New Block-Based Statistical Timing Analysis Approaches Without Moment Matching*

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Abstract

With aggressive scaling down of feature sizes in VLSI fabrication, process variation has become a critical issue in designs. We show that two necessary conditions for the "Max" operation are actually not satisfied in the moment matching based statistical timing analysis approaches. We propose two correlation-aware block-based statistical timing analysis approaches that keep these necessary conditions, and prove that our approaches always achieve *tight* lower bound and upper bound of the yield. Especially, our approach always gets the tight upper bound of the yield irrespective of the distributions that random variables have.

1 Introduction

With aggressive scaling down of feature sizes in VLSI fabrication, process variation has become a critical issue in designs. The corner-based deterministic static timing analysis (STA) becomes pessimistic and inefficient because of the complicated correlations among component delays and the huge number of corners.

The emerging statistical static timing analysis (SSTA) approaches [1–7] greatly speed up the analysis by propagating the distributions instead of single values. An essential problem in SSTA is how to compute the maximal of random variables. Assuming that process variations are not very prominent, [3] and [4] used the Clark's approach [8] to approximate the maximal of two random variables with Gaussian distribution as a Gaussian variable, and achieved good efficiency and accuracy. Random variables are represented in a linear canonical form, and the first two moments (the mean and the variance) of the outputs are matched.

The delay of a gate or a wire is affected by more than one types of process variations, and a linear form may not be accurate enough to capture the important information. So [5–7] extended the linear model to non-linear models. For example, the random variables in [7] are represented in a quadratic model. These approaches are shown to be more accurate than those based on the linear model.

With the development of SSTA tools, many statistical timing optimization works also emerged. These works optimize the timing yield–the probability that a circuit satisfies timing constraints –using the SSTA approaches to compute the timing information. Although it is shown that the SSTA approaches have good accuracy, it is not guaranteed that the computed yield is either lower or higher than the actual yield. Without this information, the designers have to over design in order to make sure that the yield is satisfied. So the computation of the lower bound and the upper bound of the yield is desired. Agarwal *et al.* [9, 10] proposed techniques to compute the bounds of the yield, but they did not consider the correlations ([9] ignored the correlations between the components, while [10] ignored the correlations due to the path re-convergence), and it is not clear whether the computed bounds are close to the actual yield when correlations are considered.

In this paper, we consider how to compute the lower bound and the upper bound of the yield. The existing SSTA works use the linear model or the second order model to *approximate* process variations, so even the yield computed by the Monte Carlo simulation is not the exact yield. But the designers can select parameters in the models such that the described process variations are the lower bound or the upper bound of the actual process variations. Then the accurate computation of the lower and the upper bound of the yield can tell whether the objective yield can be satisfied. Enforcing two necessary conditions for the statistical "max" operation that are not satisfied by moment-matching based approaches [3,4], our approaches achieve the tight bounds of the yield. Furthermore, for upper bound computation, our approach can also be used with the second-order model.

The rest of this paper is organized as follows. Section 2 briefly reviews the existing works on SSTA. Section 3 presents the relations between the results and the operands in the statistical "Max" operation, and the problem in moment matching based approaches. Section 4 presents our correlation-aware approaches for the statistical "Max" operation. The experiments on the proposed approaches and their comparison with the Monte Carlo simulation are reported in Section 5. Finally, the conclusions are drawn in Section 6.

2 Preliminary

The combinational circuit is represented by a directed acyclic graph (DAG) $G(V, E)$ with a vertex (or node) set V and an edge set E. Each vertex represents a primary input, a primary output or a gate, each edge represents an interconnection from the source vertex to the target vertex, and the edge weight is its delay. Two dummy nodes s and t are introduced into the graph: s is connected to all the primary inputs, and t is connected from all the primary outputs. The weights of the edges from s to PIs or from POs to t are zero,

All the delays (or weights), slacks and arrival time are represented in a first-order canonical form as in [3]:

$$c_0 + \sum_{i=1}^{n} c_i X_i,$$

where c_0 is the mean value, X_i's are principal components [11], and c_i's are the coefficients. Principal component analysis [11] can be performed to get this canonical form [3].

We define the followings for two Gaussian random variables X and Y with correlation coefficient ρ.

