A Binary-Constraint Search Algorithm for Minimizing Hardware during Hardware/Software Partitioning

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Abstract

Partitioning a system's functionality among interacting hardware and software components is an important part of system design. We introduce a new partitioning approach that caters to the main objective of the hardware/software partitioning problem, i.e., minimizing hardware for given performance constraints. We demonstrate results superior to those of previously published algorithms intended for hardware/software partitioning. The approach may be generalizable to problems in which one metric must be minimized while other metrics must merely satisfy constraints.

1 Introduction

Combined hardware/software implementations are common in embedded systems. Software running on an existing processor is less expensive, more easily modifiable, and more quickly designable than an equivalent application-specific hardware implementation. However, hardware may provide better performance. A system designer's goal is to implement a system using a minimal amount of application-specific hardware, if any at all, to satisfy required performance. In other words, the designer attempts to implement as much functionality as possible in software.

Deficiencies of the much practiced ad-hoc approach to partitioning have led to research into more formal, algorithmic approaches. In the ad-hoc approach, a designer starts with an informal functional description of the desired system, such as an English description. Based on previous experience and mental estimations, the designer partitions the functionality among hardware and software components, and the components are then designed and integrated. This approach has two key limitations. First, due to limited time, the designer can only consider a small number of possible partitionings, so many good solutions will never be considered. Second, the effects that partitioning has on performance are far too complex for a designer to accurately estimate mentally. As a result of these limitations, designers often use more hardware than necessary to ensure performance constraints are met.

In formal approaches, one starts with a functional description of the system in a machine-readable language, such as VHDL. After verifying, usually through simulation, that the description is correct, the functionality is decomposed into functional portions of some granularity. These portions, along with additional information such as data shared between portions, make up an internal model of the system. Each portion is mapped to either hardware or software by partitioning algorithms that search large numbers of solutions. Such algorithms are guided by automated estimators that evaluate cost functions for each partitioning. The output is a set of functional portions to be mapped to software and another set to be mapped to hardware. Simulation of the designed hardware and compiled software can then be performed to observe the effects of partitioning. Figure 1 shows a typical configuration of a hardware/software partitioning system.

![Figure 1: Basics parts of a hw/sw partitioning system](image)

The partitioning algorithm is a crucial part of the formal approach because it is the algorithm that actually minimizes the expensive hardware. However, current research into algorithms for hardware/software partitioning is at an early stage. In [1], the essential criteria to consider during partitioning are described, but no particular algorithm is given. In [2], certain partitioning issues are also described but no algorithm is given. In [3], an approach based on a multi-stage clustering algorithm is described, where the closeness metrics include hardware sharing and concurrency, and where particular clusterings are evaluated based on factors including an area/performance cost function. In [4], an algorithm is described which starts with most functionality in hardware, and which then moves portions into software as long as such a move improves the de-

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sign. In [5], an approach is described which starts with all functionality in software, and then moves portions into hardware using an iterative-improvement algorithm such as simulated annealing. The authors hypothesize that starting with an all-software partitioning will result in less final hardware than starting with all-hardware partitioning (our results support this hypothesis). In [6], simulated annealing is also used. The focus is on placing highly utilized functional portions in hardware. However, there is no direct performance measurement. The common shortcoming of all previous approaches is the lack of advanced methods to minimize hardware.

We propose a new partitioning approach that specifically addresses the need to minimize hardware while meeting performance constraints. The novelty of the approach is in how it frees a partitioning algorithm, such as simulated annealing, from simultaneously trying to satisfy performance constraints and to minimize hardware, since trying to solve both problems simultaneously yields a poor solution to both problems. Instead, in our approach, we use a relaxed cost function that enables the algorithm to focus on satisfying performance, and we use an efficiently-controlled outer loop (based on binary-search) to handle the hardware minimization. As such, our approach can be considered as a meta-algorithm that combines the binary-search algorithm with any given partitioning algorithm. The approach results in substantially less hardware, while using only a small constant factor of additional computation and still satisfying performance constraints (whereas starting with an all-software partition and applying iterative-improvement algorithms often fails to satisfy performance). Such a reduction can significantly decrease implementation costs. Moving one of the subproblems to an outer loop may in fact be a general solution to other problems in which one metric must be minimized while others must merely satisfy constraints.

