Chebyshev Affine Arithmetic Based Parametric Yield Prediction Under Limited Descriptions of Uncertainty

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ABSTRACT

In modern circuit design, it is difficult to provide reliable parametric yield prediction since the real distribution of process data is hard to measure. Most existing approaches are not able to handle the uncertain distribution property coming from the process data. Other approaches are inadequate considering correlations among the parameters. This paper suggests a new approach that not only takes care of the correlations among distributions but also provides a low cost and efficient computation scheme. The proposed method approximates the parameter variations with Chebyshev Affine Arithmetics (CAA) to capture both the uncertainty and the nonlinearity in Cumulative Distribution Functions (CDF). The CAA based probabilistic presentation describes both fully and partially specified process and environmental parameters. Thus we are capable of predicting probability bounds for leakage consumption under unknown dependency assumption among variations. The end result is the chip level parametric yield estimation based on leakage prediction. The experimental results demonstrate that the new approach provides reliable bound estimation while leads to 20% yield improvement comparing with interval analysis.

1. INTRODUCTION

Due to the hard-to-measure distribution of the real process data, it is difficult to provide accurate parametric yield prediction for IC design. One example is leakage power which becomes a major issue when scaling down to submicron technology node. To be more specific, the 30% variation in the effective channel length could cause over 20x fluctuations in leakage current [1][2][3]. Therefore, it is very crucial to develop an efficient and accurate performance bound estimation framework to provide yield prediction.

One obstacle in yield prediction is the mismatch between the fundamental feature of practical IC design processes and the assumptions made to carry on research in the related area. The real process data is neither complete nor well understood. It may not behave as normal distribution as assumed in a number of papers. Wang and Orshansky were the first ones pointed out this problem [4]. The authors addressed the limitations of the existing methods and suggested a probabilistic interval analysis method which has the capability of preserving the notion of probability comparing with the pure interval analysis approaches.

However, as pointed out in [5], the interval analysis approach uses first order approximation to estimate the impact of variations. Not only the probability information is not incorporated in each operation, nonlinear dependency of performance on parametric variations is always either overestimated or underestimated. Moreover, correlation situations are handled in over optimistic manner. Such situation is getting worse in the leakage power prediction case. To illustrate, the leakage current, in general, is modeled as an exponential function of variation parameters, e.g. effective channel length L_{eff} and threshold voltage V_{th}. An interval analysis with first order approximation of an exponential function will definitely lead to inaccurate predictions. Regarding the leakage current equation as an operation redefined in the format of interval analysis, the above induced error will further worsen the overall yield prediction even with probability included in every step.

This paper overcomes the above-mentioned limitations by introducing a new Chebyshev affine arithmetic (CAA) based formulation that allows the preservation of probability while captures nonlinearity of the predicted performance, i.e. leakage consumption. The new method enables robust prediction of timing- and power-limited parametric yield. In addition, CAA based method also allows representations describing correlated, fully or partially specified parameter variations. It approximates a nonlinear function by a piece-wise linear function with the representation of a random variable as a family of distributions, i.e., bounds for cumulative distribution functions (CDF), and therefore can work with a wider class of uncertainty models. During each operation, the new approach finds the upper and lower bounds of CDF instead of computing probabilities by discretized P-box as in [4][6][7][8]. The CAA based approach divides the whole range of a random variable into several intervals and approximates the random variable's CDF in piece-wise linear formulation. We name the probability box as Piece-wise Linear Probability Bounds (PLPB). The experimental results are promising. In most cases, not only the accuracy is guaranteed because of the high accuracy of CAA approximation, the efficiency is also improved as we avoid performing optimization procedure. The experimental results show that our CAA based method is able to reliably predict the chip-level parametric yield and improve the estimation by evaluating the impact of environmental variations. The

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proposed method leads to 21.7% improvement in the mean value prediction of total leakage current, compared with interval analysis.

The paper is organized as follows. Section 2 describes the new CAA based robust estimation methodologies. Section 3 presents the new mathematical formulation of CDF computation procedure. Then Section 4 demonstrates how to applying the new CAA based method on chip-level leakage model and yield estimation. Experimental results are included in Section 5. Finally Section 6 concludes the paper.