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2), \quad (1)$$

$$\Phi(y) = \int_{-\infty}^{y} \phi(x)dx, \quad (2)$$

$$\theta_{XY} = \sqrt{\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y}, \quad (3)$$

$$\alpha_{XY} = \frac{\mu_X - \mu_Y}{\theta}. \quad (4)$$

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Given any two random variables X and Y, [4] defined the *tightness probability* T_X of X as the probability that it is larger than Y, and $T_Y = 1 - T_X$. Thus,

$$T_X = \Phi(\frac{x_0 - y_0}{\theta_{XY}}),$$

when $X \neq Y$.

Let $Z = \max(X, Y)$. In block-based SSTA, the moment matching is performed to compute the canonical form representing $\max(X, Y)$. For example, [4] matches the mean, variance and covariance, while [6] matches the raw moments.

Chang *et al.* [3] compute the maximal of two Gaussian random variables as follows. Suppose

$$\begin{aligned} A &= a_0 + \sum_{i=1}^{n} a_i X_i, \\ B &= b_0 + \sum_{i=1}^{n} b_i X_i. \end{aligned}$$

Let C represent $\max(A, B)$. Then according to [8],

$$\begin{aligned} \mu(C) &= T_A \mu(A) + (1 - T_A)\mu(B) + \theta\phi(\alpha), \quad (5) \\ \sigma^2(C) &= [\sigma^2(A) + \mu^2(A)]T_A \\ &\quad + [\sigma^2(B) + \mu^2(B)](1 - T_A) \\ &\quad + [\mu(A) + \mu(B)]\phi(\alpha) - \mu^2(C). \quad (6) \end{aligned}$$

Through moment matching, [3] gets

$$C = \mu(C) + \frac{\sigma(C)}{s_0} \sum_i \beta_i X_i,$$

where

$$\beta_i = T_A a_i + (1 - T_A) b_i,$$

and

$$s_0 = \sqrt{\sum_i \beta_i^2}.$$

3 Statistical "Max" operation

We introduce two concepts for the random variables.

Definition 1 (Dominance relation) *Suppose A and B are two random variables, then A dominates B iff*

$$Pr(A \geq B) = 1.$$

Definition 2 (Comparison relation) *Suppose A, B and C are random variables. If*

$$\begin{aligned} Pr(C > A) &= Pr(B > A), \\ Pr(C > B) &= Pr(A > B). \end{aligned}$$

are satisfied, C has the comparison relations with A and B.

The following theorem shows that both the dominance relation and the comparison relation are necessary conditions for the statistical "Max" operation.

Theorem 1 *Suppose A and B are random variables. If $C = \max(A, B)$, C dominates A and B, and has the comparison relations with A and B.*

Proof: Since C is the maximal of A and B, $C \geq A$ and $C \geq B$, so C dominates A and B.

$$\begin{aligned} Pr(C > A) &= Pr(\max(A, B) - A > 0) \\ &= Pr(\max(A - A, B - A) > 0) \\ &= Pr(\max(0, B - A) > 0) \\ &= Pr(B - A > 0) \end{aligned}$$

Similarly, we can prove

$$Pr(C > B) = Pr(A > B).$$

□

The block-based SSTA approaches [3, 4] assume that all the random variables have Gaussian distribution. They use a canonical form

$$c_0 + \sum_{i=1}^{n} c_i X_i$$

to represent a random variable, where c_0 is the nominal value, and X_i's are independent random variables with standard normal distribution. When they compute the maximal of Gaussian variables, they use Clark's approach [8] to match the mean and the variance. But during this match, the dominance and comparison relations are not kept. For example, compute the maximal of the following two Gaussian random variables using the approach [4]:

$$A = 30 + x_1,$$

and

$$B = 30.5 + 0.5x_1.$$

Suppose $C = max(A, B)$, then theoretically,

$$Pr(C \geq A) = 1 \textit{ and } Pr(C \geq B) = 1.$$

But the results computed from the moment matching based approach in [4] are

$$Pr(C \geq A) = 89.46\% \textit{ and } Pr(C \geq B) = 62.57\%.$$

So the dominance relations are not kept. Also theoretically,

$$Pr(A > B) = 15.84\%,$$

but the moment matching based approach gets

$$Pr(C > B) = 62.57\%,$$

which should be equal to $Pr(A > B) = 15.84\%$. So the comparison relations are not kept either.

