Advanced tools for simulation and design of oscillators/PLLs

Xiaolue Lai and Jaijeet Roychowdhury
Department of Electrical and Computer Engineering, University of Minnesota
Email: {laixl, jr}@umn.edu

***Abstract*— We present a robust, automated oscillator macromodeling technique for extracting comprehensive phase and amplitude macromodels from oscillators' SPICE circuit descriptions. The macromodels are able to correctly predict oscillator response in the presence of interference at far lower computational cost than that of full SPICE-level simulation, while retaining the simulation accuracy. We find many applications for the proposed macromodeling technique, which include injection locking prediction, fast simulation of coupled oscillating systems, and fast PLL transient simulation and jitter analysis. We demonstrate the applications on some oscillator-based systems, and compare results against full SPICE-level simulation. Experimental results show that the macromodels capture the behavior of the oscillator systems accurately, and provide speedups of over three orders of magnitude.**

I. Introduction

Oscillators are critical components in electronic and optical systems. They are often used, for example, for frequency-translation of information signals in communication systems and as clocks in digital systems. Phase-locked loops (PLLs) [1], which feature voltage-controlled oscillators as key components, are widely used in both digital and analog circuits for clock generation and recovery, frequency synthesis, signal conditioning and synchronization, *etc.*. The design of oscillators and oscillator-based systems (*e.g.*, PLLs, coupled oscillators) is an important part of overall system design; however, simulating oscillator-based systems presents unique challenges because of their fundamental property of neutral phase stability, often accompanied (especially in high-Q oscillators) with very slow amplitude responses that border on instability.

The traditional approach for simulating circuit systems is to use simulators such as SPICE [2]. However, SPICE-level transient simulation is far from ideally suited for simulating oscillator-based systems. Direct time-domain simulation of oscillator-based systems at SPICE-level is typically impractical because of its great inefficiency. The transient response can last hundreds of thousands of cycles, with each cycle requiring many small timesteps for accurate simulation of the embedded oscillator. In addition, transient simulation algorithms tend to accumulate phase error unboundedly for oscillators due to oscillators' neutral phase stability, resulting in severe accuracy problems. To alleviate the accuracy problem, smaller timesteps have to be taken in each oscillation cycle, exacerbating the inefficiency already associated with large oscillator-based systems with widely separated time scales of activity.

To improve the simulation efficiency of oscillator-based systems at the SPICE level, specialized techniques based on using macromodels (*e.g.*, [3]–[7]) have been proposed for the simulation of oscillator-based systems. The models are all based on linear integration of a perturbing input to generate output phase deviation. However it has been shown that linear models are unable to capture some nonlinear phenomena, such as injection locking/pulling in oscillators [8]. In addition, those models do not capture amplitude variations in oscillators, which can be important for second-order effects.

We have developed methods for extracting comprehensive phase and amplitude macromodels of oscillators from their SPICE-level circuit descriptions. The macromodels are based on combining a scalar, nonlinear time-shifted phase equation [9]–[12] with a small linear periodic time-varying (LPTV) system to capture slowly-dying amplitude variations. The nonlinear time-shifted phase macromodel we extract can be shown to be exact for capturing oscillator phase responses to small perturbations. The amplitude macromodel is built via *Floquet* decomposition [13] (made efficient using *Krylov* subspace methods) on top of the nonlinear phase macromodel. The comprehensive macromodel is able to replicate the behavior of oscillators at *far lower computational cost* than full SPICE-level simulation of the original circuit. Applying the oscillator macromodel, we have developed methods for predicting injection locking in oscillators, estimating phase errors in PLLs due to loop non-idealities, and simulating the transient behaviors of PLLs and coupled oscillators. These phenomena, all of great importance in RF, mixed-signal and analog design, are particularly difficult to simulate effectively using prior techniques.

Injection locking is a nonlinear dynamical phenomenon that is often exploited in electronic and optical oscillator design (*e.g.*, injection locked lasers, ultra-high-frequency frequency dividers). We show that the nonlinear time-shifted phase macromodels are able to capture injection locking, while providing significant computational speedups over full SPICE-level circuit simulation. We also show that our approach is equally effective for capturing the dynamics of transition to locking, including unlocked tones and phase jump phenomena. Furthermore, we develop a general analytical equation for predicting injection locking in oscillators. Compared to traditional approaches, such as *Adler's equation* [14], which is limited to ideal LC oscillators, our approach is more accurate and considerably more generally applicable.

Timing jitter caused by power supply fluctuations and substrate interference is an important concern in phase-locked loop (PLL) design; indeed, PLL malfunction due to loop non-idealities is one of the most important factors in re-fabs of SoCs. As a result, fast simulation of PLLs to predict PLL behaviors in the presence of non-idealities, such as power supply interference, non-ideal filters and loop stability issues, is an immediate, pressing need in the semiconductor design industry. We present a nonlinear-oscillator-macromodel-based PLL simulation technique that is considerably more accurate than prior linear PLL simulation techniques. Our method is able to accurately capture transient behavior and faithfully estimate timing jitter in noisy PLLs. We evaluate the proposed technique on ring and LC voltage-controlled oscillator (VCO) based PLLs, and show that, unlike prior linear macromodel based approaches, the proposed nonlinear technique captures the dynamics of complex phenomena such as phase pulling, cycle slipping and power supply noise induced PLL jitter, replicating qualitative features from full SPICE simulations accurately while providing great speedups.

