Algorithmic Aspects of Three Dimensional MCM Routing

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Abstract- In this paper, we present a new routing approach for MCMs in which the routing space is partitioned into several towers. The routing is carried out in three steps. In the first step, the routing density is uniformly distributed over the three dimensional routing space. In the next step, the exact locations of nets on the faces of each tower are determined. Finally, the exact paths for the nets in each tower are determined and routed. Unlike the traditional MCM routing which converts the three dimensional routing problem into a set of two dimensional routing problems, our approach decomposes the problem into a set of smaller, yet three dimensional tower routing problems. Experimental results show the validity of our methodology.

1 Introduction

The routing environment for MCMs and multi-layer PCBs can be characterized as three dimensional routing medium due to the large number of layers present. Traditionally, the MCM routing is converted into a set of two dimensional routing problems, since two dimensional routing problems have been well studied. For example, in [3, 7], the routing phase is decomposed into a pin redistribution phase, layer assignment phase, and the detailed routing phase which is carried out either on a layer or a layer pair. An integrated pin redistribution and routing approach is adapted in the SLICE router [8]. The SLICE router performs planar routing on one layer at a time. Later, the V4R router was developed which improved upon the SLICE router [9]. The V4R router uses two adjacent layers and routes one column at a time from left to right using four vias for most of the nets.

Rather than converting the three dimensional routing problem into two dimensional routing problems, we introduce a new routing approach which maintains the three dimensional characteristics while decomposing the complex three dimensional problem into a set of simpler tower routing problems. By maintaining the three dimensional characteristics of the original problem, our routing approach can effectively utilize the three dimensional routing space while satisfying the delay, noise, and fabrication constraints.

For the sake of brevity, the details of several algorithms and proofs of the theorems are omitted. In this paper, we discuss the Z-dimension routing distribution, terminal assignment, and tower routing phases in detail. Complete details of our approach is presented in [14].

The rest of this paper is organized as follows. An overview of our approach is given in section 2. We discuss the Z-dimension routing distribution phase in Section 3.

In Section 4, we discuss the terminal assignment phase. The tower routing phase is presented in Section 5. Finally, experimental results and conclusion are discussed in Sections 6 and 7 respectively.

2 Overview

In our approach, the routing process is decomposed into several phases. In the first phase, the substrate is partitioned into several tiles and the terminals are mapped to the tile edges. In the next phase, the routing density is uniformly distributed in two-dimensional XY-plane as well as in the Z-dimension. During the twodimensional routing distribution phase, the loose route of each net is determined. A global route of a net is a Steiner tree in which each demand point or Steiner point corresponds to a tile. The problem of two-dimensional routing distribution for k nets can be defined as the problem of finding k-Minimum Steiner Trees (k-MST). We have extended the hierarchical routing algorithm of Burstein and Pelavin [1] to solve the k-MST problem. In our algorithm, the k-MST problem on a $m \times n$ grid is solved hierarchically. At each level of hierarchy, the problem is to find a solution to k-MST problem on a $2 \times n$ grid. During the next phase of Z-dimension routing distribution, a layer is assigned to each terminal of each net on the faces of the towers. The objective of the Z-dimension routing distribution phase is to uniformly distribute the routing density along the Z-dimension while minimizing the number of vias used. At the end of the routing distribution phases, the number of layers required for routing is determined and the length of the nets is estimated accurately. This characteristic allows us to estimate the routing resources (e.g. the number of layers and the length of the nets) before actually carrying out the routing itself which may be expensive in terms of time and space. In the next phase, the exact locations for the terminals of the nets on the faces of each tower are assigned. This phase is referred to as the terminal assignment phase. The objective of the terminal assignment phase is to maximize the number of planar nets. At the end of the terminal assignment phase, the original three dimensional routing problem is reduced to that of routing several smaller three dimensional towers. As the routing problem in each tower is independent, all the towers can be processed in parallel. A tower is partitioned recursively if it is too large or its routing density is too high. In this way, our approach allocates more computing resources to the regions which require them. During the tower routing phase, we find a maximum set of planar nets for each layer which can be directly routed

on the layer by using an approximation algorithm which guarantees 60 % of the optimal solution. The remaining nets are routed by using a three dimensional router.