The paper is organized as follows. Section 2 gives a definition of the hardware/software partitioning problem. Section 3 describes previous hardware/software partitioning algorithms, an extension we have made to one previous algorithm to reduce hardware, and our new hardware-minimizing algorithm based on constraint-search. Section 4 summarizes our experimental results on several examples.

2 Problem Definition

While the partitioning subproblem interacts with other subproblems in hardware/software codesign, it is distinct, i.e., it is orthogonal to the choice of specification language, the level of granularity of functional decomposition, and the specific estimation models employed.

We are given a set of functional objects \( O = \{ o_1, o_2, \ldots, o_n \} \) which compose the functionality of the system under design. The functions may be at any of various levels of granularity, such as tasks (e.g., processes, procedures or code groupings) or arithmetic operations. We are also given a set of performance constraints \( Cons = \{ C_1, C_2, \ldots, C_m \} \), where \( C_j \subseteq \{ G, timecon \} \), \( G \subseteq O \), and \( timecon \in Positive \). \( timecon \) is a constraint on the maximum execution-time of the all functions in group \( G \). It is simple to extend the problem to allow other performance constraints such as those on bitrates or inter-operation delays, but we have not included such constraints in order to simplify the notation.

A hardware/software partition is defined as two sets \( H \) and \( S \), where \( H \cap S = O \), \( H \cup S = O \), \( H \cap S = \emptyset \). This definition does not prevent further partition of hardware or software. Hardware can be partitioned into several chips while software can be executed on more than one processor. The hardware size of \( H \), or \( |size(H)| \) is defined as the size (e.g., transistors) of the hardware needed to implement the functions in \( H \). The performance of \( G \), or \( Performance(G) \), is defined as the total execution time for the group of functions in \( G \) for a given partition \( H, S \). A performance satisfying partition is one for which \( Performance(C_j, G) \leq C_j.timecon \) for all \( j = 1 \ldots m \).

Definition 1: Given \( O \) and \( Cons \), the Hardware/Software Partitioning Problem is to find a performance satisfying partition \( H, S \) such that \( |size(H)| \) is minimal. In other words, the problem is to map all the functions to either hardware or software in such a way that we find the minimal hardware for which all performance constraints can still be met. Note that the hardware/software partitioning problem, like other partitioning problems, is NP-complete.

The all-hardware size of \( O \) is defined as the size of an all-hardware partition, or in other words as \( |size(O)| \). Note that if an all-hardware partition does not satisfy performance constraints, no solution exists.

To compare any two partitions, a cost function is required. A cost function is a function \( Cost(H, S, Cons, I) \) which returns a natural number that summarizes the overall goodness of a given partition, the smaller the better. \( I \) contains any additional information that is not contained in \( H, S \) or \( Cons \). We define an iterative improvement partitioning algorithm as a procedure \( PartAlg(H, S, Cons, I) \) which returns a partition \( H', S' \) such that \( Cost(H', S', Cons, I) \leq Cost(H, S, Cons, I) \). Examples of such algorithms include group migration [7] and simulated annealing [8].

Since it is not feasible to implement the hardware and software components in order to determine a cost for each possible partition generated by an algorithm, we assume that fast estimators are available [9, 10, 11].

3 Partitioning solution

3.1 Basic algorithms

One simple and fast algorithm starts with an initial partition, and moves objects as long as improvement occurs. The algorithm presented in [4], due to Gupta and DeMicheli, and abbreviated as GD, can be viewed as an extension of this algorithm which ensures performance constraints are met. The algorithm starts by creating an all-hardware partition, thus guaranteeing that a performance
satisfying partition is found if it exists (actually, certain functions which are considered unconstrained are initially placed in software). To move a function requires not only cost improvement but also that all performance constraints still be satisfied (actually they require that maximum interfacing constraints between hardware and software be satisfied). Once a function is moved, the algorithm tries to move closely related functions before trying others.