Coupled self-oscillating systems appear in diverse natural and physical systems. Coupled oscillating system models arise in contexts ranging from the nanoscale (*e.g.*, Josephson junction arrays, spintronics) to the cosmic scale (*e.g.*, gravitational interactions between systems of galaxies). Fast and accurate simulation of the dynamics of coupled oscillating system is therefore of significant theoretical and practical interest not only for circuit applications, but in a multitude of other disciplines (*e.g.*, biochemical and nanotechnological systems)

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as well. Using the nonlinear oscillator macromodels, we are able to predict the behavior of coupled oscillating system far more effectively than prior techniques, including SPICE-level simulation. Using these macromodels in place of the original SPICE-level circuits enables large speedups, while capturing phase and amplitude dynamics, during simulation of collective behavior of coupled oscillating systems. We demonstrate the technique on the simulation of biological pattern formation [15], [16] and nano-scale cellular nonlinear network (CNN) [17], [18], obtaining speedups of more than 3 orders of magnitude over full SPICE-level simulations.

The remainder of the paper is organized as follows: in Section II, we present the oscillator macromodeling technique, including the nonlinear time-shift phase macromodel and the LPTV amplitude macromodel. In Section III, Section IV, and Section V we demonstrate the applications of the proposed technique, including injection locking prediction, PLL simulation and jitter analysis, and the simulation of coupled oscillating systems.

II. Nonlinear Oscillator Macromodeling Technique

In this section, we briefly introduce the macromodeling technique, including the nonlinear time-shifted phase macromodel and the linear time-varying amplitude macromodel.

A. Nonlinear Oscillator Phase Macromodel

A general oscillator under perturbation can be described by an ODE equation

$$\dot{x} + f(x) = b(t), \tag{1}$$

where $b(t)$ is a vector of perturbation signals applied to the free running oscillator (we use the ODE form here just for simplicity and clearness, the theory can apply to DAE systems as well). The corresponding linear periodic time-varying (LPTV) system can be obtained by linearizing this oscillator about its steady state orbit:

$$\begin{aligned} \dot{w}(t) \approx & -\frac{\partial f(x)}{\partial x}|_{x_s(t)} w(t) + b(t) \\ =& A(t)w(t) + b(t), \end{aligned} \tag{2}$$

where $x_s(t)$ is the steady state orbit of the oscillator. Via Floquet decomposition of the homogeneous part of this LPTV system, a series of Floquet exponents and corresponding eigenvectors can be obtained. Since phase deviation of the oscillator never vanishes, it should correspond to the Floquet exponent with the value of 0. [9] showed that the phase sensitivity waveform of the oscillator can be extracted from the eigenvector associated with the Floquet exponent 0, and that the phase deviation $\alpha(t)$ is governed by a one-dimensional nonlinear time-shifted differential equation [9]

$$\dot{\alpha}(t) = v_1^T(t + \alpha(t)) \cdot b(t), \tag{3}$$

where $v_1(t)$ is the perturbation projection vector (PPV). $v_1(t)$ is a vector with the size of system size n; Each element in $v_1(t)$ represents the oscillator's phase sensitivity to the perturbation applied to the corresponding node. The PPV, or the phase sensitivity vector, has periodic waveforms that have the same frequency as that of the oscillator. $b(t)$ is a vector of size n, representing the perturbations applied to each oscillator equation. The dot product translate these two vectors into a scalar, as a result, we obtain a simple one-dimensional differential phase equation. Various methods [9]–[12], both in the time domain and the frequency domain, have been presented for calculating the PPV from SPICE-level circuit descriptions of oscillators. In (3), the phase deviation $\alpha(t)$ has units of time. To obtain the phase deviation in radians, we need to multiply $\alpha(t)$ by the oscillator's free-running frequency ω_0.

B. Amplitude Macromodel

Once the phase deviation $\alpha(t)$ is obtained by solving (3), a macromodel for dominant amplitude components can be built as well, by linearizing the oscillator over its perturbed time-shifted orbits $x_s(t+\alpha(t))$. In [19], a method is presented to construct amplitude macromodels of oscillators. The oscillator is first linearized on $x_s(t+\alpha(t))$:

$$\begin{aligned} \dot{y}(t) &\approx -\frac{\partial f}{\partial x}|_{x_s(t+\alpha(t))} y(t) + b(t) \\ &= A(x_s(t+\alpha(t)))y(t) + b(t), \end{aligned} \tag{4}$$

where $x_s(t)$ is the oscillator's steady-state orbit, $\alpha(t)$ is the phase deviation due to perturbation $b(t)$, and $y(t)$ is a small amplitude deviation from the phase-shifted orbit, due to the perturbation $b(t)$. By introducing a new variable $\hat{t} = t + \alpha(t)$ and defining $\hat{y}(\hat{t}) = y(t)$ and $\hat{b}(\hat{t}) = b(t)$, we obtain a linear periodic time-varying (LPTV) system

$$\dot{\hat{y}}(\hat{t}) = A(x_s(\hat{t}))\hat{y}(\hat{t}) + \hat{b}(\hat{t}). \tag{5}$$

Applying Floquet decomposition, the LPTV system can be decomposed into a diagonalized LTI system with periodic input/output vectors:

$$\hat{y}(\hat{t}) = \sum_{i=1}^{n} u_i(\hat{t}) \int_0^{\hat{t}} \exp(\mu_i(\hat{t} - \tau)) v_i^T(\tau) \hat{b}(\tau) d\tau, \tag{6}$$

where μ_i are Floquet exponents, and $v_i(t)$ and $u_i(t)$ are periodic input/output vectors. By dropping the Floquet exponent corresponding to phase and other less important Floquet exponents, we obtain a reduced amplitude macromodel. For large circuits, the Floquet decomposition could be very inefficient. Hence, we developed a technique [20] to extract the amplitude macromodel using the Krylov-subspace model order reduction (MOR) method.