2. CAA BASED RANDOM VARIABLE PRESENTATION AND COMPUTATION

Affine Arithmetic (AA) [10] is a methodology for range analysis, which is used to solve range estimation problems in the presence of uncertainties. In this work, we apply AA to handle random variable computation and nonlinearity estimation. In affine arithmetic, the uncertainty of a random variable x is represented by a first order *affine form* $\hat{x}$:

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n. \quad (1)$$

The quantity x_0 is called the *central value* (mean), and each ε_i is called an *uncertainty symbol* or a *variation symbol*, which stands for an independent component of the total fluctuation from the central value. The coefficient x_i is so-called *partial deviation* which gives the magnitude of corresponding component ε_i. Note here that in our yield estimation framework, *variation symbols* will represent parameter variations, including process variations and environmental variations, and *central value* will denote the nominal value of corresponding parameter. Following this presentation, we can conveniently transform parameter variations into affine forms.

With random variables represented in affine forms, we need to further evaluate the arithmetical operations upon these affine formed variables. In general, AA operations could be classified into two categories: affine operations and non-affine operations.

Let's observe specifically a bivariate AA operation $z \leftarrow f(\hat{x}, \hat{y})$. This is a procedure that returns an affine form for z in terms of $\hat{x}, \hat{y}$:

$$\begin{aligned} z &= f(\hat{x}, \hat{y}) \\ &= f(x_0 + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n,\ y_0 + y_1\varepsilon_1 + \cdots + y_n\varepsilon_n). \quad (2) \end{aligned}$$

Our task is to find an affine form $\hat{z} = z_0 + z_1\varepsilon_1 + \cdots + z_n\varepsilon_n$ to substitute $f(\hat{x}, \hat{y})$ in (2).

If the operation f itself is an affine function of $\hat{x}$ and $\hat{y}$, then (2) could be simply expanded into an affine combination of the variation symbols ε_i. Explicitly, for any constant value α, ζ,

$$\begin{aligned} \hat{x} \pm \hat{y} &= (x_0 + y_0) + (x_1 + y_1)\varepsilon_1 + \cdots + (x_n + y_n)\varepsilon_n, \\ \alpha\hat{x} &= (\alpha x_0) + (\alpha x_1)\varepsilon_1 + \cdots + (\alpha x_n)\varepsilon_n, \quad (3) \\ \hat{x} \pm \zeta &= (x_0 \pm \zeta) + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n. \end{aligned}$$

An affine operation directly return an affine form variable without any introduced computation error.

We now turn to the non-affine case. For a non-affine operation $\hat{z} \leftarrow f(\hat{x}, \hat{y})$, $\hat{z}$ is described by:

$$\begin{aligned} z &= f(x_0 + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n,\ y_0 + y_1\varepsilon_1 + \cdots + y_n\varepsilon_n) \\ &= f^*(\varepsilon_1, \cdots, \varepsilon_n). \quad (4) \end{aligned}$$

If f^* itself is not affine, then z cannot be exactly an affine combination of the variation symbols ε_i. In this situation, we need to find a suitable and effective approximation for f^*. Therefore, we choose certain affine function in terms of ε_i, namely

$$f^a(\varepsilon_1, \cdots, \varepsilon_n) = z_0 + z_1\varepsilon_1 + \cdots + z_n\varepsilon_n$$

to approximate $f^*(\varepsilon_1, \cdots, \varepsilon_n)$ over a given domain. Besides, we will then introduce an extra independent term to represent the *approximation error*. Therefore we have:

$$\hat{z} = f^a(\varepsilon_1, \cdots, \varepsilon_n) = z_0 + z_1\varepsilon_1 + \cdots + z_n\varepsilon_n + z_k\varepsilon_k \quad (5)$$

where $z_k\varepsilon_k$ denotes above-mentioned *approximation error*:

$$e^*(\varepsilon_1, \cdots, \varepsilon_n) = f^*(\varepsilon_1, \cdots, \varepsilon_n) - f^a(\varepsilon_1, \cdots, \varepsilon_n).$$

Variation symbol ε_k (guaranteed to lie in the interval $[-1, 1]$) is induced in the approximation procedure, thus should be independent of all other existing variation symbols.

Then we focus on how to decide the most appropriate approximation expression. In general, for the purpose of simplicity and efficiency, we only consider the approximations f^a which themselves are affine combinations of input arguments $\hat{x}$ and $\hat{y}$ [10], that is,

$$f^a(\varepsilon_1, \cdots, \varepsilon_n) = \alpha\hat{x} + \beta\hat{y} + \zeta \quad (6)$$

Thus, we only need to determine the optimal coefficients in (6), α, β and ζ. This is the subject of *Chebyshev approximation theory* [10]. Chebyshev approximation is actually the optimal affine approximation that minimizes the maximum absolute error.

Chebyshev approximations theory is a well-developed field in a number of literatures. In particular, for univariate functions $\hat{z} \leftarrow f(\hat{x})$, [11] states the existence of Chebyshev affine approximation. It can be easily extended to multi-variate cases. Furthermore, [10] provides an algorithm to find the optimal coefficient α and ζ for Chebyshev approximation.