Greedy algorithms, such as the one described above, suffer from the limitation that they are easily trapped in a local minimum. As a simple example, consider an initial partition that is performance satisfying, in which two heavily communicating functions o1 and o2 are initially in hardware. Suppose that moving either o1 or o2 to software results in performance violations, but moving both o1 and o2 results in a performance satisfying partition. Neither of the above algorithms can find the latter solution because doing so requires accepting an intermediate, seemingly negative move of a single function.

To overcome the limitation of greedy algorithms, others have proposed using an existing hill-climbing algorithm such as simulated annealing. Such an algorithm accepts some number of negative moves in a manner that overcomes many local minima. One simply creates an initial partition and applies the algorithm.

In [5], such an approach is described by Ernst and Henkel that uses an all-software solution for the initial partition. A hill-climbing partitioning algorithm is then used to extract functions from software to hardware in order to meet performance. The authors reason that such extraction should result in less hardware than the approach where functions are extracted in the other direction, i.e., from hardware to software.

Cost function

We now consider devising a cost function to be used by the hill-climbing partitioning algorithm. The difficulty lies in trying to balance the performance satisfiability and hardware minimization goals. The GD approach does not encounter this problem since performance satisfiability is not part of the cost function. The cost function is only used to evaluate partitions that already satisfy the performance constraints. The algorithm simply rejects all partitions that are not performance satisfying. We saw that this approach will become trapped in a local minimum. The Ernst/Henkel approach does not encounter this problem since hardware size is not part of the cost function; instead, it is fixed beforehand, by allocating resources before partitioning. This approach requires the designer to manually try numerous hardware sizes, reapplying partitioning for each, to try to find the smallest hardware size that yields a performance satisfying partition.

We propose a third solution. We use a cost function with two terms, one indicating the sum of all performance violations, the other the hardware size. The performance term is weighted very heavily to ensure that a performance satisfying solution is found, so minimizing hardware is a secondary consideration. The cost function is:

\[
\text{Cost}(H, S, Cons) = k_{perf} \times \sum_{j=1}^{m} \text{Violation}(C_j) + k_{area} \times H_{size}(H)
\]

where \( \text{Violation}(C_j) = \text{Perform}(x(C_j, G) - \text{timecon}) \) if the difference is greater than 0, else \( \text{Violation}(C_j) = 0 \). Also, \( k_{perf} >> k_{area} \), but \( k_{perf} \) should not be infinity, since then the algorithm could not distinguish a partition which almost meets constraints from one which greatly violates constraints.

We refer to this solution as the PWHC (performance-weighted hill-climbing) algorithm. We shall see that it gives excellent results as compared to the GD algorithm, but there is still room for improvement. In particular, when starting with an all-software partition, hill-climbing algorithms often fail to find a performance-satisfying solution.

3.2 A new constraint-search approach

While incorporating performance and hardware size considerations in the same cost function, as in PWHC, tends to give much better results than previous approaches, we have determined a superior approach for minimizing hardware. Our approach involves decoupling to a degree the problem of satisfying performance from the problem of minimizing hardware.

3.2.1 Foundation

The first step is to realize that the difficulty experienced in PWHC is that the two metrics in the cost function, performance and hardware size, directly compete with each other. In other words, decreasing performance violations usually increases hardware size, while decreasing hardware size usually increases performance violations. An iterative-improvement algorithm has a hard time making significant progress towards minimizing one of the metrics since any reduction in one metric’s value yields an increase in the other metric’s value, resulting in very large “hills” that must be climbed. To solve this problem, we can relax the cost function goal. Rather than minimizing size, we just wish to find any size below a given constraint \( C_{size} \).

\[
\text{Cost}(H, S, Cons, C_{size}) = k_{perf} \times \sum_{j=1}^{m} \text{Violation}(C_j) + k_{area} \times \text{Violation}(H_{size}(H), C_{size})
\]

It is no longer required that \( k_{perf} >> k_{area} \). We set \( k_{perf} = k_{area} = 1 \). The effect of relaxing the cost function goal is that once the hardware size drops below the constraint, decreasing performance violations does not necessarily yield a hardware-size violation; hence the iterative-improvement algorithm has more flexibility to work towards eliminating performance violations.
The hardware minimization problem can now be stated distinct from the partitioning problem.