When both the phase shift $\alpha(t)$ and amplitude variations $y(t)$ are available, the oscillator's orbit under perturbation can be obtained by the equation

$$x_p(t) = x_s(t + \alpha(t)) + y(t), \tag{7}$$

where $x_s(t)$ is the steady state orbit of the oscillator, and $x_p(t)$ is the orbit of the oscillator under perturbation.

Using the macromodeling technique, the complex oscillator equations in oscillator-based systems can be replaced with the nonlinear time-shifted phase equation (3), and the reduced amplitude macromodel. The resulting reduced system can be simulated using any transient simulator, with great speedups compared to the full system. Moreover, since the system is simulated in the phase domain directly, the simulation efficiency can be further improved by using larger timesteps and simpler integration methods, without appreciable loss of accuracy.

III. Injection Locking Prediction

Injection locking is an interesting and useful phenomenon universally observed in all kinds of physical oscillators. The term refers to the fact that, under certain conditions, when an oscillator is perturbed by an external weak signal that is close (but not identical) to the oscillator's natural frequency, the oscillator's frequency changes to become identical to that of the perturbing signal – *i.e.*, it "locks" to the external signal. In electronic system, injection locking is a well known and practical technique for phase locked loops (PLLs) to increase pull-in range and reduce output phase jitter [21].

Despite its widespread use in circuits, the simulation of injection locking presents challenges. Direct simulation of oscillators at the SPICE level is usually inefficient and inaccurate. Our approach is to replace the full oscillator circuit with the nonlinear time-shifted phase equation. We derived a phase criterion for predicting if an oscillator is in lock or not. With this criterion, we can only simulate the phase deviation of the oscillator due to injected signal using the phase macromodel, and predict injection locking and unlocked pulling behavior of oscillators without doing full circuit simulation.

If an oscillator locks to an external signal, the phase shift $\phi(t)$ should satisfy

$$\omega_0 t + \phi(t) = \omega_1 t + \theta,$$

or

$$\phi(t) = (\omega_1 - \omega_0)t + \theta, \tag{8}$$

where ω_0 is the frequency of the free-running oscillator, ω_1 is the frequency of the injected signal, and θ is a constant which represents the phase difference between the locked oscillator and the injected signal. (8) encapsulates the basic intuition that if the oscillator locks to an injected signal, the phase shift due to the injected signal should grow with time linearly with a slope of $\omega_1 - \omega_0$.

This phase shift can be calculated by solving the phase equation (3). Since the phase shift $\alpha(t)$ in (3) has units of time, we can multiply the phase shift $\alpha(t)$ by free-running frequency ω_0 and obtain the phase shift in radians

$$\phi(t) = \omega_0 \alpha(t). \tag{9}$$

Substituting (9) in (8), we have

$$\omega_0 \alpha(t) = (\omega_1 - \omega_0)t + \theta \tag{10}$$

or

$$\alpha(t) = \frac{\Delta\omega_0}{\omega_0} t + \frac{\theta}{\omega_0}, \tag{11}$$

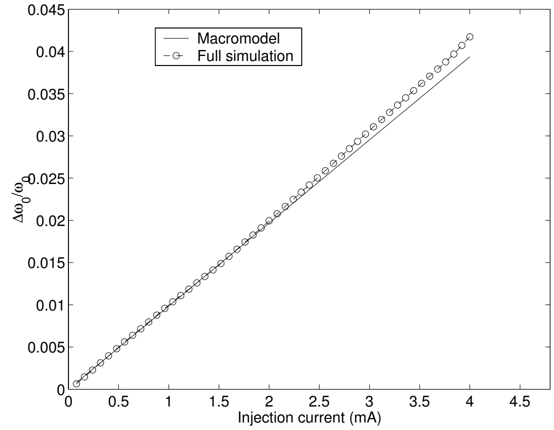
where $\Delta\omega_0 = \omega_1 - \omega_0$. So the phase shift $\alpha(t)$ should change with time linearly with a slope of $\frac{\Delta\omega_0}{\omega_0}$ when the oscillator is in lock.

This relationship provides a direct method for predicting locking behavior in oscillators, by simulating (3) and checking the slope of the phase deviation $\alpha(t)$. For example, if solving (3) for an oscillator injected with a perturbation signal of frequency 10% higher than its free-running frequency results in a phase shift $\alpha(t)$ that increases linearly with a slope of 0.1, we can conclude that the oscillator is locked by the injected signal. Because (3) is a simple one-dimensional differential equation that can be solved efficiently by numerical methods, this approach offers large speedups over full circuit simulations.

