Macromodelling Oscillators Using Krylov-Subspace Methods

Xiaolue Lai and Jaijeet Roychowdhury
Department of Electrical and Computer Engineering
University of Minnesota
Email: {laixl, jr}@ece.umn.edu

***Abstract*— We present an *efficient* method for automatically extracting unified amplitude/phase macromodels of arbitrary oscillators from their SPICE-level circuit descriptions. Such comprehensive oscillator macromodels are necessary for accuracy when speeding up simulation of higher-level circuits/systems, such as PLLs, in which oscillators are embedded. Standard MOR techniques for linear time invariant (LTI) and varying (LTV) systems are not applicable to oscillators on account of their fundamentally nonlinear phase behavior. By employing a cancellation technique to deflate out the phase component, we restore the validity and efficacy of Krylov-subspace-based LTV MOR techniques for macromodelling oscillator amplitude responses. The nonlinear phase response is re-incorporated into the macromodel after the amplitude components have been reduced. The resulting unified macromodels predict oscillator waveforms, in the presence of any kind of input or interference, at far lower computational cost than full SPICE-level simulation, and with far greater accuracy compared to existing macromodels. We demonstrate the proposed techniques on LC and ring oscillators, obtaining speedups of 30-120× with no appreciable loss of accuracy, even for small circuits.**

I. Introduction

Oscillators are important building blocks in electronic and optical systems. For example, they are often used for frequency-translation of information signals in communication systems. Voltage-controlled oscillators (VCOs) are key components of phase-locked loops (PLLs), which are widely used in both digital and analog circuits for clock generation and recovery, frequency synthesis, *etc.* Despite their widespread use, the simulation of oscillators and oscillator-based systems still poses significant challenges.

Traditional circuit simulators, such as SPICE [1], are far from ideally suited for simulating oscillators. One key problem is that transient simulation accumulates numerical phase errors without limit; furthermore, it is also difficult to extract phase information from time-domain voltage/current waveforms accurately. To improve phase accuracy, many timesteps need to be taken in each oscillation cycle, with transient simulations of high-Q oscillators requiring many thousands of cycles, hence suffering from great inefficiency.

Since phase responses are of major concern for oscillators, various specialized and approximate techniques (*e.g.*, [2]–[7]) have been developed for predicting phase information directly without relying on transient simulation of the full circuit. Simulation in the phase domain can result in great speedups. However, phase macromodels do not capture amplitude variations at the output of the oscillator, which can be important in many situations. For example, in pico-radio systems, "radio nodes" must be ultra-low power, leading to novel, very simple PLL systems where VCO outputs are in essence directly fed to analog mixers, with no intervening amplitude stabilization or clipping. VCO amplitude variations in such systems change gains of PLL loops dynamically, thereby affecting important phenomena such as jitter, lock/capture behavior, *etc.* Similar design philosophies are emerging for a variety of low power systems in wireless communication and mobile systems.

In [8], a technique was presented to macromodel both phase and amplitude in oscillators under perturbation. The phase deviation was calculated via a nonlinear phase equation [9], while the amplitude macromodel was extracted by full Floquet decomposition [10] of the linearized, time-varying oscillator. The method applies to any kind of oscillator, including LC, ring, *etc.* However, it has a drawback: Floquet decomposition becomes very computationally expensive, and can also suffer from numerical errors, as system sizes increase. An oscillator macromodelling technique that scales gracefully with circuit size is of great practical interest, given that on-chip RF oscillators today can have many thousands of nodes.

In this paper, we present a novel method to circumvent this issue. Instead of performing full Floquet decomposition on the linearized oscillator equations, we apply the time-varying Padé (TVP) method [11], to reduce the oscillator system to a smaller LPTV system which captures the important amplitude-variation components of the original oscillator accurately. The transfer function of the LPTV oscillator system is expanded into matrix forms using time- or frequency-domain methods (such as FDTD or harmonic balance), and then reduced using Krylov-subspace methods [12]–[14]. A major issue faced for oscillators, that we solve in this paper, is that the oscillator's LPTV system incorporates both phase and amplitude information, leading to inaccurate amplitude macromodels if the phase component is not separated out correctly. Moreover, because oscillators are fundamentally phase-unstable [9] and sustained small perturbations can change the phase of the oscillator unboundedly, additional issues are faced if the phase component is not dealt with specially.