We also use the approach in [6] to approximate the maximal of two Gaussian variables as a non-Gaussian variable, and find that neither the dominance nor comparison relation is kept. For example, using the approach in [6], for the dominance relations, we get

$$Pr(C \geq A) = 63.43\% \textit{ and } Pr(C \geq B) = 49.17\%,$$

and for the comparison relations, we get

$$Pr(C > B) = 49.17\% \neq Pr(A > B) = 15.84\%.$$

Thus, the existing approximation approaches have not kept the necessary conditions in the statistical "Max" operation.

Timing analysis approach will be eventually used in timing optimizations. In statistical timing optimization, we need

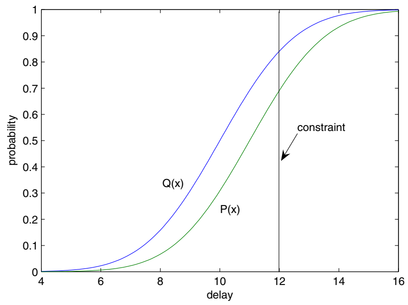

Figure 1: CDF $Q(x)$ is an upper bound of CDF $P(x)$.

to compute the yield, that is, the probability that the constraint is satisfied. The moment matching based SSTA approaches [3, 4] are approximation approaches, and it is not guaranteed whether they are conservative or optimistic. For example, given a timing constraint for the maximal delay from the primary input to the primary output, we do not know if the computed yield is higher or lower than the actual yield.

Definition 3 *For any two cumulative distribution functions $P(x)$ and $Q(x)$, $Q(x)$ is the upper bound of $P(x)$ if and only if $\forall x : Q(x) \geq P(x)$.*

As shown in Fig. 1, using the upper bound of $P(x)$, the yield $Pr(x \leq constraint)$ according to the upper bound of $P(x)$ is higher than the yield according to $P(x)$.

Definition 4 *For any two cumulative distribution functions $P(x)$ and $Q(x)$, $Q(x)$ is the lower bound of $P(x)$ if and only if $\forall x : Q(x) \leq P(x)$.*

The yield $Pr(x \leq constraint)$ is lower when the lower bound of $P(x)$ is used to compute the yield.

We will show later that the approaches based on the dominance relations or the comparison relations give the lower bound and the upper bound of the yield respectively.

4 SSTA without moment matching

"Max" and "Add" are two fundamental operations in timing analysis. In SSTA, all random variables are represented in the canonical form. The "Add" operation is easy. For the "Max" operation, we want to maintain the dominance relations or the comparison relations.

4.1 Theory

Our SSTA approach traverses a circuit in the topological order, and computes the distribution of the arrival time at each node. Depending on what relations the procedure keeps, our approach has two variants. The first one, denoted as LBDomSSTA, keeps the dominance relations, while the second one, denoted as UBCompSSTA, keeps the comparison relations.

Note that the theory in this subsection holds for random variables of any distributions, not only limited to Gaussian.

For the dominance relation, we have the following theorem:

Theorem 2 *In a combinational circuit, when a "max" operation is encountered, if we always use a random variable that dominates the operands to represent their maximal, the computed yield is the lower bound of the actual yield.*

Proof: Suppose A and B are operands of the "max" operation, and C dominates A and B. Then $C \geq A$ and $C \geq B$, so $C \geq \max(A, B)$. So if we use C as the maximal of A and B, the computed maximal delay is no less than the actual maximal delay, so the yield is not higher than the actual yield. $\square$

Therefore, LBDomSSTA always gets the lower bound of the yield.

The comparison relations can be transformed into

$$\begin{aligned} Pr(C \leq A) &= Pr(B \leq A), && (7)\\ Pr(C \leq B) &= Pr(A \leq B). && (8) \end{aligned}$$

For the comparison relation, we have

Theorem 3 *Suppose A and B are two random variables. Let*

$$C = \beta A + (1 - \beta)B,$$

where $\beta \in [0, 1]$, then C always satisfies the comparison conditions:

$$\begin{aligned} Pr(C \leq A) &= Pr(B \leq A),\\ Pr(C \leq B) &= Pr(A \leq B). \end{aligned}$$

Now we prove the following lemma.