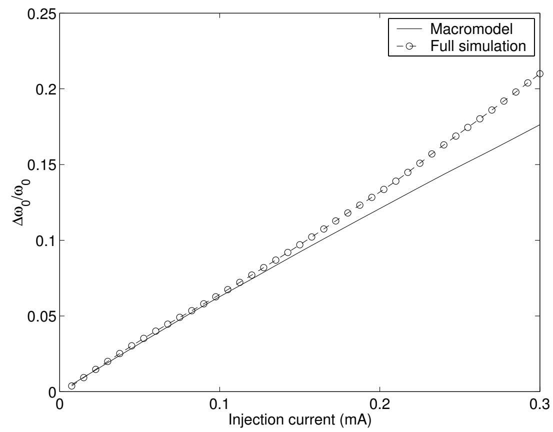
We apply this method to analyze the locked and unlocked behavior, and phase jump behavior of LC and ring oscillators. For locked cases, we use the nonlinear phase macromodel (3) to plot the maximum lock range of the oscillators, and compare the results with SPICE-level full circuit simulation in Figure 1; For unlocked case, we plot the spectrum of the output waveforms and make comparison in Figure 2. For phase jump case, we first apply an injected signal to make the oscillator in lock. We then apply another noise signal to initialize a phase jump. We use full simulation to plot the time domain waveform, use macromodel to plot the phase domain waveform, and find consistency in these two plots, as shown in Figure 3. In all cases, the macromodel has very good match to full circuit simulation, with 90 times speedup.

IV. PLL Simulation Simulation And Jitter Analysis

PLLs have many applications in both mixed-signal and digital systems, while the design of the PLL still presents significant challenges. Modern PLL design involves trade-offs between various conflicting design metrics such as phase noise/jitter, lock and capture range, acquisition time, *etc.*, for different kinds of applications. Simulation tools are extensively used in design processes for finding



(a) LC oscillator



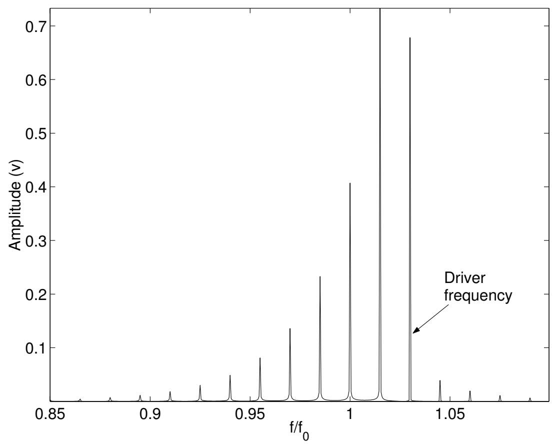
(b) Ring oscillator

Fig. 1. Maximum locking range plots of oscillators

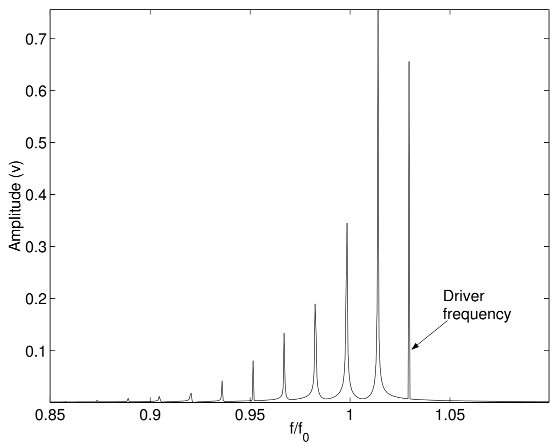
a good balance between design metrics to meet the performance requirements. Unfortunately, existing full circuit simulation tools (such as SPICE), are very inefficient for the simulation of PLLs at the transistor level [22]; and this problem worsens when dealing with frequency synthesizers with large divide ratios. It is not uncommon for many months to be required to finalize the design of today's advanced PLLs. As a result, a fast and accurate method for PLL simulation and jitter analysis is of great practical importance for the semiconductor design industry.

To improve the efficiency of simulating PLLs at the SPICE level, a popular approach towards approximate PLL simulation involves the use of phase domain macromodels (*e.g.*, [23]). In this approach, each building block of a PLL is represented approximately using small, simple macromodels, and the system of macromodels simulated. Furthermore, in contrast to SPICE-level circuits which use voltage/current domain device models, the PLL block macromodels used are typically in the phase domain. For example, in traditional approaches, the VCO is represented as a simple linear integrator that converts input voltages to output phases; similar simple macromodels of other blocks are also employed. The use of such macromodels can lead to dramatic speedups (of many orders of magnitude over the SPICE level); however, such speedups are obtained at the expense of accuracy.

The accuracy problem of the linear VCO model arises due to the non-idealities and nonlinearity in PLLs. Linear model considers everything in PLLs is ideal. However, this assumption is generally not true for real PLLs: power, ground and substrate noise, high-frequency

