On Locally Optimal Breaking of Nondisjoint Cyclic Vertical Constraints in VLSI Channel Routing

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Abstract

Locally optimal breaking strategy was already developed for disjoint directed circultics in the vertical constraint graph. Paper reports extensions to two classes of nondisjoint circuits: with a common vertex, and with a common path. Significance: demonstration of general applicability of the locally optimal breaking concept opens a new approach to improving the channel router heuristics for automatic and interactive routers, using parallel architectures.

I. Introduction

There is a feeling among researchers that VLSI Channel Routing is an overworked field. In so far as this is true, it regards the algorithmic side; theoretical part of the field is covered by a comparatively modest number of fundamental results, which concern mainly proving the NP-completeness of the problem and the bounds on channel width; some results, applicable to routing decisions in a general case can be found in [1,2,3]. This paper presents a contribution to the theory of locally optimal breaking of cyclic vertical constraints [4].

The existence of vertical constraints is the main obstacle to achieving optimal results in channel routing. The original Left Edge algorithm of Hashimoto and Stevens was proven to be optimal for channels without vertical constraints; on the other hand, in the presence of vertical constraints the problem has been proven NP-complete for all significant routing models. Presence of directed circuits (DC) and directed paths in the vertical constraint graph (VCG) introduces additional difficulty and may cause an increase in channel width over the channel density bound. Two classes of routing algorithms have been developed for channels with vertical constraints: for channels with, and without DC’s in the VCG; the reason for it being, that directed circuits, often referred to as cyclic vertical constraints (CVC), present a more difficult problem to channel routers than the directed paths do. Altogether, breaking the CVC’s is the toughest problem in channel routing, for which little theory has been developed, and no deterministic algorithmic solutions have been published that guarantee a degree of optimality.

In the light of the proven NP-completeness of the problem, globally optimal routing is not likely to be guaranteed by any algorithms. The local optimality concept has been introduced to support what can be achieved towards the goal of optimality. The concept relies on theory which is based on a new VCG augmented by geometric concepts[4]. New VCG has been proven to contain a complete description of the vertical constraint structure, as opposed to the classical VCG, that has been proven incomplete, alone and together with the horizontal constraint graph [5].

Basic theory of locally optimal breaking was developed for disjoint DC’s [4]. First examples indicated the usefulness of the weighted VCG in constructing deterministic partial routing algorithms with the property of local optimality. Two directed circuits in the VCG with a common vertex, or a common path, are the next level of complexity of vertical constraints after the vertex disjoint DC’s. For those two classes, results of this paper guarantee the existence of a locally optimal routing, and of a deterministic procedure for its characterization. This proves beyond reasonable doubt that extensions of the theory of locally optimal breaking to the classes of connected DC’s are feasible, and available at the cost of a reasonable research effort.

As to the impact of the theory on the VLSI channel routing strategies, the deterministic methods for locally optimal breaking of DC’s will allow a shift in the level of heuristic decision making, from the primarily low level routing decisions concerning a single signal net, or a geometrically not clearly related group of signal nets, to a higher level where final routing of all signal nets relies on combining the locally optimal partial routings of groups of vertically constrained nets. It is highly unlikely that an optimal
routing of a channel will consist entirely of locally non optimal partial routings of nets involved in CVC’s. On the other hand, a search through the space of locally optimal partial routings has a good chance to find a desirable combination. Deterministic quality of the breaking procedures provides a low cost quality to the locally optimal partial solutions. They can be also heuristically combined with deterministic procedures for solving the other problem posed by complex vertical constraints: breaking long directed paths.

Locally optimal breaking of DC’s has a potential for improving automatic and interactive routers. For automatic routers, a bounded search space is provided by the set of available alternative locally optimal partial routings, that lends itself to efficient search using parallel computer architectures. In interactive systems, designers can be presented graphical representations of available locally optimal routings, what would allow them to use human intuition for choosing the combination they prefer.

Section 2 introduces definitions of terms. Section 3 presents contributions to the theory of locally optimal breaking of two nondisjoint DC’s. Section 4 contains concluding remarks.