Lemma 1 *Suppose A and B are two random variables. Let*

$$C = \beta A + (1 - \beta)B,$$

where $\beta \in [0, 1]$, then

$$\max(A, B) \geq C.$$

Proof:

$$\begin{aligned} \max(A, B) - C &= \max(A - C, B - C)\\ &= \max(A - (\beta A + (1 - \beta)B),\\ &\qquad B - (\beta A + (1 - \beta)B))\\ &= \max((1 - \beta)(A - B), \beta(B - A)) \end{aligned}$$

Thus, if $A \geq B$, $(1 - \beta)(A - B) \geq 0$, so $\max(A, B) \geq C$; if $A \leq B$, $\beta(B - A) \geq 0$, so $\max(A, B) \geq C$. $\square$

According to Lemma 1, we know that the "max" of two random variables as computed in Theorem 3 is not greater than their actual maximal. So

Lemma 2 *The maximal delay from the primary inputs to the primary outputs computed in UBCompSSTA is not greater than the actual maximal delay.*

This lemma can be easily proved based on the monotonic property of the "max" operation.

For two random variables A and B, if $P_r(A \leq B) = 1$, then $P_r(A \leq D) \geq P_r(B \leq D)$, where D is a constant. Thus, we have the following theorem.

Theorem 4 *The yield computed by UBCompSSTA gives the upper bound of the actual yield.*

4.2 Lower bound

Most of the existing SSTA approaches assume that the random variables have the Gaussian distributions. In this subsection, we consider the LBDomSSTA under this assumption. Suppose A and B are two Gaussian variables. Let

$$A = a_0 + \sum_{i=1}^{n} a_i x_i, \quad B = b_0 + \sum_{i=1}^{n} b_i x_i,$$

and

$$C = \max(A, B) \approx c_0 + \sum_{i=1}^{n} c_i x_i.$$

Unfortunately, we have the following theorem.

Theorem 5 *Let A and C be two Gaussian variables represented in the first order canonical form. Then*

$$Pr(C \geq A) = 1$$

cannot be satisfied unless $C = A + d$, where d is a non-negative constant number.

Proof: Suppose

$$A = a_0 + \sum_{i=1}^{n} a_i X_i, C = c_0 + \sum_{i=1}^{n} c_i X_i.$$

If $C = A + d$,

$$Pr(C \geq A) = Pr(A + d \geq A) = Pr(d \geq 0) = 1,$$

If $C \neq A + d$, obviously $\sum_{i=1}^{n}(a_i - c_i)^2 \neq 0$. So

$$\begin{aligned} Pr(C \geq A) &= \Phi(-\mu_{A-C}/\sigma_{A-C}) \\ &= \Phi(-\frac{a_0 - c_0}{\sqrt{\sum_{i=1}^{n}(a_i - c_i)^2}}) \end{aligned}$$

But $\Phi(x)$ is not equal to 1, so

$$Pr(C \geq A) < 1.$$

We can similarly prove the theorem holds on the reverse direction. □

So it is impossible to find a Gaussian random variable to dominate all the operands simultaneously for most cases. But if the dominance relation is relaxed to

$$Pr(C \geq A) \geq \eta, \quad Pr(C \geq B) \geq \eta, \tag{9}$$

where $0 < \eta < 1$, it is possible to find a C satisfying this condition. If η is higher, the confidence that the computed yield is the lower bound increases.

Clark [8] stated that the covariance between $C = max(A, B)$ and any random variable Y can be expressed in terms of covariances between A and Y and between B and Y, that is,

$$Cov(C, Y) = Cov(A, Y)T_A + Cov(B, Y)(1 - T_A).$$

As suggested in [4], in order to preserve the covariance, let $Y = x_i$, and we get

$$c_i = a_i T_A + b_i(1 - T_A) \quad i = 1, 2, \ldots n. \tag{10}$$

We adjust the mean value (c_0) such that the dominance relation is satisfied.

$$Pr(C \geq A) = \Phi(\frac{c_0 - a_0}{(1 - T_A)\sqrt{\sum_i (a_i - b_i)^2}}) \geq \eta \tag{11}$$

$$Pr(C \geq B) = \Phi(\frac{c_0 - b_0}{T_A\sqrt{\sum_i (a_i - b_i)^2}}) \geq \eta \tag{12}$$

Our objective is to compute the minimal c_0 such that these two inequalities are satisfied.