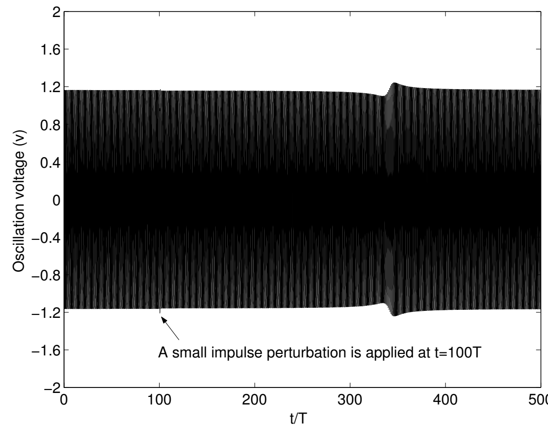


(a) Full simulation

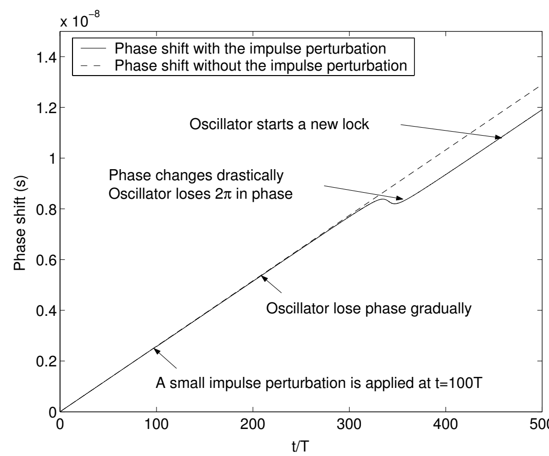


(b) Nonlinear macromodel

Fig. 2. Output spectra of the unlocked driven LC oscillator.



(a) Full simulation



(b) Nonlinear phase macromodel

Fig. 3. The simulation of phase jump in the injection-locked oscillator.

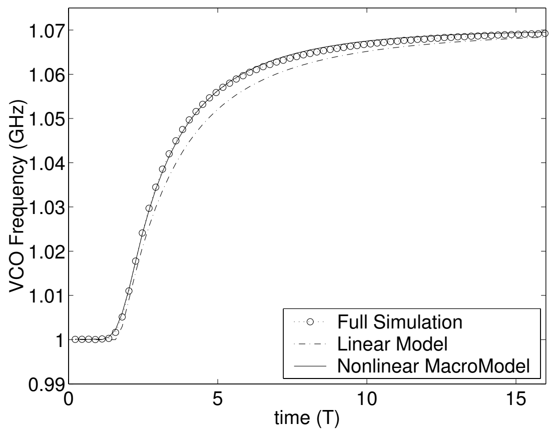
AC noise leaked from low pass filter and reference signal noise are facts that cannot be ignored. These non-idealities affect the response of the PLL via some nonlinear phase effects, such as phase pulling, which cannot be captured by any linear phase models [8].

Since the nonlinear time-shifted phase macromodel is proven to be able to capture the nonlinear phase effects in oscillators, we can apply this macromodel to PLLs for a better macromodeling accuracy. The key advantage of this technique is that it considers non-idealities and nonlinearities in the PLL loop. As a result, the transient behavior of PLLs can be simulated far more accurately than with linear macromodels: phase noise in the PLL's reference signal, high-frequency components from the phase/frequency detector, power supply, ground and substrate interference are all accounted for correctly.

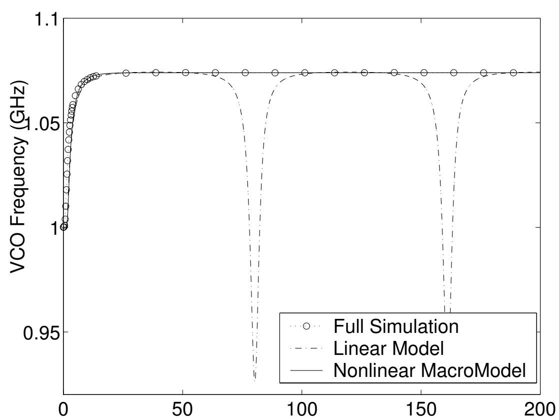
To demonstrate the capabilities of the proposed technique, we apply it to estimating and simulating step response and cycle slipping of a high-bandwidth PLL. We provide comparison against simulations with linear macromodels and full simulation, demonstrating that the new technique provides accuracies essentially equivalent to full SPICE-level simulation, but with speedups of over two orders of magnitude. Furthermore, we demonstrate how the same simulations, but using linear macromodels, can completely fail to predict important phenomena such as cycle slips.

Figure 4 depicts the step input response of the PLL under different reference frequencies. We first make the PLL locks to frequency f_0, which is the central frequency of the VCO. We then switch the reference frequency to $1.07 f_0$, which introduce a step input to the PLL. We use full simulation, linear VCO model and the nonlinear time-shifted phase macromodel to simulate the step input response of the PLL, and plot the results in Figure 4(a). In this case, three methods have very close results. The nonlinear macromodel has better match to the full simulation, but the difference is negligible. In this simulation, we obtain about 150 times speedup by using the macromodel. In Figure 4(b), we increase the step size by switching the reference frequency from f_0 to $1.074 f_0$. In this case, we can see big difference between the linear model and the nonlinear macromodel: the linear model gives totally wrong result.