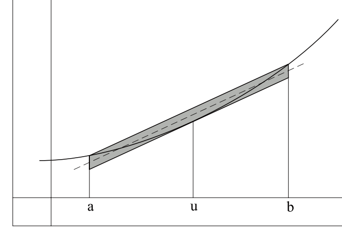

Figure 1: Geometry of Chebyshev Approximations

Figure 1 illustrates the geometrical interpretation of univariate Chebyshev approximations. The shaded parallelogram is exactly the range of univariate Chebyshev approximation. The dash line denotes the approximation equation $\alpha\hat{x} + \zeta$, with a perturbation of approximation error $z_k\varepsilon_k$ in (5). Clearly, Chebyshev approximation guarantees the smallest area of this parallelogram by minimizing the approximation error δ. According to (5), we can express $\hat{z}$ in following affine form explicitly:

$$\begin{aligned} \hat{z} &= \alpha\hat{x} + \zeta + \delta\varepsilon_k \\ &= (\alpha x_0 + \zeta) + (\alpha x_1)\varepsilon_1 + \cdots + (\alpha x_n) + \delta\varepsilon_k \quad (7) \end{aligned}$$

where ε_k is a new variation symbol, introduced to represent approximation error.

3. CAA BASED DEPENDENCY BOUNDS COMPUTATION

In this section we develop a robust estimation framework, which enables reliable computation of distribution functions of random variables. As indicated before, in order to deal with correlations among variations, bounds computations

will be performed under the assumption of unknown dependency. As a first step, we apply Chebyshev approximations to extract appropriate probabilistic representation which results in a very low computation cost.

3.1 P-box Representation

In our prediction scheme, we first need to work out an appropriate probabilistic representation for the uncertainty of variables. Most generally, a fully specified random variable is represented by its CDF. While for a partially specified random variable, the most general representation is a family of cumulative distribution functions, or a *p-box* [9].

Definition 1 [9]: $\overline{F}$ and $\underline{F}$ are non-decreasing functions, and $\underline{F} \leq \overline{F}$. A *p-box*, denoted by $[\underline{F}, \overline{F}]$, is defined as a set of imprecisely known cumulative distribution functions, $F(x) = P(X \leq x)$, where $\underline{F}(x) \leq F(x) \leq \overline{F}(x)$.

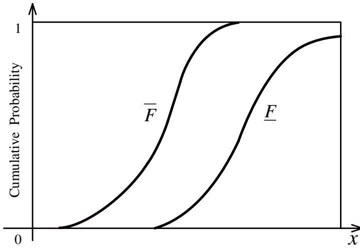

Figure 2: p-box is a probability box consisting of a left and right bound

A p-box actually represents upper and lower bounds for cumulative distribution function of a random variable. It is possible to provide a robust description of random variable even on the condition of uncertain distributions. For a partially specified random variable, given necessary statistical metrics (i.e. mean value and variance) as well as variable range, we can successfully construct a p-box by applying one-sided Chebyshev inequality and Cantelli inequality [4]. Therefore we are able to deal with a variety of non-Gaussian distribution.

3.2 P-box Linearization using Chebyshev Approximations

P-box is the basic notion of our probability bounds computation framework, all parameter variabilities will be transformed into p-box representations, also all the operations are performed upon p-boxes of random variables. Based on Chebyshev Approximations, we propose a novel piece-wise linearization method to obtain appropriate p-boxes, which are convenient for calculating dependency bounds. Different from traditional p-box discretization schemes (e.g.[4][6][7][8]), we divide the whole range of a certain random variable into several intervals, apply Chebyshev approximations to probability bounds for each interval, and construct the CDF bounding functions in piece-wise linear formation. Here we name it Piece-wise Linearized Probability Bounds (PLPB).

We here claim that PLPB is a better representation for probability in comparison with traditional discretized p-box. For better illustration, we draw in Figure 3 both discretized p-box and PLPB expression for the same probability box. We see that PLPB preserves more useful probability information than simply discretized p-box, which is in fact a constant value for each interval. In contrast, PLPB holds the property of continuity. Moreover, since continuous CDF generally exhibits a slowly increasing property, we can always extract a corresponding PLPB with few number of divided interval. It will be clear in next part that such

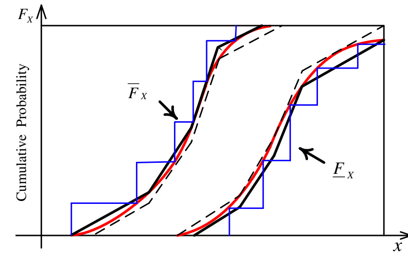

Figure 3: Discretized p-box vs. PLPB

piece-wise linear form provides an impressive low computation cost as well as high accuracy.