2. Definitions

Problem is considered in the grided Manhattan model. The top and bottom row sets of terminals are denoted by $T_T$ and $T_B$. The weighted VCG $G_w(V,E_w,W_w)$ has been formally defined in [4] as an extension of the classical VCG $G(V,E)$: its edge set $E_w$ contains an edge for every column that induces a vertical constraint, and it includes a set $W_w$ of edge weights which are equal to the order numbers of the columns that induce the edges. Graphical representations of $G_w$ in the paper have the edge weights indicated next to the edges. Characterizations of the locally optimal breaking patterns of routing depend critically on the geometry of columns that induce the CVC’s. $G_w$ with secondary vertical constraints included, is denoted by $G_{w}$. Secondary vertical constraints were recognized, but not conceptually differentiated from the commonly known, primary, already in [1,3]; they are induced by the geometry of terminals and the geometry of routing: at a column that contains signal net terminals, but not a terminal of the net whose vertical jog is placed in the column. Terminal column of a signal net is a column that contains a terminal of the net. Terminal column switching, often referred to as terminal doglegging, places the switching jog in a terminal column of the switching net; its virtue that it does not induce secondary vertical constraints, and therefore secondary DC’s also, has been recognized in [1,3]. Column interval $Q_{Cl}$ of a DC $C_i(V_{Cl},E_{Cl},W_{Cl})G_w$ is the set of columns $Q_{Cl} = \{ q_k \mid \min_{j \in W_{Cl}} q_j - \min_{j \in W_{Cl}} q_j \leq q_k \leq \max_{j \in W_{Cl}} q_j \}$. Tight set of DC’s is a set of DC’s whose column interval contains only columns that induce vertical constraints represented by the edges of the DC’s in the set. Tight channel is one whose all columns belong to the column intervals of tight sets of DC’s. Tightness concept for DC’s was introduced in [2]. Breaking a cyclic vertical constraint is the ensemble of all routing decisions that must be taken in the process of avoiding a violation of vertical constraints due to the existence of a set of CVC’s. Locally optimal breaking of a CVC is a routing of the nets involved in a DC $C_{w}G_w$ that occupies a minimal number of tracks inside the $C_i$ column interval.

3. Locally optimal breaking of two connected DC’s

Breaking the CVC’s involves a set of decisions that includes:

- selecting the track-switching signal net(s), at least one per set of connected DC’s, at most one per DC,
- selecting the switching column(s), at least one for every switching net,
- determining a feasible ordering of horizontal segments of nets on tracks, for all nets involved in CVC’s.

The results of these decisions can be routinely transformed into the exact geometry of wiring. Characteristic properties of CVC’s that govern the decision process, and the rules for making the decisions that yield locally optimal breaking will be developed.

The worst case will be considered only, characterized by the channels being tight. It has been chosen for presentation because it also provides a base for the nontight channel cases, and to show that there is always a locally optimal way of breaking nondisjoint DC’s. Presence of additional columns, which make a channel nontight, adds some more breaking patterns on top of the two which exist in the worst case, and some of those may replace the tight channel pattern as the locally optimal breaking pattern.

3.1 Breaking two DC’s with a common vertex

A tight channel whose vertical constraint graph $G_w$ contains two directed circuits with a common vertex is shown in Fig.1. One DC is of length two and the other of length three, what makes the example concise, yet sufficiently general.

The switching net $N_i$ and the switching columns for the locally optimal breaking of two edge disjoint DC’s with a common vertex are determined by Theorem 1.

**Theorem 1:** When the vertical constraint graph $G_w$ contains two DC’s $C_i \cap C_2 = (v_i, \emptyset)$, with a single common vertex
Fig. 1 Two patterns of locally optimal breaking two DC’s with a common vertex in a tight channel. (a) Switching in columns 2 and 4. (b) $G_w$ of the channel. (c) Switching in columns 1 and 5.

Both DC’s can be broken in the locally optimal way if the net $N_v$, represented in $G_w$ by $v_5$, is switched twice in a couple of its terminal columns characterized by the following two conditions:

a) they must not induce vertical constraints represented by the edges of only one of $C_1$ and $C_2$, b) one of them must have a terminal of $N_v$ in $T_{c_1}$ and the other in $T_{c_2}$.

Proof: Vertex $v_5$ is of degree four, so signal net $N_v$ has four terminal columns: two of which induce the vertical constraints involved in $C_1$, and the other two the constraints involved in $C_2$. This means that $N_v$ has two pairs of terminal columns in which it can be switched to break both $C_1$ and $C_2$; what further guarantees that both DC’s can be broken without creating a secondary DC. Out of the four combinations of two switching columns only two combinations will simultaneously break $C_1$ and $C_2$; the other two are not usable because $N_v$ has in them both terminals in $T_{c_1}$ or in $T_{c_2}$. The above conditions completely determine the two feasible column combinations.

QED

Weighted VCG, $G_w$ is the best suited data structure for recognizing the feasible column combinations by visual inspection or by a computer search: the columns of a feasible combination induce vertical constraints represented by the edges that are incident at the common vertex $v_5$, directed one into and the other out of $v_5$, and belong to different DC’s. An example of the two combinations is shown in Fig. 1: in 1(a) the switching columns are 2 and 4, in 1(c) the switching columns are 1 and 5.

Ordering of nets of tracks is determined by Theorem 2.

Theorem 2: The only two locally optimal orderings of nets on tracks for the nets involved in $C_1$ and $C_2$ are determined by the following rules,

a) nets involved in each of $C_1$ and $C_2$, are separately assigned to sets of adjacent tracks $T_{c_1}$ and $T_{c_2}$, by the rules for disjoint DC’s from [4],

b) the two orderings are then combined according to the selected feasible column combination as follows:

- $T_{c_1}$ goes on top of $T_{c_2}$, when the edge directed into $v_5$ belongs to $C_1$,

- $T_{c_2}$ goes on top of $T_{c_1}$, when the edge directed into $v_5$ belongs to $C_2$.