The PLL is a phase estimator which has multiple steady states with an interval of 2π. Once a PLL enters its locking mode, it tends to maintain a steady state where the phase error lies between its slip boundaries. Cycle slipping happens when the phase error accumulates to such an extent that it exceeds the slip boundary and the PLL jumps from one steady-state to another. In Figure 5, we simulate the cycle slipping phenomenon in a PLL, and compare results to full simulation and the linear model. In Figure 5(a), we first make the PLL in lock, and apply a relatively strong interference signal to the PLL to introduce a phase error. We then stop the interference signal and check if the PLL can fix this phase error. In this case, the PLL is unable to fix the phase error, its phase changes 2π: cycle slipping happens. Both the linear model and the nonlinear macromodel are able to predict this phenomenon, but the nonlinear macromodel has better accuracy. In Figure 5(b), we apply a weak interference to the

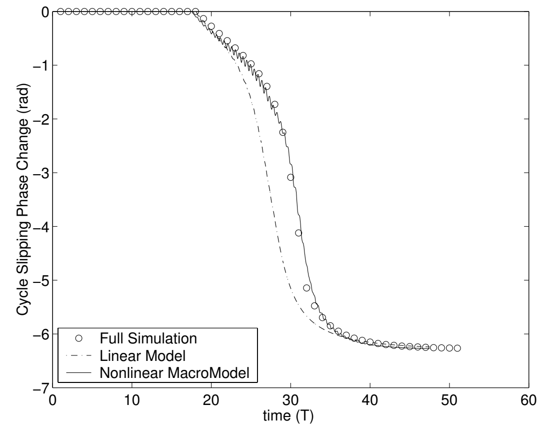


(a) $f_{ref} = 1.07f_0$

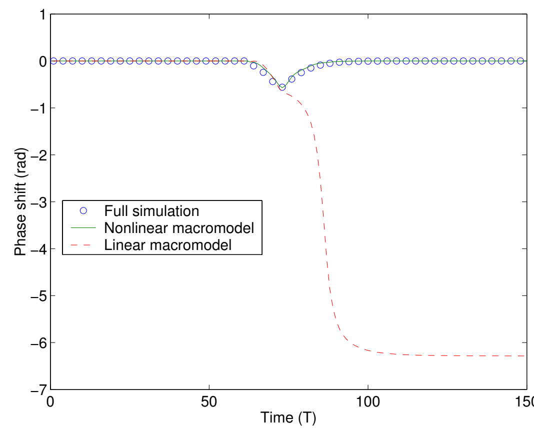


(b) $f_{ref} = 1.074f_0$

Fig. 4. The step response of the PLL under different reference frequencies.



(a) Noise amplitude is $5mA$



(b) Noise amplitude is $3mA$

Fig. 5. Cycle slipping in the PLL under different noise amplitudes.

PLL, which introduces a small phase error. This time, the PLL can correct this small phase error, from the results of the full simulation and nonlinear macromodel, while the Linear model gives totally wrong prediction.

In the last example, we simulate the phase error of a ring VCO based PLL under periodic power supply interference with different amplitudes, and plot results in Figure 6. The nonlinear macromodel provides very good results, while the linear macromodel fails in this case.

V. Simulation Of Coupled Oscillating System

Coupled oscillating systems model a variety of physical systems and phenomena. For example, in nanoelectronics, the tunneling phase logic (TPL) [24] device, which makes use of the bistability of single-electron tunneling oscillation to realize logic in phase are proposed for large scale circuits, due to its extremely high gate density and ultra low power dissipation. This concept has been applied [25] to implement cellular nonlinear networks (CNN) [17], [18], with ultra-high integration levels far beyond even DSM CMOS. Such CNN systems, consisting of large populations of interacting TPL oscillators, constitute a promising approach for implementing future large-scale high performance image processing systems.

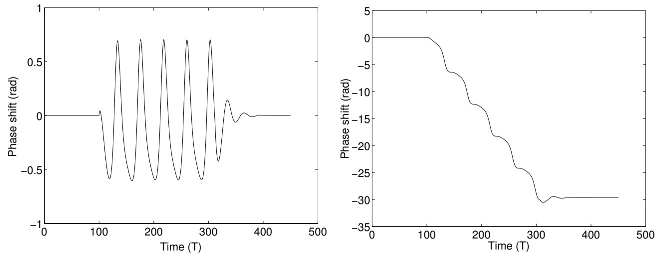
Patterns widely exist in many biological systems, such as animal furs and human fingerprints. The pattern formation process, which is important for the understanding of biological mechanisms in biological systems, can be modeled as a reaction-diffusion system (RDS) [26]. In such systems, coupled chemical oscillators interact with each other, forming patterns from an initially uniform state.

The simulation of these biochemical and nanotechnological systems presents huge challenges for direct simulation methods, as the system sizes are very large. With our nonlinear oscillator macromodeling technique, such systems can be simulated with very small computational cost. In our approach, the nonlinear phase domain macromodels of oscillators in such biochemical and nanotechnological systems are extracted, and used to replace the complex oscillator equations in the coupled oscillating systems. The interaction between oscillators can be modeled as perturbations applied to the oscillators. Using this technique, we can reduce the original huge nonlinear systems to smaller systems with only simple phase equations. Those phase equations have very good numerically stability, so that they can be simulated using relatively simple ODE solvers, such as the Runge-Kutta method, resulting in great speedups.

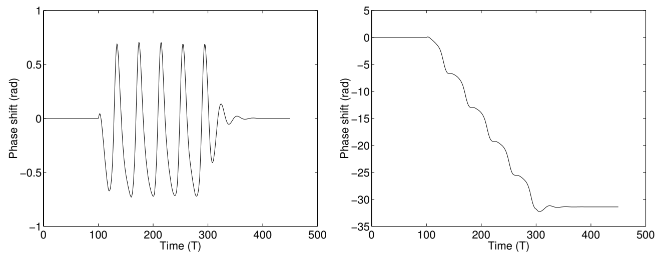
In this section, we present two examples of coupled oscillating system simulation: one is the biochemical pattern formation simulation, the other is the tunneling phase logic based cellular nonlinear network (TPL-CNN).