If we are given a fully specified random variable, which means its CDF is determined, we can directly apply Chebyshev approximation on its CDF to extract PLPB formulation (See Figure 4).

3.3 Dependency Bounds Computation

Having CAA representation for uncertainty and PLPB representation for probability, we are now capable of calculating probability bounds under AA. We will see in Section 4.2 that, in Affine Arithmetic, bounds computation only needs to be executed for “Add” and “Subtract” operations on random variables. [7] indicates that CDF bounds computation could be effectively performed under these two operations, when there exists unknown dependency between random variables.

We now discuss how to compute dependency bounds. Given two random variables X and Y in p-box representations, i.e., with upper bound $\overline{F}_X$ and lower bound $\underline{F}_X$ for random variable X, while $\overline{F}_Y$ and $\underline{F}_Y$ for B respectively. We need to compute dependency bounds $[\underline{F}_Z, \overline{F}_Z]$ for $Z = X \pm Y$. It is pointed out in [7] that by applying duality theorem we can express the inverses of F_Z bounds in terms of the inverses of F_X and F_Y bounds. Specifically, for “Add” operation:

$$\underline{F}_Z^{(-1)}(p) = \underline{F}_{X+Y}^{(-1)}(p)$$

$$= \begin{cases} \min\limits_{u\in[p,1]} [\underline{F}_X^{(-1)}(u) + \underline{F}_Y^{(-1)}(p-u+1)], & \text{if } p \neq 0 \\ \underline{F}_X^{(-1)}(0) + \underline{F}_Y^{(-1)}(0), & \text{if } p = 0 \end{cases} \quad (8)$$

$$\overline{F}_Z^{(-1)}(p) = \overline{F}_{X+Y}^{(-1)}(p)$$

$$= \begin{cases} \max\limits_{u\in[0,p]} [\overline{F}_X^{(-1)}(u) + \overline{F}_Y^{(-1)}(p-u)], & \text{if } p \neq 1 \\ \overline{F}_X^{(-1)}(1) + \overline{F}_Y^{(-1)}(1), & \text{if } p = 1 \end{cases} \quad (9)$$

Likewise, for “Subtract” operation, we have

$$\underline{F}_{X-Y}^{(-1)}(p) = \min_{u\in[p,1]} [\underline{F}_X^{(-1)}(u) - \overline{F}_Y^{(-1)}(u-p)],\ p \neq 0. \quad (10)$$

$$\overline{F}_{X-Y}^{(-1)}(p) = \max_{u\in[0,p]} [\overline{F}_X^{(-1)}(u) - \underline{F}_Y^{(-1)}(u-p+1)],\ p \neq 1. \quad (11)$$

The associated derivations and their proofs are provided in [7]. From (8)-(11), it is shown that for $Z = X \pm Y$, Z and X, Y exhibits a function relationship in their respective inverses of CDF bounds. This observation provides us an approach to work out the dependency bounds for F_Z.

Take $\underline{F}_Z^{(-1)} = \underline{F}_{X+Y}^{(-1)}$ for instance. For a fixed input probability p, $\underline{F}_{X+Y}^{(-1)}(p)$ is actually the minimum of the summation of two inverse CDF functions over the range $[p, 1]$. From this point of view, it seems to be attributed to an optimization problem. However, with our PLPB representation of random variables, we will show that the dependency bounds computation does not require complicated optimization procedure and thus has very low computation cost.

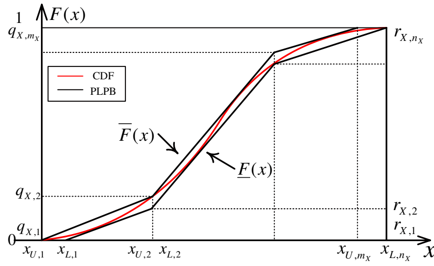

Figure 4: PLPB representation for a fully-specified random variable

Given a random variable A in PLPB representation (shown in Figure 4, fully specified in this case). we have:

$$\overline{F}_X(x) = \begin{cases} 0, & \text{for} \quad x < x_{U,1} \\ k_{X,i} \cdot x + u_{X,i}, & \text{for} \quad x_{U,i-1} \leq x < x_{U,i}, \\ & \qquad i = 2, \cdots, m_X \\ 1, & \text{for} \quad x \geq x_{U,m_X} \end{cases} \tag{12}$$

$$\underline{F}_X(x) = \begin{cases} 0, & \text{for} \quad x < x_{L,1} \\ k_{X,j} \cdot x + l_{X,j}, & \text{for} \quad x_{L,j-1} \leq x < x_{L,j}, \\ & \qquad j = 2, \cdots, n_X \\ 1, & \text{for} \quad x \geq x_{L,n_X} \end{cases} \tag{13}$$

Due to the non-decreasing property of cumulative probability, clearly $k_{X,i}$ and $k_{X,j}$ values are positive.

