Frequency Selective Model Order Reduction via Spectral Zero Projection

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Abstract— As process technology continues to scale into the nanoscale regime, interconnect plays an ever increasing role in determining VLSI system performance. As the complexity of these systems increases, reduced order modeling becomes critical. In this paper, we develop a new method for the model order reduction of interconnect using frequency restrictive selection of interpolation points based on the spectral-zeros of the RLC interconnect model's transfer function. The methodology uses the imaginary part of spectral zeros for frequency selective projection and provides stable as well as passive reduced order models for interconnect in VLSI systems. For large order interconnect models with realistic RLC parameters, the results indicate that our method provides more accurate approximations than techniques based on balanced truncation and moment matching with excellent agreement with the original system's transfer function.

I. Introduction

Aggressive feature scaling and increasing operating frequencies are greatly impacting the performance of high speed integrated circuits [1], Motivated by the expanding complexity of nanoscale integrated circuits, the model order reduction (MOR) of RLC interconnect models has been the focal point of substantial research efforts over the last decade [2, 3, 4]. The methods used for the MOR of interconnects fall into two main categories: Singular Value Decomposition (SVD) methods and Krylov subspace projection methods. In SVD methods for linear systems, Hankel-norm approximation, balanced truncation, and singular perturbation methods are well known. The most popular technique is balanced approximation, in which the primary aim is to generate a balanced representation of the system with same degree of reachability and observability. Balanced approximation based algorithms such as PMTBR [5, 6], PRTBR [7] and FABT [8, 9] have been proposed for interconnect model order reduction. However, SVD methods have high computational complexity with dense computations of order n^3 and storage of order n^2. As such, these methods are not suited for reducing systems with high complexity.

For large scale systems, moment-matching projection methods [10] provide a tractable alternative to SVD methods. In general, projection methods consist of iterations involving the controllability and reachability subspaces (also known as Krylov subspaces) spanned by a sequence of vectors. These methods provide an iterative approximation of the eigenvalues in order to match the system's moments. A number of Krylov-based model order reduction implementations [11, 3] have been proposed. These methods are attractive due to their iterative nature and are computationally less expensive than balanced approximation. However, unlike balanced approximation methods, there is typically no guarantee of stability or an upper error bound.

High performance interconnects have a frequency dependent resistance and reactance that give rise to complex frequency responses [9, 12]. The complex frequency response usually gives rise to high frequency variations. To match these variations, a high order system is typically required to approximate the original system response. This is computationally expensive and may not be suitable for large scale VLSI systems. In order to efficiently approximate large variations in the frequency response, a frequency selective model order reduction technique is well-suited to provide low order approximations and to match the original system response over a given range of frequencies.

In this paper, we develop a passivity preserving model order reduction method with frequency selective interpolation points derived from imaginary part of the spectral zeros of the system. The reduced model preserves the properties of the original system as well as the stability and passivity. The preservation of passivity is guaranteed by selecting interpolation points as a subset of the stable system's spectral zeros. The MOR technique is implemented using Krylov projection methods to ensure low complexity. The results indicate that our method provides more accurate approximations than techniques based on balanced truncation and moment matching.

II. Model Order Reduction Problem Formulation

An RLC representation for a linear dynamic system can be written using following state space representation:

$$\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $E \in \Re^{nxn}$, $A \in \Re^{nxn}$, $B \in \Re^{nxm}$, $C \in \Re^{pxn}$ and $D \in \Re^{pxm}$ are the matrices defining the linear maps between inputs, outputs, and internal state variables generated using common techniques such as modified nodal analysis. Without loss of generality, we will assume E to be identity for the purpose of theoretical analysis. We represent the state space system as a linear dynamic system using the following representation

$$\Sigma = \left[\frac{A|B}{C|D}\right] \in \Re^{(n+p)x(n+m)}. \tag{2}$$

We then approximate the system using the following reduced model order formulation

$$\Sigma = \left[\frac{\hat{A}|\hat{B}}{\hat{C}|\hat{D}}\right] \in \Re^{(k+p)x(k+m)} \tag{3}$$

where $k << n$. When W^* and V are orthogonal (W^*V=I_k) and span the reduced order subspace, the reduced system is of the form