A. Simulation of Pattern formation in a Brusselator biochemical network

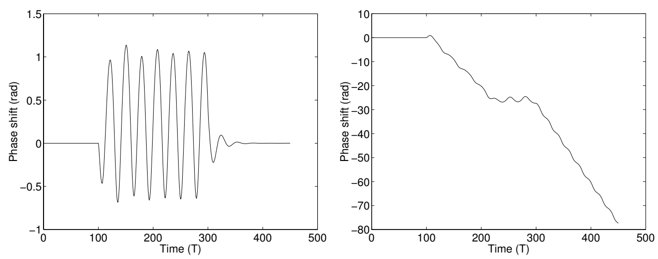
In Figure 7, we simulate a biochemical pattern formation system consists of Brusselator oscillators using the nonlinear macromodeling technique. In this simulation, we simulate a network of oscillators with size of 400×400, 160 thousands oscillators in total. Direct



(a) Full simulation (injection amplitude = 0.01mA) (b) Full simulation (injection amplitude = 0.02mA)



(c) Nonlinear Macromodel (injection amplitude = 0.01mA) (d) Nonlinear Macromodel (injection amplitude = 0.02mA)



(e) Linear model (injection amplitude = 0.01mA) (f) Linear model (injection amplitude = 0.02mA)

Fig. 6. Phase shift of the PLL under different injection amplitudes, using different simulation methods.

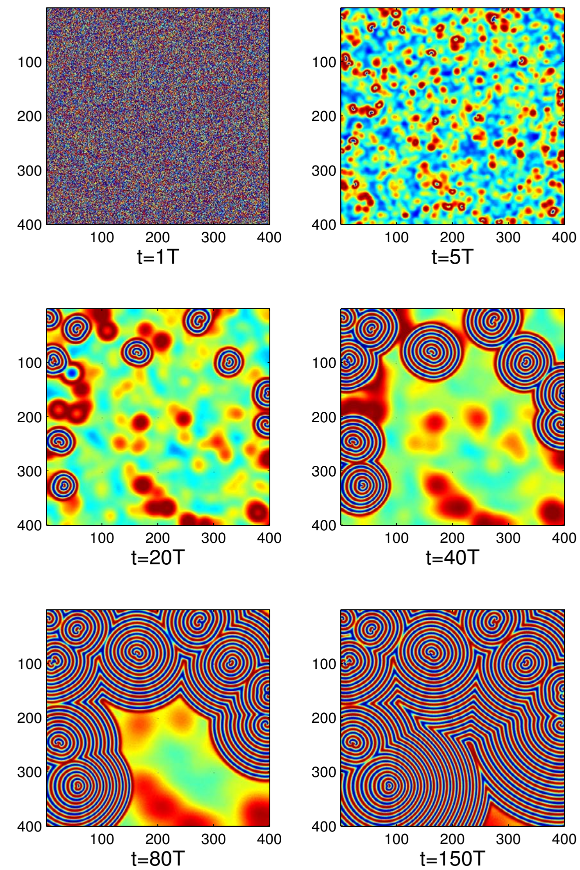


Fig. 7. Pattern formation in an unforced biochemical system.

simulation of such system is impossible in Matlab, as the size is too large. However, using our macromodeling technique, we can run hundreds cycles simulation of such system in hours, the speedup is more than 1000×. In this figure, we use colors to represent the phase of oscillators, *i.e.*, different colors indicate oscillators with different phases. In the beginning ($t = 0$), all oscillators are given a random phase: hence we cannot see any pattern. After 5 oscillating cycles ($t = 5T$), the collective synchronization phenomenon is clearly seen: oscillators synchronize their phase with the phase of their neighbors. As a result, we can see many color spots in the figure. After 20 oscillating cycles, some small target patterns appear, and a spiral wave pattern forms on the right side of the figure. From $t = 40T$ onwards, we can see those patterns grow, and merge together. Finally, after 150 cycles, we obtain a complex figure which combines both target pattern and spiral pattern.

In Figure 8, we investigate the patterns of this biochemical system under different coupling strength. In the first figure, coupling is strong ($R = 10$), all oscillators lock to same phase, so we cannot see any pattern. In the second figure, we increase the coupling resistance to $R = 12.5$, a spiral wave pattern forms. We keep increasing the coupling resistance and obtain different kinds of patterns. When the coupling is very week, the system runs into chaos, as the last picture in Figure 8.

B. Simulation of a Tunneling Phase Logic Based Cellular Nonlinear Network

Figure 9 depicts a basic tunneling phase logic unit and its oscillating waveform. A basic TPL unit consists of an ultra-small SET junction with capacitance C_j, a DC bias V_{DC} and a pump voltage V_p. The SET junction has the property that when its voltage increases to a threshold V_T, single-electron tunneling occurs and the capacitor C_j is discharged. With the DC bias V_{DC} providing a bias current, the SET junction behaves as shown in Figure 9(b). The AC pump provides a sinusoidal voltage with amplitude V_p, which runs two times faster than the SET frequency. Therefore, if the SET is super-harmonically locked by the pump voltage, it has two steady states, with the phase difference π. If the phase of the SET oscillator is set to represent the logical values 0 and 1, we can realize logic in phase, instead of voltage as in traditional CMOS circuits. Using the tunneling junction oscillators, we can realize the cellular nonlinear network (CNN) in nano-scale.