Definition 2: Given \( O, Cons, PartAlg() \) and \( Cost() \), the Minimal Hardware-Constraint Problem is to determine the smallest \( C_{\text{size}} \) such that \( \text{Cost}(\text{PartAlg}(H, S, Cons, C_{\text{size}}, \text{Cost}()), Cons, C_{\text{size}}) = 0 \). In other words, we must choose the smallest size constraint for which a performance satisfying solution can be found by the partitioning algorithm.

Theorem 1: Let \( \text{PartAlg() be such that it always finds a zero-cost solution if one exists. Then} \) \( \text{Cost}(\text{PartAlg}(H, S, Cons, C_{\text{size}}, \text{Cost}()), Cons, C_{\text{size}}) = 0 \) implies \( \text{Cost}(\text{PartAlg}(H, S, Cons, C_{\text{size}} + 1, \text{Cost}()), Cons, C_{\text{size}} + 1) = 0 \).

Proof: we can create a (hypothetical) algorithm which subtracts 1 from its hardware-size constraint if a zero-cost solution is not found. Given \( C_{\text{size}} + 1 \) as the constraint, then if a zero-cost is not found, the algorithm will try \( C_{\text{size}} \) as the constraint. Thus the algorithm can always find a zero-cost solution for \( C_{\text{size}} + 1 \) if one exists for \( C_{\text{size}} \).

The above theorem states that if a zero-cost solution is found for a given \( C_{\text{size}} \), then zero-cost solutions will be found for all larger values of \( C_{\text{size}} \). Also, from this theorem we see that the sequence of cost numbers obtained for \( C_{\text{size}} = 0, 1, \ldots \), AllHardwareSize consists of x non-zero numbers followed by \( C_{\text{size}} = x \) zeroes, where \( x \in \{0, \text{AllHardwareSize}\} \). Let CostSequence equal this sequence of cost numbers. Figure 2 depicts an example of a CostSequence graphically. (It is important to note that CostSequence is conceptually; it describes the solution space, but we do not actually need to generate this sequence to find a solution). We can now restate the minimal hardware-constraint problem as a search problem:

![CostSequence Example](image)

Figure 2: An example cost sequence

Definition 3: Given \( H, S, Cons, PartAlg() \) and \( \text{Cost}() \) the Minimal Hardware-Constraint Search Problem is to find the first zero in CostSequence.

Given this definition, we see that the problem can be easily mapped to the well-known problem of Sorted-array search, i.e., of finding the first occurrence of a key in an ordered array of items. The main difference between the two problems is that whereas in sorted-array search the items exist in the array beforehand, in our constraint-search problem an item is added (i.e., partitioning applied and a cost determined) only if the array location is visited during search. In either case, we wish to visit as few array items as possible, so the difference does not affect the solution. A second difference is that the first \( x \) items in CostSequence are not necessarily in increasing or decreasing order. Since we are looking for a zero cost solution, we don’t care what those non-zero values are, so we can convert CostSequence to an ordered sequence by mapping each non-zero cost to 1.

The constraint corresponding to the first zero cost represents the minimal hardware, or optimal solution to the partitioning problem. Due to the NP-completeness of partitioning, it should come as no surprise that we can not actually guarantee an optimal solution. Note that we assumed in the above theorem that PartAlg() finds a zero-cost solution if one exists for the given size constraint. Since partitioning is NP-complete, such an algorithm is impractical. Thus PartAlg() may not find a zero-cost solution although one may exist for a given size constraint. The result is that the first zero in CostSequence may be for a constraint which is larger than the optimal, or that non-zero costs may appear for constraints larger than that yielding the first zero cost, meaning the sequence of zeros contains spikes. However, the first zero cost should correspond to a constraint near the optimal if a good algorithm is used. In addition, any spikes that occur should also only appear near the optimal. Thus the algorithm should yield near optimal results.

It is well-known that binary-search is a good solution to the sorted-array search problem, since its worst case behavior is \( \log(N) \) for an array of \( N \) items. We therefore incorporate binary-search into our algorithm.