$$\begin{aligned} \hat{A} &= W^*AV, \\ \hat{B} &= W^*B, \\ \hat{C} &= CV, \\ \hat{D} &= D \end{aligned} \tag{4}$$

where order of $\hat{A} << A$. The most popular technique for computing the orthogonal projectors used to generate the reduced system is the Block Arnoldi method, which has been the basis for several previous techniques [3, 13, 14]. The Block Arnoldi method iteratively produces basis vectors of the following form

$$AV_k = V_k H_k + r_k e_k^* \tag{5}$$

where V_k are the orthogonal basis vectors generated at iteration k of the algorithm. Arnoldi implicitly forms the basis vectors that span the Krylov subspace defined as

$$\mathcal{K}(A, B) = [B, AB, A^2B \cdots A^{k-1}B]. \tag{6}$$

Several issues complicate the implementation of Krylov-based projection methods. Numerical issues may cause the basis vectors (H_k) to lose orthogonality, thereby corrupting the projectors and mitigating the effectiveness of the reduced system approximation. The reduced system may also not be stable or passive unless certain methods are employed [3, 14]. Since the Arnoldi iteration first approximates the eigenvalues associated with the high-frequency poles of the system, it may produce reduced order models that do not match the original system's response in the frequencies of interest. By applying shifted-Arnoldi to provide a multi-point interpolation of reduced systems at a different frequency points, the function can be approximated across a wide-range of frequencies [15]. However, choosing these interpolation points to minimize the computational complexity may be problematic.

In the following sections, we derive and implement a new method for choosing interpolation points based on the spectral zeros of the system's transfer function to match over the desired frequency range. The method preserves the passivity of the reduced order system by construction.

III. Model Order Reduction with Preservation of Passivity

Our proposed method of model order reduction with preservation of passivity is based on the frequency selective positive real interpolation of linear time invariant systems. The approach is inspired by frequency selection and projection of systems with the desired dominant frequency band of interest. The choice of interpolation points will guarantee the preservation of passivity and can produce a lower order model when compared with techniques such as balanced truncation and moment matching.

It is known that the passivity of a linear time invariant system is preserved if its transfer function, $G(s)$, is *positive real.* The positive realness of the Σ system is achieved if the transfer function satisfies the following conditions [16, 17]:
1. $G(s)$ is analytical for $Re(s) > 0$;
2. $G(s) = \overline{G(s)}$ for all $s \in C$; and
3. $G(s) + G(s)^* \geq$ for $Re(s) > 0$.

The second condition is satisfied for real systems and the third condition implies the existence of a rational function with a stable inverse. Therefore, there exists a set of projectors, V and W^* with $VW^* = 0$ with $VW^* \neq 0$ obtained by interpolating the transfer function so that the projected system is both stable and passive. As a result, we seek interpolation points that are positive real. In the linear system Σ defined in (2), (A, B) are reachable and (C, A) are observable. The matrix A is also assumed to be stable with eigenvalues residing in the open left-hand plane. The system passivity is then equivalent to the positive realness of the associated transfer function

$$G(s) = D + C(sI - A)^{-1}B. \tag{7}$$

The *spectral zeros* of a system are defined as the zeros of the quantity $G(s) + G(-s)$. Furthermore, it is true that

$$G(s) + G(-s) = \frac{r(s)r(-s)}{d(s)d(-s)}, \tag{8}$$

where $d(s)$ and $n(s)$ are the denominator and numerator of the transfer function, respectively, and

$$r(s)r(-s) = n(s)d(-s) + d(s)n(-s). \tag{9}$$

The roots of the polynomial $r(s)$ are in the closed left-hand plane and the coefficients are real. This means that the spectral zeros cannot be purely imaginary. Additionally, the *stable spectral zeros* are defined as the roots of $r(s)$. In terms of the system matrices, the stable spectral zeros are all λ that satisfy

$$\Upsilon - \lambda\Phi = 0 \tag{10}$$

where

$$\Upsilon = \begin{bmatrix} A & 0 & B \\ 0 & -A^* & -C^* \\ C & B^* & D + D^* \end{bmatrix}$$
$$\Phi = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