A major potential application of CNNs is in image processing [27]. Here we present an example to show the image processing ability of TPL-CNNs: we use a TPL-CNN to detect the image edge. We extract the nonlinear phase macromodel for the tunneling oscillator, use the macromodel to replace the oscillators in the CNN, and run the simulations in phase domain. The original images are shown in Figure 10(a) and Figure 10(b). We transfer these images into two-

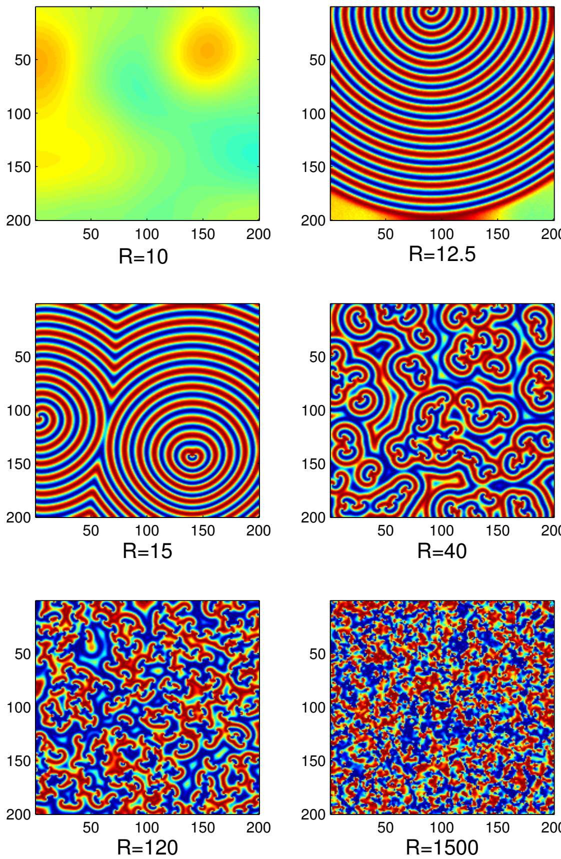


Fig. 8. Biological patterns under different coupling strength.

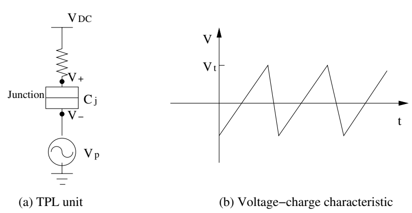
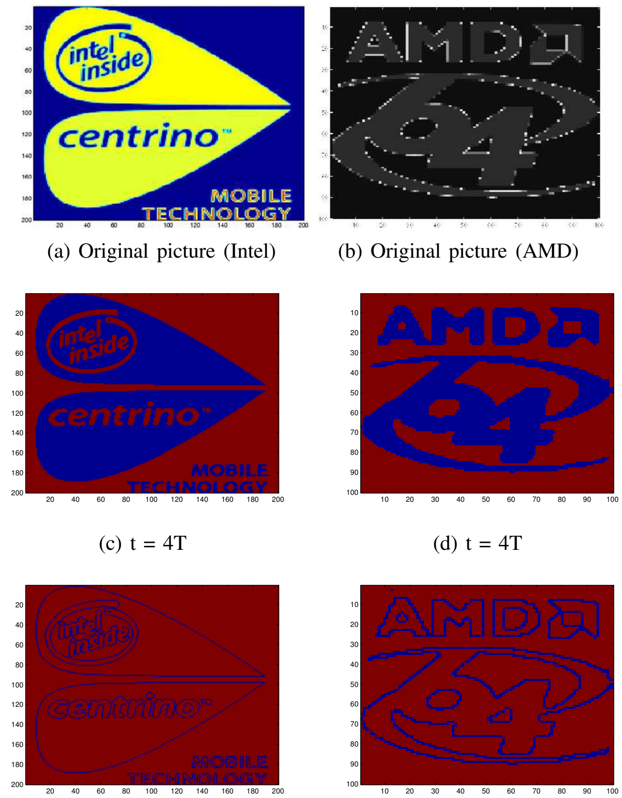


Fig. 9. TPL unit and its voltage-charge characteristic.

color mode, apply them as inputs to the CNN network, and use our macromodeling technique to simulate the TPL-CNN under this input. In Figure 10(c) and Figure 10(d), we can see tunneling occurs and the inputs are replicated at the outputs of the CNN after 4 oscillating cycles. At $t = 8T$, the edges of the images are detected, as shown in Figure 10(e) and Figure 10(f).

VI. Conclusions

We have presented a nonlinear oscillator macromodeling technique which combining a nonlinear time-shifted phase equation and a simple LPTV amplitude macromodel, and successfully applied the proposed technique for investigating injection locking, transient response and collective behavior in oscillator-based systems. Experimental results show that our technique offers great speedups over SPICE-level full simulation, without appreciable loss of accuracy.



(a) Original picture (Intel) (b) Original picture (AMD)

(c) t = 4T (d) t = 4T

(e) t = 8T (f) t = 8T

Fig. 10. Image edge detection performed by a TPL-CNN.

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