Our approach is similar to aviation flight scheduling technique, in which the routes of nets are distributed in the x-y plane as well as along the z-dimension. The uniform distribution of the routes of the nets result in better utilization of the routing space and increases the fabrication yield of the MCM substrate. Moreover, the routing distribution allows us to control the net length and the highest layer to which a net can reach, therefore meeting the delay constraint for the net. On the other hand, the terminal assignment allows us to specify the separation value between any two nets so that these two nets cannot be routed close to each other. The separation constraint is also considered in the tower routing phase to maintain a minimum separation between the critical nets. As a result, noise due to crosstalk is minimized. Noise due to reflection is minimized because of the planar routing and via minimization techniques. As a result, our routing approach can satisfy the delay and noise constraints that are critical in the high performance systems. In addition, since the tiles can be partitioned recursively, we can predict the outcome of routing resources more accurately by further partitioning the tiles and therefore spending more computing resources at the required place. This characteristic is desirable in the design of MCMs as it uses less computing resources in early design and uses more computing resources to produce more accurate results in the later stages of MCM design.

3 Z-Dimension Routing Distribution

In this section, we discuss the Z-dimension routing distribution phase which assigns the layer on which each net passes through each tower face. The objective of this phase is to uniformly distribute the routing density along z-dimension while minimizing the total number of vias for each net.

The uniform distribution of routing density is guaranteed because we assign a capacity to each edge e on each layer l which is the maximum number of nets that can pass the edge e on layer l. Let C(e, l) and c(e, l)be the capacity and the number nets that have already been assigned to edge e on layer l respectively. Clearly for each edge e, only those layers l where c(e, l) < C(e, l)can be assigned to a net. The available layers at each edge are modeled as shown in Figure 1(b). Note that the edges that a net passes through have been determined during the xy-plane routing distribution phase which is as shown in Figure 1(a). The number of vias required for the net in a tower is $|t_i - t_j|$ where t_i and t_j are the layers that have been assigned to the net on the edges iand j in the tower. Assuming that the edges which a net passes through are $1, 2, \ldots, p$, the problem of assigning layers to a net is to find t_i for $i = 1, 2, \ldots, p$ such that $c(i, t_i) < C(i, t_i)$ and $\sum_{i=1}^{p-1} (|t_i - t_{i+1}|)$ is minimized. It can be easily seen that the above problem can be solved optimally by using the multistage graph technique [6]. Note that the problem stated above is for a two-terminal net. The technique can be easily extended to a multiterminal net by exploring all combinations of paths at the branching point of the global route.

Instance: Minimum Via Layer Assignment Problem (MVLA): Given a set of edges i = 1, 2..., p and



Figure 1: (a) Global route of a net, b) Layer assignment for the net.

c(i, j), C(i, j) for i = 1, 2..., p, j = 1, 2..., k. Question: Does there exist $1 \le t_i \le k$ for i = 1, 2..., p such that $c(i, t_i) < C(i, t_i)$ and $\sum_{i=1}^{p-1} (|t_i - t_{i+1}|)$ is minimized?

Theorem 1 MVLA problem can be solved in $O(k^2p)$ time.

4 Terminal Assignment

In this section, we discuss the terminal assignment phase which finds the exact locations of the terminals of each net in the towers through which the net passes by using a recursive top-down approach. The objective of terminal assignment phase is to maximize the number of planar nets in each tower while satisfying the pathseparation constraint. We assume that all the nets are of type two-terminal. The proposed algorithms can be easily extended to handle multi-terminal nets.