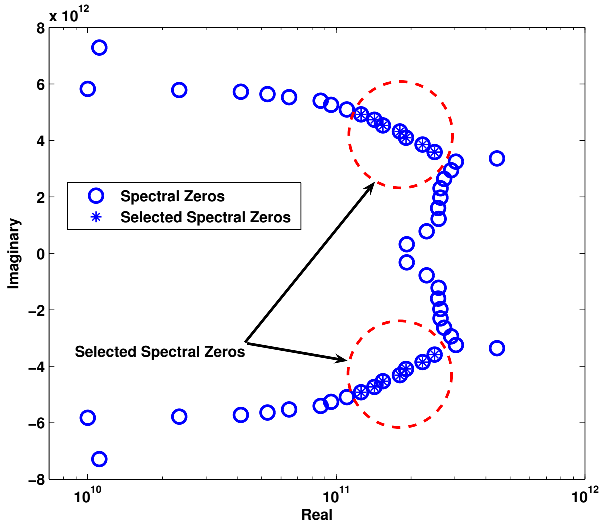

Fig. 1. Spectral zeros of an RLC circuit model with frequency selective spectral zeros maked by $*$.

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Algorithm: Frequency Selective Passivity Preserving MOR (Input: C, G, B, L, w
Output: Cn, Gn, Bn, Ln)
Find zeros of G(s)+G(-s) as given in Section IV
Initialize: N array (1    n    N) of imaginary part of Spectral Zeros
Select Spectral Zeros
    for each n     S do
        if f1 < imag(Si) < f2
        Wi=1    /* Select Spectral zero*/
        end
    end for
Generate S matrices (singular values) based on selected Spectral Zeros
V = XQS^-1
W = YQS^-1
Generate Reduce Order Model
Gn = W'GV
Bn = W'*B
Ln = LV
Cn = W'CV
Dn = D
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Fig. 2. Pseudocode of frequency selective passivity preserving MOR algorithm.

TABLE I
DESIGN PROBLEM CHARACTERISTICS

	RLC	Inductor
Order	79	1008
Permuted Order	77	981
Reduced Order	28	64
Minimum Frequency (rads/sec)	10^9	10^9
Maximum Frequency (rads/sec)	10^{14}	10^{14}
Condition number $(E^{-1}G)$	4.0189x10^{10}	2.5309x10^{13}
Condition number A	801.1024	1.2909x10^5
Condition number E	∞	∞

Moreover, if $D + D^*$ is invertible, these numbers are the generalized eigenvalues of the following *Hamiltonian* matrix:

$$\begin{bmatrix} A & 0 \\ 0 & -A^* \end{bmatrix} - \begin{bmatrix} B \\ -C^* \end{bmatrix} (D + D^*)^{-1}(CB^*). \quad (12)$$

The critical result is that if the interpolation points are chosen to be the spectral zeros of the original system Σ, the reduced system is both stable and passive. Equation (11) shows that the method can also be applied to systems in descriptor form. In our implementation, we have used Implicitly Restarted Arnoldi to solve the generalized eigenvalue problem to handle higher order systems [18]. This makes the computational cost comparable to Krylov methods ($O(kN^2)$) and much less than balanced truncation methods ($O(N^3)$). The eigenvalues are the spectral zeros and lie on the left-half plane since $G^*(-\lambda_i) + G^*(\lambda_i) = 0$.

The generalized eigenvalues λ defined in (10) are the spectral zeros of a given system. In order to obtain an accurate approximation of the original system for the frequencies of interest, we need to choose appropriate finite spectral zeros to use as interpolation points. Figure 1 shows the spectral zeros of an RLC circuit model and the selected spectral zeros. We select the spectral zeros with imaginary part near the frequency of interest to match the response close to the selected frequency. In addition, for each spectral zero selected, we selected its complex conjugate. Therefore, each spectral zero selected as an interpolation point increases the reduced system order by 2. Once we have chosen the spectral zeros, we use the eigenvectors (Q) corresponding to the selected spectral zeros to construct the projectors V and W. In the implementation of the proposed scheme, the state dimension of the model will be reduced to k with $k \ll n$. Assuming $VW^* \neq 0$, the projected system Σ can interpolate the system transfer function at $s_i, i = 1, ..., 2k$. The projectors V and W are used to reduce the system in the following manner:

$$\hat{B} = W^*B, \hat{C} = CV, \hat{E} = W^*EV, \hat{D} = D. \quad (13)$$

The algorithm for the selection of spectral zeros is shown in Figure 2. The frequency selection is performed using the absolute value of the imaginary part of the spectral zeros. The selected spectral zeros are then used to identify corresponding eigenvectors, which are used to construct the projectors V and W of the system. These projectors are orthogonal and dynamically generate reduced order model.