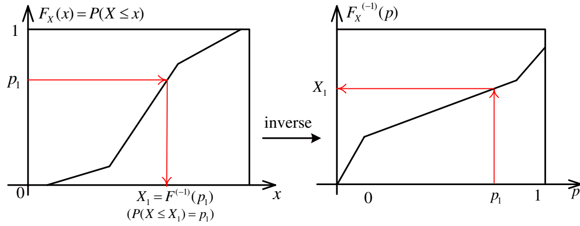

Figure 5: The inverse of CDF for a random variable

For the inverse function of CDF, the input argument is a probability value, while the output argument is the corresponding quantile. With PLPB representation for a random variable, we can conveniently derive the inverses of its CDF bounds. Because PLPB formed CDF is continuous and non-decreasing, and is a linear function for each interval, its inverse function should also be in piece-wise linear formation (shown in Figure 5). We then define the inverse of PLPB:

$$\overline{F}_X^{(-1)}(w) = \begin{cases} x_{U,1}, & \text{if} \quad 0 \leq w \leq q_{X,1} \\ \frac{1}{k_{X,i}}(w - u_{X,i}), & \text{if} \quad q_{X,i-1} < w \leq q_{X,i}, \\ & \qquad i = 2, \cdots, m_X \\ x_{U,m_X}, & \text{if} \quad q_{X,m_X} < w \leq 1 \end{cases} \tag{14}$$

$$\underline{F}_X^{(-1)}(w) = \begin{cases} x_{L,1}, & \text{if} \quad 0 \leq w \leq r_{X,1} \\ \frac{1}{k_{X,j}}(w - l_{X,j}), & \text{if} \quad r_{X,j-1} < w \leq r_{X,j}, \\ & \qquad j = 2, \cdots, n_X \\ x_{L,n_X}, & \text{if} \quad r_{X,n_X} < w \leq 1 \end{cases} \tag{15}$$

where $0 \leq q_{X,1} \leq q_{X,2} \leq \cdots \leq q_{X,m_X} \leq 1$, and $0 < r_{X,1} \leq r_{X,2} \leq \cdots \leq r_{X,n_X} \leq 1$. Random variable Y has similar expressions for piece-wise linearized bounding functions.

We now focus on how to construct lower dependency bound $\underline{F}_{X+Y}$. Under unknown dependency assumption, (8) gives:

$$\underline{F}_{X+Y}^{(-1)}(p) = \min_{u \in [p,1]} [\underline{F}_X^{(-1)}(u) + \underline{F}_Y^{(-1)}(p-u+1)], \quad p \neq 0. \tag{16}$$

For a fixed probability value p, $\underline{F}_Y^{(-1)}(p-u+1)$ can be regarded as a function with respect to u, and we let $F'(u) = \underline{F}_Y^{(-1)}(p-u+1)$, where $p \leq u \leq 1$. According to (15):
$F'(u) = \underline{F}_Y^{(-1)}(p-u+1)$

$$= \begin{cases} y_{L,1}, & \text{if} \quad p < u \leq p+1-r_{Y,n_Y} \\ \frac{-u+p+1-l_{Y,j}}{k_{Y,j}}, & \text{if} \quad p+1-r_{Y,j} < u \leq p+1 \\ & \qquad -r_{Y,j-1}, \; j = 2, \cdots, n_Y \\ y_{L,n_Y}, & \text{if} \quad p+1-r_{Y,1} \leq u \leq p+1 \end{cases} \tag{17}$$

For any $p \neq 0$, our purpose is to find the minimum value of $\underline{F}_X^{(-1)}(u) + F'(u)$ over the range $[p, 1]$, where $\underline{F}_X^{(-1)}(u)$ and $F_1(u)$ are both piece-wise linear functions (expressed in (15), (17)). The probability range for $\underline{F}_X^{(-1)}(u)$ is divided by a set of transition points $\{0, \; r_{X,1}, \cdots, \; r_{X,n_X}\}$, while $F'(u)$ is divided by set $\{p, p+1-r_{Y,n_Y}, \; p+1-r_{Y,n_Y-2}, \cdots, 1\}$. Therefore we can redivide the probability range by constructing a combinational set consisting of the transition points of $\underline{F}_X^{(-1)}(u)$ and $F'(u)$. Specifically, let

$$S = \{r_{X,1}, \; \cdots, \; r_{X,n_X}, \; 1, \; p, \; p+1-r_{Y,n_Y}, \; \cdots, \; 1\}.$$

