough cells in the row assignment phase. Finally, \( K \) subnets lists will be linearized as \( K \) rows of standard cells to minimize the total channel density in the multi-row linear placement phase for a standard cell placement.

In this paper, we model the row assignment phase in the partitioning-based approach to minimize the number of feedthrough cells in a standard cell placement. Furthermore, an integer linear programming (ILP) optimal approach is proposed to minimize the number of feedthrough cells for the row assignment in a standard cell placement. Finally, two standard cell benchmarks, Primary1 and Primary2, have been tested on the proposed approach for the assignment of different number of rows, and the experimental results show that the ILP approach is efficient for the assignment of cell rows.

2. Problem Modeling

Before we model the row assignment problem of minimizing the number of feedthrough cells in a standard cell placement, some important assumptions will be described as follows:

First, assume that each feedthrough cell only contains one vertical net segment crossing one row of standard cells. Therefore, the number of vertical net segments crossing one row of standard cells will be the number of feedthrough cells in the row of standard cells. Second, in general, the pins of a net are fully distributed onto two adjacent rows, the net is a local net. In contrast, the pins of a net are not fully distributed onto two adjacent rows, the net is a global net. Although the connection of a local net may contains some feedthrough cells after the pin assignment, it will be reasonable that the connection of a local net is free of any feedthrough cell in the placement phase. Hence, it is assumed that the connection of a local net is free of any feedthrough cell. In contrast, the connection of a global net is obtained by adding feedthrough cells. Furthermore, although the pins of a global net are distributed into different groups by a multi-way partitioning, it will be assumed that each global net generates at most one feedthrough cell in each row of standard cells after the row assignment. Hence, for a global net, the net will be separated into several local nets by assigning at most one feedthrough cell in each row of standard cells. The connection of a global net and the assignment of a feedthrough cell will be shown in Fig. 1(c).

As mentioned above, given the number of rows in a standard cell placement, if the standard cells are partitioned into desirable groups by a multi-way partitioning, the number of feedthrough cells will be further minimized by an optimal row assignment to reduce the size of the standard cell layout. Hence, these partitioned groups of standard cells will construct a separation graph for the row assignment in a standard cell placement as follows:

**Definition 1**: For a \( k \)-way partitioning of a standard cell netlist, a separation graph \( G(V, E) \) is an undirected edge-weighted graph defined as follows: any cluster in the \( k \)-way partitioning is represented as one vertex of \( V \). Each edge \((u, v) \) in \( E \) if and only if two clusters corresponding to \( u \) and \( v \) are connected by at least one net, and the weight \( w(u, v) \) of \((u, v) \) is the number of nets connecting the two clusters corresponding \( u \) and \( v \).

![Fig. 2](image)

(a) Four partitioned groups of standard cells.
(b) A separation graph for Fig. 2(a).

Four partitioned groups of standard cells and a separation graph are shown in Fig. 2, where \( V = \{v_1, v_2, v_3, v_4\} \), \( E = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3), (v_2, v_4), (v_3, v_4)\} \) and \( w(e_{12}) = 1, w(e_{13}) = 2, w(e_{14}) = 4, w(e_{23}) = 3, w(e_{24}) = 1, \) and \( w(e_{34}) = 5 \). Furthermore, due to the vertical linear structure of a standard cell placement, the row assignment in a standard cell placement will belong to the linear embedding of a separation graph. In order to solve the row assignment in a standard cell placement, the linear embedding of a graph will be defined as follows:

**Definition 2**: For an undirected graph \( G(V, E) \), a linear embedding \( E(G) \) of \( G \) is defined as a linear ordering \( \{v_1, v_2, \ldots, v_n\} \) of \( V \). If \( (v_i, v_{i+1}) \), \( 1 \leq i \leq n-1 \), is in \( E \), the edge \((v_i, v_{i+1}) \) is called as an embedding edge. In contrast, for a linear embedding \( E(G) \), if any edge \((u, v) \) in \( E \) is not an embedding edge, the edge \((u, v) \) is called as a non-embedding edge.