IV. Results

The first structure simulated is a simple RLC circuit representing an interconnect wire. The circuit used to model the structure is presented in Figure 3(a). The model takes into account the resistance, inductance, self-capacitance, coupling-capacitance, and mutual-inductance between the segments in the interconnect wire. Note that in Figure 3(a), not all of the coupling-capacitances and mutual inductances are shown. In creating state space models for the simulated circuit, we used the field solvers FastCap and FastHenry for capacitance and inductance extraction [19, 20] and finally combine the results using modified nodal analysis [3]. The complexity of the initial system is $n = 77$. The original system is compared against

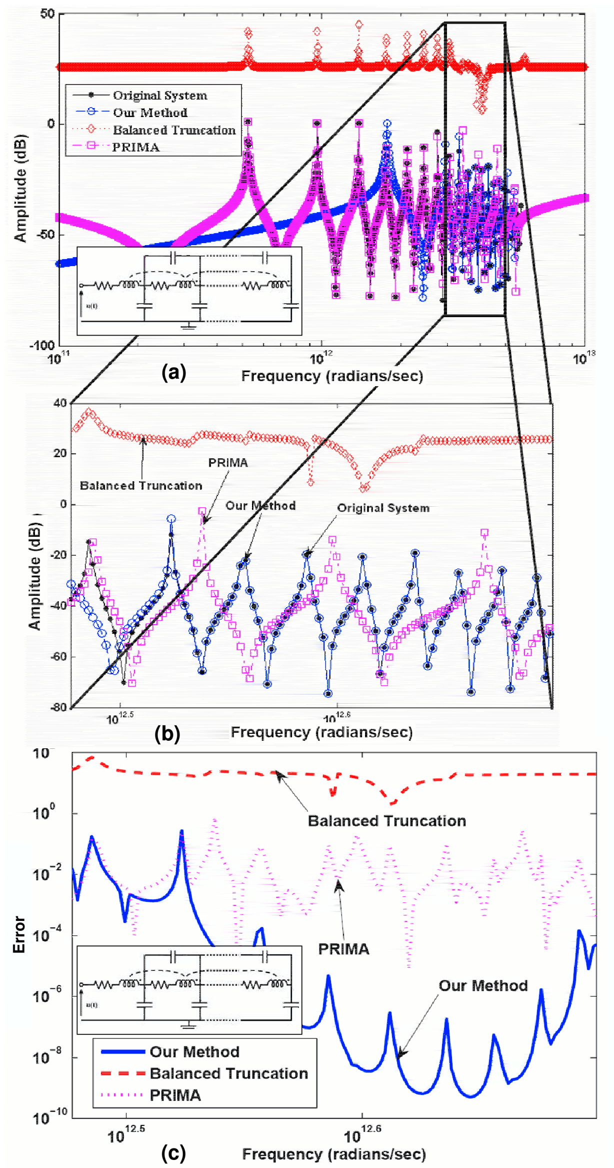

Fig. 3. System frequency response (a, b) and error (c) for an RLC network with original and reduced models of order $n = 79$ and $n = 28$, respectively.