Rearrange the elements of S in ascending order, i.e., from smallest to largest and rename them $s_1, s_2, \cdots$. For each interval $[s_i, s_{i+1}]$, $\underline{F}_X^{(-1)}(u) + F'(u)$ is a summation of two linear functions and the resulting function is a linear function with respect to u. That is, the minimum function value for each interval must be determined by either starting point or end point. We conclude that the value of $\min[\underline{F}_{X+Y}^{(-1)}(p)]$ can be directly determined by the interval endpoints, since the global minimum of $\underline{F}_X^{(-1)}(u) + F'(u)$ must be achieved at certain transition points over range $[p, 1]$. Thus, we avoid performing optimization procedure which may induce expensive computation cost.

Other cases for $\overline{F}_{X+Y}$, $\underline{F}_{X-Y}$, and $\overline{F}_{X-Y}$ can be derived in the same way.

4. PARAMETRIC YIELD ESTIMATION PROCEDURE

In this section an effective CAA based method to predict full-chip parametric yield is represented. We apply the leakage current model proposed in [1][4], where leakage current is modeled as an exponential function of several key parameters. We first use CAA described in Section 2 to transform the leakage model into affine operations. Then we can perform dependency bounds computation under affine operations. Finally, we provide a reliable performance bound estimation for leakage current.

4.1 Leakage Current Model

In [1], Rao *etal.* presented a leakage current model for yield analysis. In this model, subthreshold and gate leakage current are functions of different process parameters. The total leakage current is a sum of these two components:

$$I_{total} = I_{sub} + I_{gate} \tag{18}$$

In previous work [1], only process parameters are incorporated in the modeling function. [4] improves the model by taking into account environmental parameters. In general, the variations of following key parameters are considered: process variations including effective channel length ($\triangle L$), threshold voltage ($\triangle V_{th}$) and oxide thickness ($\triangle T_{ox}$), as well as environmental uncertainty including power supply voltage ($\triangle V_{dd}$) and on-chip temperature ($\triangle T$).

For a unit device transistor, the subthreshold leakage is described as a nominal value with multiplication of an ex-

ponential function in terms of L_{eff}, V_{th}, V_{dd} and T:

$$I_{sub} = I_{sub,nom} \cdot e^{a\triangle L^2 + b\triangle L + c\triangle V_{th} + d\triangle V_{dd} + e\triangle T}$$

Furthermore, we decompose the variation of each process parameter into global and local components [1] and them separately:

$$I_{sub} = I_{sub,nom} \cdot e^{a\triangle L_l^2 + (2a\triangle L_g + b)\triangle L_l + c\triangle V_{th,l} + d\triangle V_{dd} + e\triangle T} \cdot e^{a\triangle L_g^2 + b\triangle L_g + c\triangle V_{th,g}} \quad (19)$$

where $I_{sub,nom}$ is the nominal value of subthreshold leakage, $(\triangle L_l, \triangle V_{th,l})$ and $(\triangle L_g, \triangle V_{th,g})$ denote intra-chip and inter-chip components of process variations respectively.

Similarly, the intra-chip gate leakage is modeled as an exponential function of $\triangle T_{ox}$ and $\triangle V_{dd}$ [4]:

$$\begin{aligned} I_{gate} &= I_{gate,nom} \cdot e^{h\triangle T_{ox} + k\triangle V_{dd}} \\ &= I_{gate,nom} \cdot e^{h\triangle T_{ox,l} + k\triangle V_{dd}} \cdot e^{h\triangle T_{ox,g}} \end{aligned} \quad (20)$$

where $I_{gate,nom}$ is the nominal value of gate leakage. This model is insensitive to on-chip temperature variation [3].

4.2 Robust Estimation Procedure

In this part, we discuss about how to estimate the chip-level parametric yield by the CAA based method proposed in previous sections. Our work focuses on predicting the probability bounds for the leakage current of a specific chip.

First of all we represent all the parameter variations in affine forms. For each parameter P with variation:

$$P = P_{nom} + \triangle P.$$

Since in our leakage model, we are only concerned about the deviation from the nominal value, $\triangle P$ is modeled as a zero mean variable. Then in Affine Arithmetic, the *central value* is clearly denoted by the nominal value, each parameter variation can be regarded as an *variation symbol*, and the *partial deviation* is simply equal to one. Let $P_0 = P_{nom}$, and ε be the corresponding parameter variation, hence we get affine representation for each parameter:

$$\hat{P} = P_0 + 1 \cdot \varepsilon.$$

In this work, we model all process variations as truncated Gaussian distributions. However, as demonstrated previously, a partially specified distribution also can be handled by constructing the PLPB from its p-box to represent the uncertainty.