Instead of using full Floquet decomposition as in [8], we use a novel alternative technique in this paper, based on canceling out components of the input that excite phase responses, that enables application of Krylov-subspace methods to oscillators. The perturbation input to the oscillator is decomposed into two parts with one contributing to phase and the other to amplitude. We change the input vector of the LPTV system to remove the input corresponding to phase and apply Krylov-subspace methods to reduce this modified system. The resulting reduced system contains no phase information since the pole corresponding to phase has a very small residual, thus having been effectively eliminated by the Krylov reduction process.

Once the reduced amplitude macromodel, without any phase information, has been obtained, we re-incorporate the phase component by coupling the amplitude macromodel with the nonlinear, scalar phase macromodel presented in [9]. The resulting unified oscillator macromodel is able to predict both phase and amplitude variations accurately at far lower computational cost than that of full SPICE-level simulation. Compared to the method described in [8], the key difference is that the method in [8] requires full decomposition of the whole LPTV system, while our method relies on Krylov-subspace methods, with far lower computational cost. Hence, our method scales well to large oscillator circuits. The generated macromodels can be easily encapsulated into other circuit simulation tools (*e.g.*, MATLAB/Simulink, Verilog-A, *etc.*) to predict the comprehensive behavior of oscillators in a variety of system-level situations.

We verify our macromodelling technique on ring and LC oscillators. Comparing simulation results between our macromodels and SPICE-level full circuit simulation, we show that our macromodels are able to reproduce the waveforms of oscillators under various perturbations accurately, while obtaining impressive speedups. Even

0-7803-9451-8/06/$20.00

with the relatively small oscillators we have used for testing purposes, we obtain speedups in the range of 1-2 orders of magnitude; we expect much greater speedups for larger circuits with complex device models, which we are currently in the process of incorporating into our simulation infrastructure.

The remainder of the paper is organized as follows. In Section II, we show that linear perturbation analysis is not valid for oscillators. In Section III, we review nonlinear perturbation analysis for oscillators and the nonlinear oscillator macromodel we employ in this work. In Section IV, we summarize the time-varying Padé technique for reducing LPTV systems. In Section V, we describe our phase-component deflation technique for macromodelling amplitude variations of oscillators. In Section VI, we present simulation results on ring and LC oscillators.

II. Linear Perturbation Analysis Is Not Suitable For Oscillators

The traditional method to analyze perturbed nonlinear system is to linearize the nonlinear equations on its unperturbed orbit. In this section, we will show that this method is not suitable for oscillators.

A general oscillator under perturbations can be described by

$$\dot{x} + f(x) = Bb(t), \tag{1}$$

where $b(t)$ is perturbation signal applied to the free running oscillator. Since perturbation signal has small amplitude, we can linearize (1) on its unperturbed steady-state orbit and get a linearized periodic time-varying system

$$\dot{z}(t) + G(t)z(t) \approx Bb(t), \tag{2}$$

where $G(t) = \frac{\partial f(x)}{\partial x}|_{x_s(t)}$ is the linearized system on oscillator's steady-state orbit $x_s(t)$.