3.2.2 Algorithm

We now describe our hardware-minimizing partitioning algorithm based on binary-search of the sequence of costs for the range of possible hardware constraints, which we refer to as the BCS (binary constraint-search) algorithm. The algorithm uses variables \( \text{low} \) and \( \text{high} \) which indicate the current window of possible constraints in which a zero-cost constraint lies, and variable \( \text{mid} \) which represents the middle of that window. Variables \( H_{\text{zero}} \) and \( S_{\text{zero}} \) store the zero-cost partition which has the smallest hardware constraint so far encountered.

```
Algorithm 3.1 BCS hw/sw partitioning

low = 0, high = AllHardwareSize
while low < high loop
    mid = floor((low + high + 1) / 2)
    H', S' = PartAlg(H, S, Cons, mid, Cost())
    if Cost(H', S', Cons, mid) = 0 then
        high = mid - 1
        H_{zero}, S_{zero} = H', S'
    else
        low = mid
end if
end loop
return H_{zero}, S_{zero}
```

The algorithm performs a binary search through the range of possible constraints, applying partitioning and
then the cost function as each constraint is “visited”. The algorithm looks very much like a standard binary-search algorithm with two modifications. First, mid is used as a hardware constraint for partitioning whose result is then used to determine a cost, in contrast to using mid as an index to an array item. Second, the cost is compared to 0, in contrast to an array item being compared to a key.

3.2.3 Reducing runtime in practice
After experimentation, we developed a simple modification of the constraint-search algorithm to reduce its runtime in practice. Let size_{best} be the smallest zero-cost hardware constraint. If a C_{i, n} constraint much larger than size_{best} is provided to PartAlg(), the algorithm usually finds a solution very quickly. The functions causing a performance violation are simply moved to hardware. If a C_{i, n} constraint much smaller than size_{best} is provided, the algorithm also stops fairly quickly, since it is unable to find a sequence of moves that improves the cost. However, if C_{i, n} is slightly smaller or larger than size_{best}, the algorithm usually makes a large number of moves, gradually inching its way towards a cost of zero. This situation is very different from traditional binary-search where a comparison of the key with an item takes the same time for any item. Near the end of binary-search the window of possible constraint values is very small, with size_{best} somewhere inside this window. Much of the constraint-search algorithm’s runtime is spent reducing the window size by minute amounts and reapplying lengthy partitioning.

In practice, we need not find the smallest hardware size to such a degree of precision. We thus terminate the binary-search when the window size (i.e., high – low) is less than a certain percentage of AllHardwareSize. This percentage is called a precision factor. We have found that a precision factor of 1% achieves a speedup of roughly 2.5; we allow the user to select any factor.

3.2.4 Complexity
The worst-case runtime complexity of the constraint-search algorithm equals the complexity of the chosen partitioning algorithm PartAlg() multiplied by the complexity of our binary constraint-search. While the complexity of the binary search of a sequence with AllHardwareSize items is log_2(AllHardwareSize), the precision factor reduces this to a constant.

Theorem 2: The complexity of the binary search of an N element CostSequence with a precision factor a is \( \log_2 \left( \frac{N}{a} \right) \).

Proof: We start with a window size of N, and repeatedly divide the window size by 2 until the window size equals \( a \times N \). Let \( w \) be the number of windows generated; \( w \) will thus give us the complexity. An equivalent variable for \( w \) is obtained by starting with a window size of \( a \times N \), and multiplying the size by 2 until the size is \( N \). Hence we obtain the following equation: \( (a \times N) \times 2^w = N \). Solving for \( w \) yields \( w = \log_2 \left( \frac{N}{a \times N} \right) = \log_2 \left( \frac{N}{a} \right) \). The complexity is therefore \( \log_2 \left( \frac{N}{a} \right) \).

We see that binary constraint-search partitioning with a precision factor has the same theoretical complexity as the partitioning algorithm PartAlg(). In practice, the binary constraint-search contributes a small constant factor. For example, a precision factor of 5% results in a constant factor of \( \log_2(20) = 4.3 \).

4 Experiments
We briefly describe the environment used to compare the various algorithms on real examples. It is important to note that most environment issues are orthogonal to the issue of algorithm design. Our algorithms should perform well in any of the environments discussed in other work such as [1, 2, 3, 4, 5]. It should also be noted that any partitioning algorithm can be used within the BCS algorithm, not just simulated annealing.