Let ζ be a constant satisfying $\Phi(\zeta) = \eta$. Then the two inequalities can be transformed to

$$\frac{c_0 - a_0}{(1 - T_A)\sqrt{\sum_i (a_i - b_i)^2}} \geq \zeta, \tag{13}$$

$$\frac{c_0 - b_0}{T_A\sqrt{\sum_i (a_i - b_i)^2}} \geq \zeta. \tag{14}$$

Solving this inequality set, we can get the minimal c_0. The dominance relations are then satisfied.

4.3 Upper bound

4.3.1 Gaussian

In this subsection, we also assume that all the random variables have the Gaussian distributions.

According to Theorem 3 and the discussion in previous subsection, if we select $\beta = T_A$, the comparison relations are kept, and the covariance is also preserved.

Now we check if the upper bound is tight or not. We compare the mean and the variance computed by our approach and by the moment matching based approach respectively.

Let C represent the maximal of the two Gaussian random variables A and B computed by our approach, and D represent $\max(A, B)$ computed by [8].

$$\begin{aligned} \mu(D) - \mu(C) &= (T_A\mu_A + (1 - T_A)\mu_B + \theta\phi(\alpha)) \\ &\quad -(T_A\mu(A) + (1 - T_A)\mu(B)) \\ &= \theta\phi(\alpha) \geq 0. \end{aligned}$$

Assuming that all the random variables have at most 10% deviation (3σ) from their nominal values, we get

$$\begin{aligned} \theta^2 &= \sigma^2(A) + \sigma^2(B) - 2\rho\sigma(A)\sigma(B) \\ &\leq \sigma^2(A) + \sigma^2(B) + 2\sigma(A)\sigma(B) \\ &\leq (0.10\mu(A)/3)^2 + (0.10\mu(B)/3)^2 \\ &\quad +2(0.10\mu(A)/3)(0.10\mu(B)/3) \\ &\leq (0.10/3)^2(\mu(A) + \mu(B))^2. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\theta^2}{\mu^2(D)} &\leq \frac{\theta^2}{(T_A\mu_A + (1 - T_A)\mu_B)^2} \\ &\leq (0.10/3)^2 \frac{(\mu(A) + \mu(B))^2}{(T_A\mu(A) + (1 - T_A)\mu(B))^2} \end{aligned}$$

Since the random variables in our problem represent delay or arrival time, if we set the arrival time at the PIs to 0, their mean values should be non-negative. Without loss of generality, we assume $\mu(A) \geq \mu(B) > 0$. So $T_A \geq 0.5$. Let $\mu(A) = \gamma\mu(B)$, so $\gamma \geq 1$. Thus,

$$\begin{aligned} \frac{\theta^2}{\mu^2(D)} &\leq (0.10/3)^2 \frac{(\gamma\mu(B) + \mu(B))^2}{(\gamma T_A\mu(B) + (1 - T_A)\mu(B))^2} \\ &= (0.10/3)^2 \frac{(1 + \gamma)^2}{(\gamma T_A + 1 - T_A)^2} \\ &\leq (0.10/3)^2 \frac{(1 + \gamma)^2}{(0.5 + 0.5\gamma)^2} \\ &= 4(0.10/3)^2 \\ &= 0.0044 \end{aligned} \tag{15}$$

In addition, $\phi(\alpha) \leq 1/\sqrt{2\pi}$, so the relative error of the mean is at most

$$\frac{\sqrt{0.0044}}{\sqrt{2\pi}} = 2.66\%.$$

From this derivation, we can see that if the variance is smaller, or the correlation is positive and larger, the result is more accurate. In practice, this relative error is even smaller because of the high positive correlation between delays and the small variance.

Now we consider the error on the variance, though the mean plays a major role on the yield.

$$\begin{aligned}\sigma^2(D)-\sigma^2(C) &= T_A(1-T_A)[\sigma^2(A)+\sigma^2(B)\\ &\quad -2\rho\sigma(A)\sigma(B)+(a_0-b_0)^2]-\theta^2\phi^2(\alpha)\\ &\quad +\theta\phi(\alpha)[(a_0-b_0)(1-2T_A)] \qquad (16)\end{aligned}$$

If $\theta = 0$,

$$\begin{aligned}0 &= \sqrt{\sigma^2(A)+\sigma^2(B)-2\rho\sigma(A)\sigma(B)}\\ &= \sqrt{\sum_i (a_i-b_i)^2}\end{aligned}$$