Following [1], given a particular chip, we compute the subthreshold leakage current for a single device (19) based on such assumption: each device has unique local variations $\triangle L_l$ and $\triangle V_{th,l}$, while sharing the same global variations $\triangle L_g$ and $\triangle V_{th,g}$. Therefore, $\triangle L_g$ and $\triangle V_{th,g}$ are assumed to be fixed for each device. We then focus on intra-chip variations $\triangle L_l$ and $\triangle V_{th,l}$. Rewrite (19) in affine form:

$$I_{sub} = I_{sub,nom} \cdot e^{a\hat{L}_l^2 + (2a\hat{L}_g + b)\hat{L}_l + c\hat{V}_{th,l} + d\hat{V}_{dd} + e\hat{T}} \cdot e^{a\triangle L_g^2 + b\triangle L_g + c\triangle V_{th,g}} \quad (21)$$

where $\triangle\hat{L}_l$ denotes the deviation from the nominal value of L_l, and simply $\triangle\hat{L}_l = 0 + 1 \cdot \triangle L_l$. Other parameters are on the analogy of this formula. According to (21), clearly the quadratic term of $\triangle\hat{L}_l$ and the exponential term are non-affine operations, and thus need to apply Chebyshev approximations. As discussed in Section 2, we perform two affine approximations $z \leftarrow \hat{x}^2$ and $z \leftarrow e^{\hat{x}}$ respectively, in order to transform them into affine counterparts. Thus we actually reduce the subthreshold leakage model to a serial of affine operations on variation symbols, i.e. parameter variations of $\triangle L_l$, $\triangle V_{th,l}$, $\triangle V_{dd}$ and $\triangle T$.

We now discuss how to compute the probability bounds for I_{sub} under affine operations. According to (3), affine operations can be generalized into three fundamental cases:

Case 1. $\hat{z} = \hat{x} \pm \zeta$. We can directly obtain probability bounds for $\hat{z}$ given $\hat{x}$ in PLPB presentation, using probability knowledge [12].

Case 2. $\hat{z} = \alpha\hat{x}$. We can also work our the probability bounds for $\hat{z}$ analytically [12].

Case 3. $\hat{z} = \hat{x} \pm \hat{y}$. In Section 3 we have represented exhaustively how to compute the dependency bounds for summation and difference of two random variables.

Now we are capable of computing the probability bounds for each affine operation. Since the subthreshold leakage equation (21) has been reduced to a serial of affine operations, we can perform the computation step by step. Clearly each step returns an affine form, thus we are able to produce the probability bounds for subthreshold leakage current with affine operations.

Also, we can estimate gate leakage current by similar procedure, regarding parameter variations $\triangle T_{ox,l}$ and $\triangle V_{dd}$ as *variation symbols* and reducing I_{gate} into affine operations of them.

$$I_{gate} = I_{gate,nom} \cdot e^{h\triangle\hat{T}_{ox,l} + k\triangle\hat{V}_{dd}} \cdot e^{h\triangle T_{ox,g}} \quad (22)$$

It is mentioned above that total leakage current I_{total} is the sum of the subthreshold and the gate leakage components. From (19) and (20), due to the introduction of environmental parameter variations, subthreshold and gate leakage currents are correlated since they both are dependent on V_{dd} variation. Thus we still use the algorithm of dependency bounds computation, proposed in Section 3, to predict the probability bounds of total leakage current for a unit device.

In the last step, to estimate the overall leakage current for a specific chip, we need to sum up the leakages for all devices on the chip. By Central Limit Theorem [12], we can use the mean value of I_{total} to approximate the sum of leakages over all devices. Having the predicted bounds of cumulative probability, we can easily deduce a bounded estimation for the mean value of chip-level leakage.

5. EXPERIMENTAL RESULTS

The experiment environment is similar to that of [4]. We use the 65nm ($L_{eff} = 24.5nm$) Technology node PTM model [13]. The parameter coefficients of the leakage model (19), (20) are extracted by SPICE simulations. The process variations are modeled as truncated Gaussian distributions. The 3σ values of L, V_{th} and T_{ox} parameters are 20%, 10% and 8% of the nominal values respectively. We assume that for all process parameters, the inter-chip and intra-chip variation each accounts for half of the total variation. For environmental uncertainty, the supply voltage variation is about 10% of the nominal value, and the on-chip temperature deviates about 10° from the nominal value.