According to Floquet theory [10], the state transition matrix of the homogeneous part of (2) can be given by

$$\Phi(t, \tau) = U(t)e^{D(t-\tau)}V^T(\tau) \tag{3}$$

where $U(t) = [u_1(t), u_2(t), ..., u_n(t)]$ and $V(t) = [v_1(t), v_2(t), ..., v_n(t)]$ are T-periodic nonsingular matrix, satisfying biorthogonality conditions $v_i^T(t)u_j(t) = \delta_{ij}$, and $D = diag[\mu_1, ..., \mu_n]$, where μ_i are Floquet exponents. The particular solution of (2) under perturbation $b(t)$ is given by

$$z(t) = \sum_{i=1}^{n} u_i(t) \int_0^t e^{\mu_i(t-\tau)} v_i^T(\tau)Bb(\tau)d\tau. \tag{4}$$

For an oscillator system, one of the Floquet exponents must be 0 [9]. Without loss of generality, we assume $\mu_1 = 0$, and $e^{\mu_1(t-\tau)}$ term vanishes. Thus, we can always choose a perturbation $b(t)$ to satisfy $v_1^T(t)Bb(t)$ has nonzero average value, then $z(t)$ will grow unboundedly on t even though $b(t)$ have very small amplitude. This contradicts the assumption that $z(t)$ is small variation, thus the linear perturbation analysis is inconsistent.

III. Previous Woek: Nonlinear Oscillator Macromodel

In [9], authors show that the Floquet exponent $\mu_1 = 0$ in (4) is corresponding to the oscillator's phase deviation, which means the oscillator's phase deviation grows unboundedly even though the perturbation signal is very small. Due to oscillator's neutral phase stability, linearizing the oscillator on its steady state is meaningless since a small perturbation will change the phase, or the steady-state orbit of the oscillator dramatically.

An idea to overcome this limitation is to linearize the oscillator circuits over its perturbed time-shifted orbit. Based on this idea, in [8], a novel approach is presented to construct the oscillator macromodels. The macromodel produced is a combination of a scalar nonlinear differential equation [9] that solves the phase deviation and a reduced linear time-varying system for predicting amplitude variation, which is computationally simpler than the original oscillator system. Here, we summarize this method.

A. Nonlinear Oscillator Phase Macromodel

According to [9], the solution of the oscillator under perturbation can be expressed as

$$x(t) = x_s(t + \alpha(t)) + y(t), \tag{5}$$

where $x(t)$ is the orbit of the oscillator under perturbation, $x_s(t)$ is the unperturbed steady-state orbit of the oscillator, $\alpha(t)$ is the phase deviation, and $y(t)$ is the amplitude variation. If we can calculate the phase deviation $\alpha(t)$ and amplitude variation $y(t)$, we can rebuild the waveforms of the oscillator under perturbations using (5).

The phase deviation $\alpha(t)$ is governed by the nonlinear differential equation [9]

$$\dot{\alpha}(t) = v_1^T(t + \alpha(t)) \cdot Bb(t), \tag{6}$$

where $v_1(t)$ is the perturbation projection vector (PPV) that corresponds to oscillator's phase sensitivity to perturbations, and $b(t)$ is the perturbation applied to the oscillator. The PPV has periodic waveforms that have the same period as the oscillator. Various methods [9], [15]–[17], both in time domain and frequency domain, have been presented for calculating the PPV from the oscillator circuit equations. When the PPV is available, the phase deviation $\alpha(t)$ can be efficiently calculated by solving this simple one-dimension differential equation.

B. Amplitude Macromodel

In [8], a method is presented to to construct the amplitude macromodel of the oscillator. the oscillator is first linearized on its perturbed time-shifted orbit

$$\begin{aligned} \dot{o}(t) &\approx -\frac{\partial f}{\partial x}|_{x_s(t+\alpha(t))} o(t) + Bb(t) \\ &= A(x_s(t + \alpha(t)))o(t) + Bb(t), \end{aligned} \tag{7}$$

where $x_s(t)$ is oscillator's steady-state orbit, $o(t)$ is small variations due to perturbation $b(t)$ and $\alpha(t)$ is phase deviation. Since $A(x_s(t + \alpha(t)))$ is not periodic, we introduce a new variable $\hat{t} = t + \alpha(t)$ and define $\hat{o}(\hat{t}) = o(t)$ and $\hat{b}(\hat{t}) = b(t)$. After dropping a quadratic term, we can obtain a linear periodic time-varying system

$$\dot{\hat{o}}(\hat{t}) = A(x_s(\hat{t}))\hat{o}(\hat{t}) + B\hat{b}(\hat{t}). \tag{8}$$