Thus

$$a_i = b_i \quad \forall i = 1\ldots n,$$

and

$$T_A = 0 \text{ or } T_A = 1.$$

So

$$\sigma^2(D)-\sigma^2(C) = 0.$$

If $\theta > 0$ (note $\theta \geq 0$),

$$\begin{aligned}\alpha^2 &= \frac{(\mu(A)-\mu(B))^2}{\sigma^2(A)+\sigma^2(B)-2\rho\sigma_A\sigma_B}\\ &\geq \frac{(\gamma-1)^2}{(0.10/3)^2(\gamma+1)^2}\\ &= 900\frac{(\gamma-1)^2}{(\gamma+1)^2}. \qquad (17)\end{aligned}$$

According to [12], when $\alpha \geq 3$, the right hand side of Eq.(16) approaches 0. So when $\gamma \geq 1.22$, the error approaches 0.

When $\gamma < 1.22$, according to [12],

$$\sigma^2(D)-\sigma^2(C) \leq 0.091\theta^2.$$

While according to Eq.(15), when $\rho \geq 0$, and $\gamma < 1.22$,

$$\theta^2 \leq 0.0022\mu^2(D).$$

Thus,

$$\begin{aligned}\sigma^2(D)-\sigma^2(C) &\leq 0.091 * 0.0022\mu^2(D),\\ &= 0.0002\mu^2(D). \qquad (18)\end{aligned}$$

Therefore,

$$\frac{\sigma^2(D)-\sigma^2(C)}{\mu^2(D)} \leq 0.02\%.$$

So the error on the variance is at most 1.41% of the mean value. If the correlation coefficient (ρ) is more positive, this error gets even smaller. For example, when $\rho = 1$, the error is at most 0.85%.

In summary, the results computed in UBCompSSTA are very close to the results from the moment-matching approach. UBCompSSTA gives a tight upper bound of the yield.

4.3.2 Non-Gaussian

In this part, we consider the cases where the random variables do not have Gaussian distributions. The delay of a gate or an interconnect may be affected by not only one kind of process variation, so there may exist non-linear relations between the delays and the process variations. For example, the delay of a wire is affected by the process variations on the wire length, the wire width and the wire thickness. Zhang *et al.* [7] has proposed a quadratic delay model for a wire. A random variable D is represented in the following quadratic model:

$$D = m + \alpha\delta + \delta^T\Upsilon\delta + \gamma^T r,$$

where $r = (R_1, R_2, \ldots R_p)^T$ represents the local variances, $\delta = (X_1, X_2, \ldots X_q)^T$ represents the global variances, α and γ are sensitivity vectors, and Υ is a sensitivity matrix. All these R_i's and X_j's are independent and have the standard Gaussian distribution. The random variables represented in this form do not have the Gaussian distribution. We will show that our approach also gets tight upper bounds of the yield for this situation.

Suppose random variables A and B are represented in the quadratic model:

$$\begin{aligned}A &= m_A + \alpha_A\delta + \delta^T\Upsilon_A\delta + \gamma_A^T r, \qquad (19)\\ B &= m_B + \alpha_B\delta + \delta^T\Upsilon_B\delta + \gamma_B^T r, \qquad (20)\end{aligned}$$

In the computation of the maximal of two random variables A and B represented in the quadratic model, Zhang *et al.* [7] approximated the random variables as Gaussian variables by moment matching and computed the skewness of the output. If the skewness is greater than a threshold, the "max" operation is delayed, otherwise, the approach got

$$\max(A,B) = m_C + \alpha_C\delta + \delta^T\Upsilon_C\delta + \gamma_C^T r,$$

where

$$\begin{aligned}m_C &= T_A m_A + (1-T_A)m_B + \theta\phi(\alpha) \qquad (21)\\ \alpha_C &= T_A m_A + (1-T_A)m_B \qquad (22)\\ \Upsilon_C &= T_A\Upsilon_A + (1-T_A)\Upsilon_A \qquad (23)\\ \gamma_C &= T_A\gamma_A + (1-T_A)\gamma_B \qquad (24)\end{aligned}$$

The output of our approach differs from the output of [7] only in the m part. Our approach gets

$$m = T_A m_A + (1-T_A)m_B.$$

Since m affects only the mean value, we only need to compute the error of the mean value of our approach. Let C and D represent the outputs of [7] and our approach respectively. The difference between $\mu(C)$ and $\mu(D)$ is $\theta\phi(\alpha)$. We can similarly prove that the relative error of the mean is at most 2.66%.