In order to minimize the number of feedthrough cells for the row assignment in a standard cell
placement, the crossing distance of any edge and the total crossing distance in a linear embedding of G will be defined as follows:

**Definition 3**: For a linear embedding \( E(G) \) of \( G(V, E) \), the crossing distance \( \text{Dist}(u, v) \) of any edge \((u, v)\) will be defined as

\[
\text{Dist}(u, v) = \omega(u, v) \ast \text{Diff}(u, v),
\]

where \( \text{Diff}(u, v) \) is the number of vertices between \( u \) and \( v \) in \( E(G) \). Furthermore, the total crossing distance will be defined as the sum of the crossing distances of all the edges in \( G \) as

\[
\text{Total Dist} = \sum_{(u,v) \in E} \text{Dist}(u, v).
\]

Based on the previous assumptions and descriptions, the row assignment problem of minimizing the number of feedthrough cells in a standard cell placement will correspond to the linear embedding problem of minimizing the total crossing distance for a separation graph. Unfortunately, the linear embedding problem with different objective functions[10] for an undirected graph is still an NP-complete problem. Clearly, the linear embedding problem of minimizing the total crossing distance for a separation graph is NP-complete.

3. Optimal ILP Approach

In this section, we propose a new formulation using the integer linear programming (ILP) technique which guarantees an optimal solution for the assignment of rows in a standard cell placement.

As mentioned above, the row assignment problem of minimizing the number of feedthrough cells in a standard cell placement will correspond to the linear embedding problem of minimizing total crossing distance for a separation graph. In this approach, first, for a separation graph \( G(V, E) \), if the graph is not a complete graph, the graph will be modified into a complete separation graph by adding the edges with the weights 0. Hence, the linear embedding problem of minimizing the total crossing distance for a separation graph is to find an permutation of \( V = \{ v_1, v_2, \ldots, v_n \} \) on \( n \) consecutive locations \( \{ 1, 2, \ldots, n \} \) of one horizontal line with minimum total crossing distance according to the computation the values of \( E = \{ c_{ij} | 1 \leq i, j \leq n, i < j \} \).

In the ILP formulation, two kinds of variables \( x_{ij} \) and \( c_{ijk} \) are introduced, where the \( c_{ijk} \) are necessary for obtaining a linear objective function.

\[
x_{ij} = 1, \quad \text{if } v_i \text{ is placed in location } j, \quad \text{otherwise } x_{ij} = 0.
\]

\[
c_{ijk} = 1, \quad \text{if the difference of the locations of } v_i \text{ and } v_j \text{ is } k, \text{ otherwise } c_{ijk} = 0.
\]

The objective function to be minimized is

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n-1} (k-1)c_{ij,k}
\]

subject to

\[
\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } 1 \leq i \leq n \tag{2}
\]

\[
\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } 1 \leq j \leq n \tag{3}
\]

\[
c_{ij,k} = c_{j,i,k} \quad \text{for } 1 \leq i, j \leq n, i < j, 0 < k < n \tag{4}
\]

\[
\sum_{j=1}^{n} c_{i,j,k} = 1 \quad \text{for } 1 \leq i, j \leq n, n < j \tag{5}
\]

\[
\sum_{j=1}^{n} c_{i,j,k} = 2 \quad \text{for } 1 \leq i, j \leq n, 0 < k < \frac{n}{2} \tag{6}
\]

\[
\sum_{j=1}^{n} c_{i,j,k} = 1 \quad \text{for } 1 \leq i, j \leq n, \frac{n}{2} \leq k < n \tag{7}
\]

\[
\sum_{i=1}^{n} k c_{i,j,k} - \sum_{k=1}^{n} k x_{ij} + \sum_{k=1}^{n} k x_{j,k} \geq 0 \quad \text{for } 1 \leq i, j \leq n, i < j \tag{8}
\]

Constraint (2) states that each vertex must be placed on one of \( n \) locations. In contrast, constraint (3) states that each location only places one of \( n \) vertices. The two constraints show a one-to-one mapping relation between the vertices and the locations. Clearly, there are \( n! \) conditions in such a one-to-one mapping function. Constraint (4) denotes the symmetric property of the distance of the locations of \( v_i \) and \( v_j \) is \( k \). It is clear that the distance from the location of \( v_i \) to the location of \( v_j \) is equal to that from the location of \( v_j \) to the location of \( v_i \). Constraint (5) states that the distance of the locations of \( v_i \) and \( v_j \) from \( 1 \) to \( n-1 \). Since \( v_i \) and \( v_j \) are located on two independent locations, the distance of the locations of \( v_i \) and \( v_j \) is unique. Constraints (6)-(7) denote the linear embedding relation between \( v_i \) and \( v_j \) with a fixed distance \( k \). For the distance \( k, 0 < k < \frac{n}{2} \), the number of the vertices the distance of which and \( v_i \) is \( k \) is at most 2. In contrast, for the distance \( k, \frac{n}{2} \leq k < n \), the number of the vertices the distance of which and \( v_i \) is \( k \) is at most 1. Finally, constraint (8) denotes the relation between \( x_{ij} \) and \( c_{ijk} \) according to the distance of the locations of \( v_i \) and \( v_j \).