Applying Floquet decomposition to this LPTV system, the solution of this system can be expressed as

$$\hat{o}(\hat{t}) = \sum_{i=1}^{n} u_i(\hat{t}) \int_0^{\hat{t}} \exp(\mu_i(\hat{t} - \tau))v_i^T(\tau)B\hat{b}(\tau)d\tau. \tag{9}$$

where $u_1 = 0$ is corresponding to oscillator's phase deviation, we need to drop it. The resulting system can be reduced by dropping some less important Floquet exponents. The limitation of this method is that Floquet decomposition is very slow and numerically unstable when the system size is large.

IV. Reduced-Order Modeling for LPTV system

In [11], the time-varying Padé (TVP) method is presented to reduce the LPTV system.

A nonlinear system can be expressed as a differential-algebraic equation (DAE)

$$\frac{\partial q(x(t))}{\partial t} + f(x(t)) = b_L(t) + Bb(t)$$

$$y(t) = d^T x(t), \tag{10}$$

where $b_L(t)$ is a large signal, which determines the system's steady-state orbit, and $Bb(t)$ is small perturbations applied to the system. B and d are input/output vectors. For small perturbation analysis, we linearize this system over the orbit generated by the large signal. The time-varying Padé (TVP) method can be applied to reduce this linearized time-varying system. Here, we summarize this method.

The transfer function of the linearized system can be written as

$$H(t_1, s) = d^T \left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right)^{-1}[B], \tag{11}$$

where $\frac{D}{dt_1}[]$ is a differential operator [11]. Assume $C(t_1)$ and $G(t_1)$ to be periodic with angular frequency w_0, and define $W(t_1, s)$ to be the operator-inverse in (11), we have

$$W(t_1, s) = \left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right)^{-1}[B] \tag{12}$$

$$\Rightarrow \left(\frac{D}{dt_1}[] + sC(t_1) + G(t_1)\right) W(t_1, s) = B. \tag{13}$$

Assume $W(t_1, s)$ is also a periodic function, we can express (13) in frequency domain

$$[sC_{FD} + J_{FD}]\vec{W}_{FD} = \vec{B}_{FD}, \tag{14}$$

where

$$J_{FD} = (G_{FD} + \Omega C_{FD}), \tag{15}$$

$$C_{FD} = \begin{pmatrix} \vdots & \vdots & \vdots \\ \ldots C_0 & C_{-1} & C_{-2} \ldots \\ \ldots C_1 & C_0 & C_{-1} \ldots \\ \ldots C_2 & C_1 & C_0 \ldots \\ \vdots & \vdots & \vdots \end{pmatrix} \tag{16}$$

$$G_{FD} = \begin{pmatrix} \vdots & \vdots & \vdots \\ \ldots G_0 & G_{-1} & G_{-2} \ldots \\ \ldots G_1 & C_0 & G_{-1} \ldots \\ \ldots G_2 & G_1 & G_0 \ldots \\ \vdots & \vdots & \vdots \end{pmatrix} \tag{17}$$

$$\Omega = j\omega_0 \begin{pmatrix} \ddots & & & & & & \\ & -2I & & & & & \\ & & -I & & & & \\ & & & 0I & & & \\ & & & & I & & \\ & & & & & iI & \\ & & & & & & \ddots \end{pmatrix} \tag{18}$$

$$\vec{B}_{FD} = [\ldots, 0, 0, B^T, 0, 0, \ldots]^T. \tag{19}$$

Then the transfer function in frequency domain can be written as

$$\vec{H}_{FD}(s) = D^T [sC_{FD} + J_{FD}]^{-1} \vec{B}_{FD}, \tag{20}$$

where

$$D = \begin{pmatrix} \ddots & & & & & \\ & d & & & & \\ & & d & & & \\ & & & d & & \\ & & & & d & \\ & & & & & \ddots \end{pmatrix}. \tag{21}$$