Proof: It has been shown [4] that the nets involved in DC $C_i$ must be placed on a set of adjacent tracks $T_{c_i}$, following the order of vertices in $C_i$, starting and ending with the switching net. The same rule applies to $C_1$ and $C_2$, separately, resulting in two sets of tracks, $T_{c_1}$ and $T_{c_2}$, which both have the switching net in the outer most tracks. One track can be saved when the two sets are arranged into one continuous set $T_{c_12}$, what is a requirement for locally optimal breaking.

QED

Both locally optimal breaking patterns are shown for the channel in Fig. 1, with the ordering of nets on tracks indicated by the net numbers placed next to the tracks.

3.2 Breaking two DC’s with a common directed path

A tight channel whose vertical constraint graph $G_w$ contains two directed circuits with a common path of length two is shown in Fig. 2. Both DC’s are of length four, what keeps the example concise while sufficiently general.

In the general case of two DC’s with a common path, three characteristic directed paths can be recognized as subgraphs of the $G_w$, which contains DC’s $C_1$ and $C_2$:

- $P_{12} = C_1 \cap C_2$, the path common to $C_1$ and $C_2$,

- $P_1 = C_1 - P_{12}$, the path exclusively in $C_1$,

- $P_2 = C_2 - P_{12}$, the path exclusively in $C_2$.

The switching net and the switching columns for this case are determined by Theorem 3.

Theorem 3 When $G_w$ contains DC’s $C_1$ and $C_2$ that have a common path $P_{12}$, both DC’s can be broken in the locally optimal way, if the nets represented in $G_w$ by the terminal vertices of $P_{12}$ are switched each in one of the two columns that are characterized by the following two conditions:

a) the columns induce vertical constraints that involve the switching nets but are not represented by the edges of $P_{12}$

b) one of the two columns must induce a vertical constraint represented by an edge of $C_1$, and the other by an edge of $C_2$.

Proof: Proof is based on the properties of the vertices
Two locally optimal patterns of breaking two DC's with a common path in a tight channel. (a) Switching in columns 2 and 5 (b) $G_w$ of the channel. (c) Switching in columns 1 and 6.

- Nets represented by the internal vertices of $P_{12}$ are not switchable because their two terminal columns induce vertical constraints involved in both $C_1$ and $C_2$.
- Nets represented by the internal vertices of the paths $P_1$ and $P_2$ are not switchable because their both terminal columns belong to the DC which should be broken by their switching.
- Nets represented by the terminal vertices of the characteristic paths are switchable because they both have three terminal columns that are evenly distributed over the paths $P_1$, $P_2$, and $P_{12}$, so that both switching nets can be broken at a terminal column represented by an edge in $P_2$, to break $C_1$, or at a column represented by an edge in $P_1$, to break $C_2$.

QED

In the channel of Fig.2, the switching nets are nets 1 and 3. In Fig.2(a) the switching columns are 2 and 5; in 2(c) columns 1 and 6.

The ordering of nets on tracks for this case is determined by Theorem 4.

**Theorem 4. The only two locally optimal orderings of nets on tracks for the nets involved in $C_1$ and $C_2$ are determined by the order of vertices in the following two Euler trails in $G_w$: $E_1 = C_1 \cup P_2$ and $E_2 = C_2 \cup P_1$.**

Proof. Two switchable nets, represented by the terminal vertices of $P_{12}$, have three terminals each, which are unevenly distributed between $T_B$ and $T_p$. Both Euler trails start at the vertex that represents the switching net with two terminals in $T_B$, and both end at the vertex that represents the net that has two terminals in $T_B$. Trail $E_1$ first follows the whole DC $C_1$, and then continues through the directed path in $C_2$. In that manner, both switching nets are placed on two tracks, and the mandatory order of nets for each individual DC [4] is preserved avoiding repetitions of the nets which are not being switched. QED

Both locally optimal breaking patterns are shown for the channel in Fig.2. Euler trails are indicated by the succession of their vertex numbers placed next to the tracks.

4. Conclusion

Applying the known basic theory of the locally optimal breaking of directed circuits in the vertical constraint graph, new results have been developed which extend the theory to two, more complex classes of cyclic vertical constraints. These results prove beyond reasonable doubt that the theory and methods of locally optimal breaking of cyclic vertical constraints are not limited to simple special cases, but that they can be successfully applied to cyclic vertical constraint structures of any complexity. Locally optimal breaking strategy supports a shift of the heuristic decision making to a higher level for both, the automatic and interactive routers.

An interesting and encouraging observation is that the increase in the complexity of vertical constraints narrows down the solution space. In the considered cases: one specific net is switchable, although in two different patterns, when two directed circuits have a common vertex; exactly two specific nets must be switched in at most two ways when two directed circuits have a common path. On the other side, in the case of disjoint directed circuits a greater number of nets may be switchable. In any case, the efficient search of the bounded solution space may take advantage of parallel computer architectures.

5. References