We take a VHDL behavioral description as input. The description is decomposed to the granularity of tasks, i.e., processes, procedures, and optionally to statement blocks such as loops. Large data items (variables) are also treated as functions. Estimators of hardware size and behavior execution-time for both hardware and software are available [9, 10]. These estimators are especially designed to be used in conjunction with partitioning. In particular, very fast and accurate estimations are made available through special techniques to incrementally modify an estimate when a function is moved, rather than reestimating entirely for the new partition. The briefness of our discussion on estimation does not imply that it is trivial or simple, but instead that it is a different issue not discussed in this paper. We also note here that ILP solutions are inadequate for use here because the metrics involved are non-linear, since they are obtained by using sophisticated estimators based on design models, rather than by using linear but grossly inaccurate metrics.

We implemented three partitioning algorithms: GD, PWHC, and BCS. The PartAlg() used in PWHC and BCS is simulated annealing. From now on, we also refer the PWHC algorithm as SA (Simulated Annealing). The precision factor used in BCS is 5%. We applied each algorithm to several examples: a real-time medical system (Volume) for measuring volume, a beam former system (Beam), a fuzzy logic control system (Fuzzy), and a microwave transmitter system (Microwave). For each example, a variety of performance constraints were input. Some examples have performance constraints on one group of tasks in the system. Others have performance constraints on two groups of tasks in the system. We tested each example using a set of performance constraints that reside between the time required for an all-hardware partition and the time required for an all-software partition. The initial partition used in each trial by SA and BCS is an all-software partition. We also ran SA and BCS starting with an all-hardware partition, but found that the results were inferior to those starting with an all-software partition.

Figure 3 summarizes the results. The Average run time is the measured CPU time running on a Sparc2. The Average hardware decrease percentage is the percentage by
which each algorithm reduces the hardware, relative to the size of an all-hardware implementation. The Best cases is the number of times the given algorithm results in a hardware size smaller than those of both other algorithms. The Worst cases is the number of times the algorithm results in a hardware size that is larger than those of both other algorithms. No solution cases is the number of times the algorithm fails to find a performance-satisfying partition.

The results demonstrate the superiority of the BCS algorithm in finding a performance-satisfying partition with minimal hardware. BCS always finds a performance-satisfying partition (in fact, it is guaranteed to do so), whereas SA failed in 5 cases, which is 13.2% of all cases. While GD also always finds a performance-satisfying partition, BCS results in a 9.5% savings in hardware, corresponding to an average savings of approximately 12983 gates [12]. An additional positive note that can be seen from the experiments is that the increase in computation time of BCS over simulated annealing is only 4.1, slightly better than the theoretical expectation of 4.3.

To further evaluate the BCS algorithm, we developed a general formulation of the hardware/software partitioning problem. This formulation enables us to generate problems that imitate real examples, without spending the many months required to develop a real example. In addition, the formulation enables a comparison of algorithms without requiring use of sophisticated software and hardware estimators, which makes algorithm comparison possible for others who do not have such estimators. We generated 125 general hardware/software partitioning problems, and then applied BCS with a 5% precision factor, SA and GD. The BCS algorithm again finds performance-satisfying partitions with less hardware. SA failed to find a performance-satisfying partition in 15 cases, which is 12.5% of all cases. While GD also always found a performance-satisfying partition, BCS resulted in nearly a 10% savings in hardware. Figure 4 summarizes the comparison of BCS to the other algorithms. Details of the formulation, the pseudo-random hardware/software problem generation algorithm, and the partitioning results are provided in [13].

Figure 4: BCS compared to other algorithms

5 Conclusion

The BCS algorithm excels over previous hardware/software partitioning algorithms in its ability to minimize hardware while satisfying performance constraints. The computation time required by the algorithm is well within reason, especially when one considers the great benefits obtained from the reduction in hardware achieved by the algorithm, including reduced system cost, faster design time, and more easily modifiable final designs. The BCS approach is essentially a meta-algorithm that removes from a cost function one metric that must be minimized, and thus the approach may be applicable to a wide range of problems involving non-linear, competing metrics.

References