We can use any Krylov-subspace method to reduce (20) and obtain a smaller LPTV system.

$$\vec{H}_q(s) = L_q^T [I_{q\times q} + sT_q]^{-1} R_q, \tag{22}$$

and the reduced system equation is

$$T_q \dot{\tilde{x}}(t) + \tilde{x}(t) = R_q u(t), \tag{23}$$

where $\tilde{x}(t)$ is a vector of size q, which is much smaller than that of the original system.

V. Reducing Oscillator System Using TVP method

Reduce-order modeling of oscillators is slightly different than that of other systems, such as mixers and converters. Oscillator systems are neutral stable, traditional TVP methods may not be able to produce stable reduced system. In this section, we present our method for modeling amplitude variations of oscillators.

A. Traditional TVP Method Is Not Suitable For Oscillators

TVP methods [11] are very useful for reducing LPTV systems, such as mixers and convertors, *etc.* However, the method is not suitable for oscillators, as oscillator systems are neutral stable. We have shown in Section II that the Floquet decomposition of oscillator system has a Floquet exponent of 0, which contributes to phase response. If we expand the oscillator transfer function to frequency domain using the method described in Section IV, the resulting frequency domain transfer function (20) has many poles on the imaginary axis, with interval $j2\pi\omega_0$. Figure 1 depicts the pole distribution of an LC oscillator. In this figure, the oscillator has poles on $\pm jn\omega_0$, some poles even have positive real part due to numerical integration error. Performing MOR methods on such a system will produce unstable reduced system, especially when the expansion point is close to the imaginary axis. Moreover, since the phase and amplitude information are mixed together in the oscillator LPTV system, the reduced system is not able to correctly represent the amplitude variation of the oscillator, even though we can obtain a stable reduced system using the TVP method.

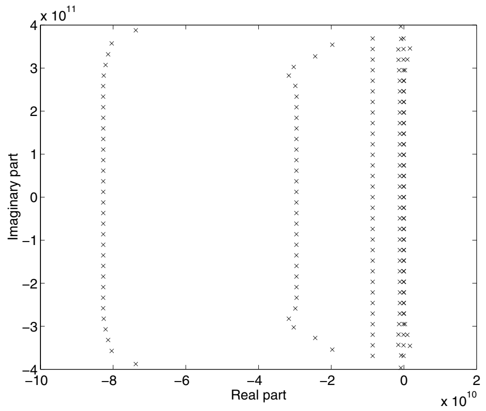


Fig. 1. Pole distribution of an LC oscillator.

B. Oscillation Pole Cancellation Method

The instability of the reduced system is due to oscillation poles in (20), or the Floquet exponent $\mu_1 = 0$ in the LPTV system (2). In [8], Floquet decomposition is used to eliminate the Floquet exponent $\mu_1 = 0$; however, this method is expensive and numerically unstable for large systems. In this work, we present a novel method to cancel the effect of μ_1 without performing the Floquet decomposition.

According to [8], we can obtain amplitude macromodel of the oscillator by linearizing the oscillator over its perturbed time-shifted orbit $x_s(\hat{t})$, where $\hat{t} = t + \alpha(t)$ and $\alpha(t)$ is the phase deviation due to perturbation. The resulting LPTV system is

$$\dot{\hat{o}}(\hat{t}) = A(x_s(\hat{t}))\hat{o}(\hat{t}) + B\hat{b}(\hat{t}). \tag{24}$$

The solution of above system can be expressed as

$$\hat{o}(\hat{t}) = \sum_{i=1}^{n} u_i(\hat{t}) \int_0^{\hat{t}} \exp(\mu_i(\hat{t} - \tau)) v_i^T(\tau) B\hat{b}(\tau) d\tau, \tag{25}$$

where μ_i are Floquet exponents, μ_1 has a value of 0 [9], $u_i(t)$ and $v_i(t)$ are T-periodic vectors, satisfying biorthogonality conditions $v_i^T(t)u_j(t) = \delta_{ij}$. In [8], the μ_1 term is dropped after the Floquet decomposition since it contributes to phase deviation.