The terminal assignment phase is carried out by bipartitioning the substrate recursively. At each level of partitioning, a set of tiles is further partitioned into two sets of tiles which are of the same size and the locations of terminals along the partition boundary is determined and assigned. We partition the substrate recursively and assign only the terminals along the partition boundary at each level of the hierarchy so as to maintain a global perspective of the net distribution. At the lowest level, the partitions correspond to tiles and all the terminals are completely assigned.

In the following discussion, we assume that the nets under consideration pass a tile edge on a layer and the terminals of these nets on the tile edge can be permuted within the tile edge. Terminals are permuted so as to minimize number of crossings as well as to balance the crossings on both the sides of the partition to satisfy the objective of the terminal assignment phase which is to maximize the the number of planar nets. If the line segments connecting the terminals of one net intersect with the line segment connecting the terminals of another net,



Figure 2: (a) & (b) Permutation of terminals results in the minimum number of crossing, (c) & (d) Permutation of terminals results in the balanced crossing on both sides.

we say that there is a crossing between these two nets and the intersection between the line segments is called *crossing*. Figure 2 illustrates the permuting steps of the terminal assignment phase. An example of minimizing the number of crossing is shown Figure 2 (a) & (b). Figure 2 (c) & (d) illustrates the idea of balancing the number of crossing on both sides of the tile edge. We prove the following theorems.

Theorem 2 There exists a permutation of the terminals such that the number of crossings between nets is minimum.

Theorem 3 There exists a permutation of the terminals such that the difference between the number of crossings between nets above and below the partition boundary is at most 1.

5 Tower Routing

The tower routing is completed in two steps: planar routing and three-dimensional routing.

- 1. **Planar Routing:** In the first step, a set of planar nets in each layer is selected and routed. Planar routing is useful in reducing the number of bends, minimizing the net lengths and avoiding the usage of large number of vias. As a result, planar routing is helpful in minimizing the noise due to reflection and minimize delay.
- 2. Residual Routing: The remaining nets are routed by using a three-dimensional router based on Soukup's style algorithm [11] while satisfying the constraints.

5.1 Planar Routing

In this section, we first consider the problem of finding a maximum planar subset for routing nets on different layers of the tower. We prove that the problem of finding such a maximum planar subset is NP-Hard and present an approximation algorithm.

We now introduce the terminology which is used in our discussion. Let k be the total number of routing layers. We consider 2 types of nets in our discussion. Let \mathcal{N}_i denote the nets that can be routed only on layer i. Let $\mathcal{N}_{i,i+1}$ denote the nets that can be routed on either



Figure 3: A tower showing different types of nets. (a) \mathcal{N}_4 , (b) \mathcal{N}_{13} , (c) \mathcal{N}_{12}

on layer i or layer i + 1. For example, \mathcal{N}_1 consists of a set of nets that can be routed only on layer one. \mathcal{N}_{12} represent a set of nets that can be routed either on layer 1 or layer 2 (see Figure 3). If two nets that are routable on layers i and i + 1 share a same pillar, then a conflict occurs if the net with the terminal on a layer less than iis assigned to layer i + 1 where the other net is assigned to layer i. Thus, if two nets share a same pillar, the net with the terminal on a layer less than i can only be assigned to \mathcal{N}_i while the other net can only be assigned to \mathcal{N}_{i+1} to avoid the possible conflicts.

Conceptually our algorithm for the planar routing step is as follows: we first generate a k-planar subset solution S_1 which consists of two maximum planar subset of nets for each layer pair starting from layer one. When k is odd, we extract a maximum planar subset for the last layer and integrate it into S_1 . Then we generate another k-planar subset solution S_2 which consists of a maximum planar subset of nets of layer one and two planar subset of nets for each layer pair thereafter from layer two onwards. If k is even we extract a maximum planar subset of the last layer and add it into S_2 . Once the solutions \mathcal{S}_1 and \mathcal{S}_2 are available, we select the maximum of \mathcal{S}_1 and \mathcal{S}_2 . We use the approach presented in [12] for finding the maximum planar subset. In order to find the two planar subset for the given pair of layers we propose an algorithm 2-RMPS described in Figure 5. Once a set of two planar subset of nets have been selected they are routed on the respective layers while satisfying the path-separation and parallel length constraints. In case the constraints cannot be satisfied for certain nets, then these nets are routed by the three dimensional router in the residual routing phase.