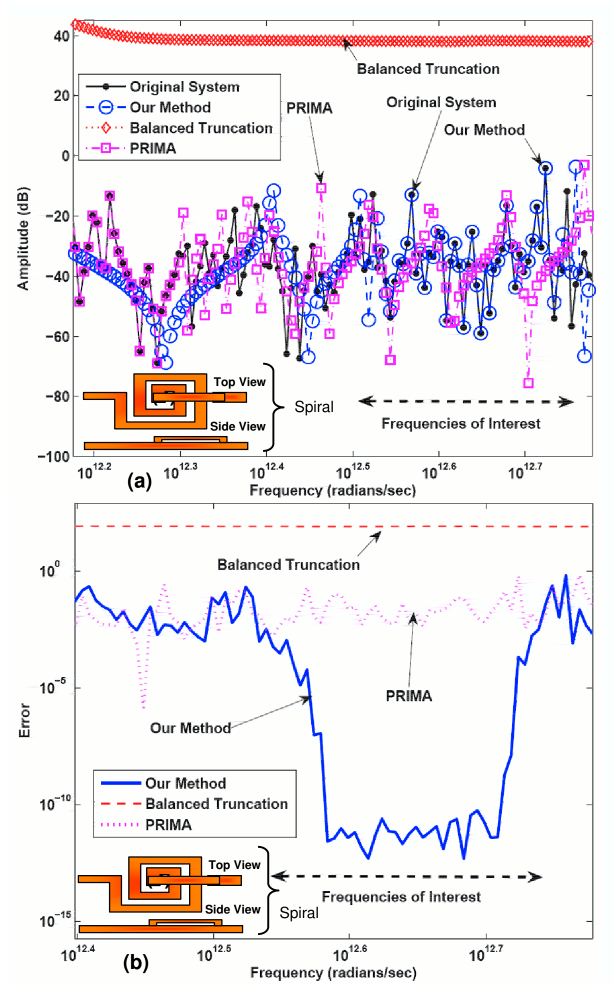

Fig. 4. System frequency response (a) and error (b) for the spiral inductor system with original and reduced models of order $n = 1008$ and $n = 64$, respectively.

model reductions performed by our proposed technique, balanced truncation, and the Arnoldi based PRIMA method [3]. The order and condition of the system matrices is given by Table I. It can be noted that matrix A is ill-conditioned, which often leads to poor performance of balance truncation method. The original system matrix E is singular and needs to be permuted to remove the singularities. The permuted system reduces the order of the system from 79 to 77. In carrying out PRIMA computation also similar ill-conditioning leads the upper Heisenberg matrix H to loose orthogonality at each iterative step. To compensate for ill-conditioning of system DGKS correction is applied at each iteration to avoid ill-conditioning and maintaining orthogonality of basis vectors. Figure 3(a) compares the frequency response of our method, PRIMA, and balanced truncation with the original system. It shows that an order 28 system generated by balanced truncation does not approximate the system. It is important to note that PRIMA matches well at low frequencies, but our method matches perfectly at the frequencies of interests as shown in Figure 3(a). The error plot is presented by Figure 3(b) and further highlights the strength of our approach. The error plot shows that the proposed method has up to 250 dB lower error compared to PRIMA and even lower error compared to balanced truncation.

The next structure modeled is a spiral inductor. Spiral inductors are important elements in the design of circuits for wireless applications [21]. The spiral inductor model considered has an order of 1008. The three aforementioned methods are used to reduce the system to order 64. The system capacitance and inductance are extracted using the field solvers FastCap and FastHenry [19, 20]. The design problem characteristics are shown by Table I. The system is highly ill-conditioned system with condition number of $E^{-1}G$ being 2.5309x10^{13}. The ill-conditioning of $E^{-1}G$ results in a nonconvergent solution of balanced truncation. The simulation of original system with a reduced order of 64 is shown in Figure 4. The results show that balanced truncation fails to converge. However, PRIMA matches well at low frequencies. The error plot shown in Figure 4(b) further highlights the strength of our method and shows low error at the frequencies of interest compared to PRIMA and balanced truncation. The example demonstrates that for large scale systems, the proposed method accurately approximates the original system.

V. Conclusion

In order to efficiently capture complex interconnect effects in high performance VLSI systems, model order reduction can provide tractable solutions to complex interconnect analysis problems. In this paper, we present new method of model order reduction for interconnects that uses the systematic selection of the interpolation points to provide the advantage of generating passive models. Our method is efficient as well as accurate and generates stable reduced systems. The simulation results using the new technique match closely with the results from the full system. Additionally, the reduced order model preserves the properties of the original system and due to its computational performance, it can be effectively applied to reduce large scale VLSI systems.

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