To avoid the Floquet decomposition, we cannot use this method to eliminate the μ_1 term; instead, we eliminate the input that contributes to phase deviation. From (25), we know that the input for the μ_1 term is $v_1^T(\tau)Bb(\tau)$. Since $u_i(t)$ and $v_i(t)$ are biorthogonal, we can eliminate this input using the projection method. We define a new input vector

$$\tilde{B} = B - v_1(\tau)Bu_1(\tau). \tag{26}$$

Using this new input vector, the input to the μ_1 term is

$$\begin{aligned} v_1^T(\tau)\tilde{B}b(\tau) &= v_1^T(\tau)Bb(\tau) - v_1(\tau)v_1(\tau)Bu_1(\tau)b(\tau) \\ &= v_1^T(\tau)Bb(\tau) - v_1(\tau)Bv_1(\tau)u_1(\tau)b(\tau) \\ &= v_1^T(\tau)Bb(\tau) - v_1^T(\tau)Bb(\tau) = 0, \end{aligned} \tag{27}$$

and inputs for other μ_is ($i > 1$) are

$$\begin{aligned} v_i^T(\tau)\tilde{B}b(\tau) &= v_i^T(\tau)Bb(\tau) - v_1(\tau)v_i(\tau)Bu_1(\tau)b(\tau) \\ &= v_i^T(\tau)Bb(\tau) - v_i(\tau)Bv_i(\tau)u_1(\tau)b(\tau) \\ &= v_i^T(\tau)Bb(\tau) - 0 = v_i^T(\tau)Bb(\tau). \end{aligned} \tag{28}$$

Hence, the new input vector $\tilde{B}$ only eliminates the effect of the Floquet exponent $\mu_1 = 0$, but preserves inputs of all other Floquet exponents in the system. We can replace the original input vector B with $\tilde{B}$ without changing the amplitude response of the LPTV system. (24) can be rewritten as

$$\dot{\hat{o}}(\hat{t}) = A(x_s(\hat{t}))\hat{o}(\hat{t}) + \tilde{B}\hat{b}(\hat{t}). \tag{29}$$

(29) is stable, because we have minimized the residuals of oscillation poles in (20), even though we do not really eliminate them. We can apply the TVP method described in [11] to reduce this modified system and obtain a stable reduced system for amplitude variations.

C. Detailed Procedure Of Oscillator MOR

In this subsection, we summarize the procedures for constructing and using the oscillator macromodel.

1) Constructing Oscillator Macromodel: Following procedure will construct the oscillator macromodel.

1) Obtain oscillator steady-state $x_s(t)$ using time domain or frequency domain methods.
2) Calculate $u_1(t) = \dot{x}_s(t)$.
3) Calculate the PPV $v_1(t)$ using numerical methods [9], [15]–[17].
4) Construct new input vector $\tilde{B} = B - v_1(t)^T Bu_1(t)$, where $\tilde{B}$ is a T-periodic vector.
5) Perform the TVP method on the new LPTV system

$$\dot{o}(t) = A(x_s(t))o(t) + \tilde{B}b(t) \tag{30}$$

and obtain the reduced system

$$T_q\dot{\tilde{o}}(t) = \tilde{o}(t) + R_q b(t), \tag{31}$$

where $\tilde{o}(t)$ has size of q, which has size smaller than that of the original system.