5.1.1 Provably Good Algorithm for 2-RMPS

In this section, we prove that restricted maximum 2planar subset problem is NP-hard, and develop an approximation algorithm for solving such a problem.

Let us start with some terminology. Given a set of nets \mathcal{N} , a set $\mathcal{N}' \subseteq \mathcal{N}$ is called a *planar subset* if the nets in \mathcal{N}' are pairwise independent. A maximum planar subset (1-MPS) is one with the maximum number of nets among all planar subsets. A k-planar subset can be defined as a set consisting of k disjoint planar subsets and a maximum k-planar subset (k-MPS) has the maximum number of vertices among all such k-planar subsets.

Planar subset problem in a switch box is polynomial time solvable [12], but for k = 2, the k-MPS is NP-complete [10]. In [2], a provably good algorithm was presented for k-MPS problem and the following theorem was proved

Theorem 4 Given a set of nets, the algorithm finds a k planar subset such that $\rho \ge (1 - (1 - \frac{1}{k})^k))$ where ρ is the ratio of solution found with optimal solution of k-MPS problem.

In the restricted maximum two planar subset problem, vertices are restricted to certain layers. We call such a problem as k-RMPS problem. Since we route on a layer pair each time, we restrict ourselves to description of 2-RMPS. Note that the problem of finding two planar subset of nets from a given two layers of a tower is equivalent to that of finding a two planar subset in a switch box. Hence algorithms for finding maximum planar subset in switch boxes can be extended to our problem of finding two planar subset. Let \mathcal{N}_1 , be the set of nets which can only be routed on layer one, \mathcal{N}_2 , be the set of nets which can only be routed on layer two, and \mathcal{N}_{12} , be the set of nets which can either be routed on layer one or layer two.

Instance: 2-Restricted Maximum Planar Subset Problem (2-RMPS): Given a switch box, and three sets $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_{12}$.

Question: Does there exist planar subsets $N_1 \subseteq \mathcal{N}_1 \cup \mathcal{N}_{12}$ and $N_2 \subseteq \mathcal{N}_2 \cup \mathcal{N}_{12}$, such that $|N_1| + |N_2|$ is maximum among all such sets ?

Theorem 5 2-RMPS is NP-Hard.

As 2-RMPS problem is NP-hard, we propose an approximation algorithm which guarantees at least 60% of the optimal solution. For a 2-restricted maximum planar subset problem, we define the *performance ratio* of 2-RMPS to be $\rho = \frac{S}{S^*}$, where S is the size of the 2-restricted planar subsets obtained by the algorithm 2-RMPS and S^* is the size of the maximum 2-restricted maximum planar subsets for a given instance.

The algorithm 2-RMPS (see Figure 5) finds a 2restricted maximum planar subset, such that first set is routable on layer i and the second set is routable on layer i+1. Algorithm 2-RMPS selects a maximum two planar subsets from three different strategies. The first strategy uses the bipartite subgraph of \mathcal{N}_{12} generated by the algorithm MBS [2] as the two planar subsets X_1 and Y_1 . The performance ratio of the algorithm MBS is at least 0.75 [2]. The second strategy finds a maximum planar subset of $\mathcal{N}_1 \cup \mathcal{N}_{12}$ and the maximum planar subset of \mathcal{N}_2 as the two planar subsets X_2 and Y_2 respectively. The subroutine $MPS(\mathcal{N}_i)$ finds the maximum planar subset for the given nets such that the set is routable on layer *i*. Subroutine MPS is based on the algorithm for finding a maximum independent set in circle graphs presented in [12]. The third strategy is similar to the second strategy. Finally, algorithm 2-RMPS returns the best result among the three strategies.