According to the definition of the ILP approach, the objective function of an ILP formulation corresponding to the linear embedding of minimizing total crossing distance for the separation graph in Fig. 3 is as \( c_{1,2} + 4c_{1,4,2} + 3c_{2,3,2} + c_{4,4,2} + 5c_{3,4,2} + 2c_{1,2,3} + 4c_{1,3,3} + 8c_{1,4,3} + 6c_{2,3,3} + 2c_{2,4,3} + 10c_{3,4,3} \). It is clear that the solution obtained by this formulation is optimal and the linear embedding of \( \{ v_1, v_2, v_3, v_4 \} \) in the separation graph is \( v_1, v_4, v_3, v_2 \). 

102
v_2. Hence, the minimum total crossing distance is 5 and the number of feedthrough cells in the mapped standard cell placement is 5.

4. Experimental Results

For the minimization of the number of feedthrough cells in a standard cell placement, the two benchmarks, Primary1 and Primary2, are applied to perform the experimental results of the two complementary approaches, the ILP optimal approach and the heuristic approach. In the experiments, first, a standard cell netlist is partitioned by a multi-way area-ratio-constrained partitioning. In the multi-way area-ratio-constrained min-cut partitioning[8], according to the number of rows, the average partitioning area (APA) in one row is obtained as

\[
\text{APA} = \text{the sum of the areas of all the standard cells},
\]

the number of rows

Furthermore, any partitioning area in one row is constrained from 80% * APA to 120% * APA as

\[80\% \times \text{APA} \leq \text{any partitioning area} \leq 120\% \times \text{APA}.\]

Therefore, according to the mentioned area constraint and different number of rows, 4, 8, 12, 16, and 20, the two standard cell netlists, Primary1 and Primary2, are first clustered into 4, 8, 12, 16, and 20 groups of smaller netlists by the multi-way area-ratio-constrained min-cut partitioning, respectively.

According to the implementation of the multi-way area-ratio-constrained min-cut partitioning for Primary1 and Primary2, the partitioning results in 20 runs of Primary1 and Primary2 are shown in Table I. Furthermore, as a multi-way partitioning of a standard cell netlist is obtained, the mapped separation graph will be constructed according to the separated clusters and the cuts between any pair of clusters. Therefore, for Primary1 and Primary2, according to the best partitioning result in 20 runs in Table I, a 4-vertex separation graph, a 8-vertex separation graph, a 12-vertex separation graph, a 16-vertex separation graph and a 20-vertex separation graph will be further constructed, respectively.

For the row assignment in a standard cell placement, the ILP optimal formulation was run using the LINDO[13] package on a SUN workstation under the Berkeley 4.2 UNIX operating system. For the separation graphs of Primary1 and Primary2, the ILP approach obtains several linear embeddings of the separation graphs and the total crossing distances of these linear embeddings are computed. According to the total crossing distances, the number of feedthrough cells for the assignment of different number of rows in a standard cell placement will be obtained. In Table II, the number of feedthrough cells for the assignment of different number of rows in a standard cell placement by the proposed ILP approach and an exhaustive approach is shown.

<table>
<thead>
<tr>
<th>Example</th>
<th>K = 4</th>
<th>K = 8</th>
<th>K = 12</th>
<th>K = 16</th>
<th>K = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best</td>
<td>Min</td>
<td>Avg</td>
<td>Best</td>
<td>Min</td>
</tr>
<tr>
<td>Primary1</td>
<td>81</td>
<td>92.3</td>
<td>173</td>
<td>197.6</td>
<td>263</td>
</tr>
<tr>
<td>Primary2</td>
<td>254</td>
<td>285.7</td>
<td>456</td>
<td>497.5</td>
<td>715</td>
</tr>
</tbody>
</table>

Table II The Results for the Number of Feedthrough Cells

<table>
<thead>
<tr>
<th>Rows</th>
<th>Primary1</th>
<th>Primary2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feedthrough</td>
<td>Feedthrough</td>
</tr>
<tr>
<td></td>
<td>Cell</td>
<td>Time(s)</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>0.1</td>
</tr>
<tr>
<td>8</td>
<td>108</td>
<td>0.5</td>
</tr>
<tr>
<td>12</td>
<td>195</td>
<td>1.5</td>
</tr>
<tr>
<td>16</td>
<td>286</td>
<td>4.3</td>
</tr>
<tr>
<td>20</td>
<td>414</td>
<td>8.3</td>
</tr>
</tbody>
</table>

References