Thus, our approach can also be applied in the situations where the variables do not have Gaussian distributions, and get an upper bound of the yield that is close to the result from [7].

5 Experimental results

We have implemented LBDomSSTA and UBCompSSTA in C++. Experiments were performed on the large cases in ISCAS85 benchmark. We use the cases where all the random variables have the Gaussian distributions as examples to show the effectiveness of our approaches. We also implemented a Monte Carlo simulation tool to compute the maximal delay from s to t. We made 10,000 trials for each test case. All the random variables have at most 10% deviation from their nominal values. All the experiments were run on a Linux PC with a 2.4 GHz Xeon CPU and 2.0 GB memory.

The comparison results of UBCompSSTA, LBDomSSTA and the Monte Carlo simulation are shown in Table 1. We perform Monte Carlo simulations to compute the 90% percentile point of the maximal delay from s to t, and select this point

Table 1: Comparison results of UBCompSSTA, LBDomSSTA and Monte Carlo simulation

name	UBCompSSTA				LBDomSSTA				Monte Carlo		
	time (s)	μ	σ	yield (%)	time (s)	μ	σ	yield (%)	μ	σ	yield(%)
c1355	0.01	1580	40	91.15	0.01	1585	40	89.07	1583	40	90.00
c1908	0.01	4000	100	91.92	0.01	4019	101	88.49	4011	100	90.00
c2670	0.01	2918	61	91.15	0.01	2926	61	89.25	2922	61	90.00
c3540	0.03	4700	120	92.22	0.02	4727	120	88.30	4715	119	90.00
c5315	0.03	4900	123	91.47	0.02	4919	123	88.69	4910	125	90.00
c6288	0.03	12400	312	92.36	0.03	12477	314	87.90	12443	313	90.00
c7552	0.05	4300	107	91.47	0.04	4320	107	88.30	4311	107	90.00

as the timing constraint. We have $\eta = 90\%$ in LBDomSSTA. The columns 2, 3, 4, and 5 show the running time, the mean of the maximal delay, the standard deviation of the maximal delay, and the yield computed by UBCompSSTA, respectively. The columns 6, 7, 8, and 9 show the running time, the mean of the maximal delay, the standard deviation of the maximal delay, and the yield computed by LBDomSSTA, respectively. The 10th and 11th columns show the mean and the standard deviation of the maximal delay from Monte Carlo simulation, respectively. The results indicate that UBCompSSTA and LBDomSSTA always get *tight* bounds of the yield. The errors on the yield are 1.68% and 1.43% on average, respectively. The relative errors on the mean and the variance are also quite small.

Fig. 2 shows the cumulative distribution functions from LBDomSSTA, UBCompSSTA and the Monte Carlo simulation for the case "c6288". The CDF from UBCompSSTA stays on the left side, while the CDF from LBDomSSTA stays on the right, and the actual CDF stays between them. It demonstrates that our approaches achieve the bounds in the whole range.

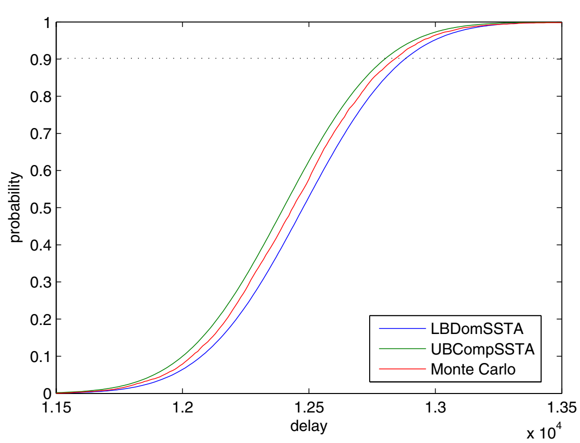

Figure 2: The CDFs from different approaches for "c6288".

6 Conclusions

The state-of-the-art statistical static timing analysis approaches cannot tell whether the computed yield is lower or higher than the actual yield. In this paper, we proposed two block-based statistical static timing analysis approaches by satisfying two necessary conditions for "max" operation. We proved that our approaches always achieve tight bounds of the yield. Furthermore, for the upper bound computation, our approach achieves the bound irrespective of the distributions of random variables.

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