The time complexity of the algorithm 2-RMPS is dominated by the time complexity of MPS and the time

Type of nets	Avg. % in a tower	Max. % in a tower	Min. % in a tower
Type I	42	66	0
Type II	24	100	0

Table 1: Percentage of different type of nets.

complexity of MBS. Since the time complexity for both MPS and MBS is $O(m^2)$, the time complexity of the algorithm 2-RMPS is $O(m^2)$ where m is the number of nets. As a result, the overall time complexity of the planar routing phase is dominated by the complexity of 2-RMPS which is $O(m^2)$. Therefore, the time complexity of the planar routing phase is $O(km^2)$.

Theorem 6 Let ρ be the performance ratio of the algorithm 2-RMPS. For any given instance of the problem, the algorithm 2-RMPS produces a solution such that $\rho \geq 0.6$.

6 Experimental Results

Our approach was implemented in C on Sun SparcStation 1+ and was tested on the benchmarks MCC1 and MCC2 as well as several randomly system generated examples. The results clearly show the advantages of our approach. For the sake of brevity, we only illustrate the results obtained on MCC1. Details of experimental results may be found in [14]. Our approach aims at routing high density MCM problem with as small number of layers as possible, while satisfying the performance constraints.

Figure 4(a) shows the interconnection for MCC1, while Figure 4(b) shows the interconnections among the terminals of the nets after the terminal assignment. It can be clearly seen that the routing has been uniformly distributed in the XY plane and significant number of nets are planar. In fact, the ratio of the number of nets of type N_i to the total number of nets shown in Table 1 is over 42%. Also shown in the table is the ratio of the number of type II nets which refers to nets that are brought to a layer and routed completely to the total number of nets. The planar nets are selected from these two types of nets by the approximation algorithm (2-RMPS). Notice that the planar nets account for more than 65% of the total number of nets, resulting in faster tower routing and satisfying the constraints.

We also tested the effects of different tiling and the effects of different values of separation between nets. Our results are based on the separation constraint being a constant. However, our algorithm has the flexibility to accept different values for separation constraint for different sets of nets (see [14] for details.

7 Conclusions

In this paper, we have proposed a new routing methodology for routing in MCMs. Our methodology is directed towards the achievement of high performance as the proposed approach can satisfy the delay, noise, and fabrication constraints. Results on known benchmarks verify the validity of our approach.



Figure 4: (a) Initial interconnection pattern of MCC1, (b) Interconnection pattern of MCC1 after terminal assignment.

$\textbf{Algorithm 2-RMPS}(\mathcal{N}_1,\mathcal{N}_2,\mathcal{N}_{12})$		
$Input: \mathcal{N}_1$: Set of nets that can be routed		
only on layer 1		
\mathcal{N}_2 : Set of nets that can be routed		
only on layer 2		
\mathcal{N}_{12} : Set of nets that can either		
be routed on layer 1 or 2		
$Output: \mathcal{R} ext{ Set of planar nets}$		
begin		
$MBS(\mathcal{N}_{12}, X_1, Y_1);$		
$\mathcal{R}_1 = X_1 \cup Y_1;$		
$MPS(\mathcal{N}_1 \cup \mathcal{N}_{12}, X_2);$		
$MPS(\mathcal{N}_2, Y_2);$		
$\mathcal{R}_2 = X_2 \cup Y_2;$		
$MPS(\mathcal{N}_1, X_3);$		
$MPS(\mathcal{N}_2 \cup \mathcal{N}_{12}, Y_3);$		
$\mathcal{R}_3 = X_3 \cup Y_3;$		
$\mathcal{R} = \text{SELECT}_{\mathcal{M}} \text{AX}(\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3);$		
return \mathcal{R} ;		
end.		

Figure 5: Algorithm 2-RMPS.

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