For a particular chip, the inter-chip variations are assumed to be fixed, which means all the devices on the chip share the same global variations. For simplicity, we first estimate the parametric yield with zero inter-chip variations. Figure 6 shows the experimental results for subthreshold leakage current (normalized to the nominal value). Since our purpose is to predict the guaranteed yield, we are only concerned about the lower probability bound. The upper bound is not shown in all figures.

We reaffirm that our CAA based method can handle arbitrary correlations among parameter variations. For the purpose of verification, we first run SPICE simulations in the very special case of independency. From Figure 6 we see the deterministic CDF obtained by Monte-Carlo simulation

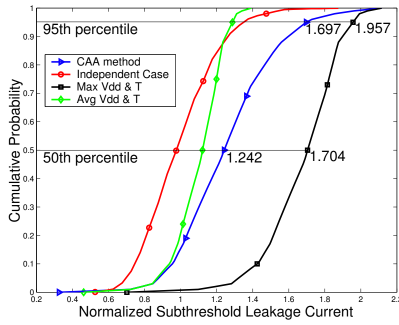

Figure 6: Subthreshold leakage current considering process and environmental variations

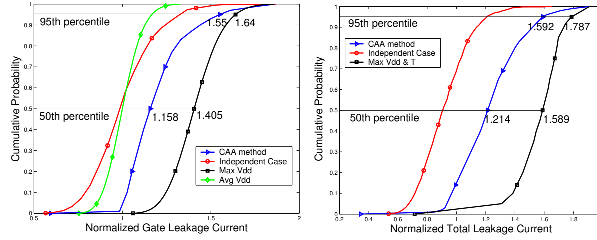

Figure 7: Gate Leakage Current **Figure 8: Total Leakage Current**

is well bounded by the dependency bound computed by our proposed algorithm. More significantly, this indicates the importance of taking into account the correlations among parameter variations. Since the simple assumption of independency tends to give an over-optimistic prediction of parametric yield, which implies a lower estimate at a specified cumulative probability.

In previous chip-level leakage analysis, e.g. [1], environmental parameter including supply voltage (V_{dd}) and on-chip temperature (T) are not taken into account and assumed to be fixed. Therefore to give a guaranteed yield, algorithm in [1] needs to select the maximum values of V_{dd} and T for prediction. To evaluate the impact of environmental variations, we also compute the (lower) probability bound based on maximum and average values of V_{dd} and T, respectively. Figure 6 shows, our reliable prediction considering environmental variations provides a tighter bound for I_{sub}. It predicts 1.697X of nominal subthreshold leakage at 95th percentile, which means that the probability of $I_{sub} \leq I_{sub,nom}$ is greater than 95%. Compared to algorithm in [1], it improves the leakage estimate by 13.3% at 95th percentile. The improvement is more significant at 50th percentile, which is 27.1%. In addition, the predicted bound based on average V_{dd} and T also gives an over-optimistic estimation. There are similar results in cases of I_{gate} and I_{total}(shown in Figure 7 and 8). The improvements are listed in Table 1.

Table 1: Estimate Improvements of Leakage Current

Estimate Improvement	I_{sub}	I_{gate}	I_{total}
50th percentile	13.29%	5.49%	10.91%
95th percentile	27.11%	17.58%	23.60%
mean value	25.07%	15.26%	21.68%

At last we estimate the (lower) probability bound of I_{total} based on different inter-chip L_{eff} variations ($\triangle L_g$). Figure 9 illustrates the 50th, 68th, 95th and 99th percentiles when $\triangle L_g = 0, \pm\sigma, \pm 2\sigma, \pm 3\sigma$. The experimental result verifies that shorter channel length L_{eff} causes more significant variation of leakage current.

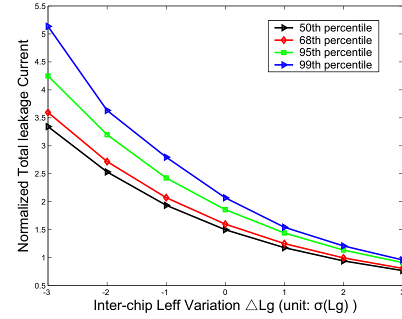

Figure 9: Contours for inter-chip L_{eff} variations

6. CONCLUSIONS

In this paper, a new parametric yield estimation framework is presented based on Chebyshev affine arithmetic. This proposed strategy is able to handle uncertain parameter distributions, and thus improve the estimation of interval analysis for partially specified parameters. Under unknown dependency assumption to deal with the correlations among parameter variations, our CAA based method is capable of predicting reliable guaranteed bounds for chip-level parametric yield, with high accuracy and low computation cost.

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