2) Using Oscillator Macromodel: To reproduce oscillator waveforms under perturbations using our macromodel, we need to integrate both the phase and amplitude equations. The phase equation (6) is solved on time t; however, the amplitude equation (29) is integrated on the shifted time $\hat{t} = t + \alpha(t)$. Below is the pseudocode to rebuild the oscillator' waveform using our macromodel.

```
1  t=0; α(0)=0; i=0
2  t=t+Δt; i=i+1
3  Calculate α(i) by solving (6) on t
4  t̂ = t+α(i)
5  Calculate amplitude variation o(t)=ô(t̂) by solving (31) on t̂
6  Rebuild oscillator's waveforms using (5)
7  goto 2
```

VI. Numerical Results

In this section, we evaluate the technique presented above using ring and LC oscillators. All simulations were performed using **MATLAB** on an Intel-architecture machine, running Linux. We constructed oscillator macromodels using the method described in Section V, simulated oscillator waveforms using the constructed macromodels, and compared the results with SPICE-like simulations of the full oscillator circuits in the same **MATLAB** environment. Experiment results show that our macromodels are able to capture the amplitude variations of oscillators accurately. The rebuilt waveforms using our macromodel match the results from full SPICE-level simulation perfectly, with about 30–120 times speedups.

A. 3-stage Single-Ended Ring Oscillator

Our first example is a 3-stage single-ended ring oscillator, as shown in Figure 2. This oscillator has a system size of 8 and an oscillation frequency of $f_0 = 1GHz$.

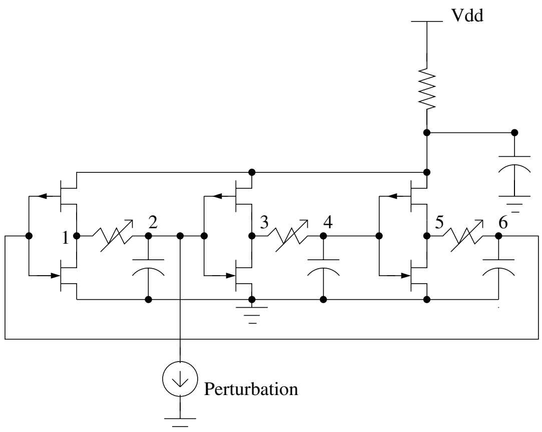
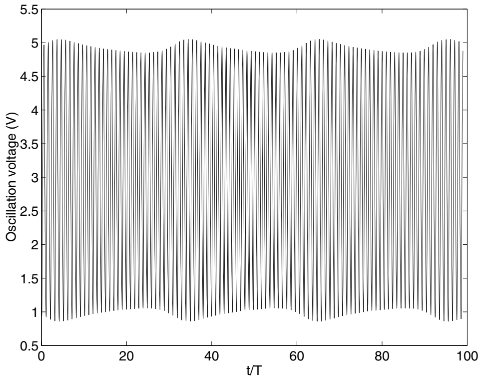


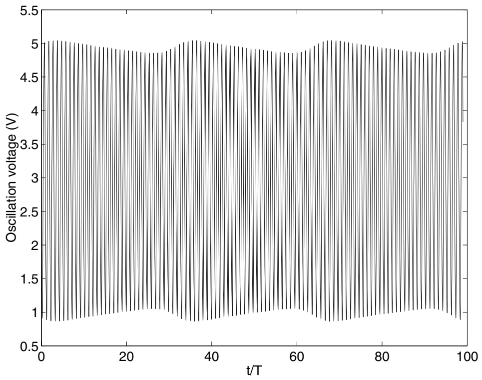
Fig. 2. A 3-stage single-ended ring oscillator.

We first calculate the steady-state of the oscillator using the harmonic balance method, and extract the PPV of the oscillator using the Monodromy method [9], [15]. We then linearize the oscillator on its steady-state orbit and reduce the resulting LPTV system using the method we presented in Section V. Since we are interested in perturbations whose frequency is close to the oscillator's oscillation frequency, we choose the Arnoldi expansion point $s_0 = 2\pi f_0$, where f_0 is the oscillator's free-running frequency. The original nonlinear system has the size of 8, using our method, we can reduce the system size to 3. The resulting small LPTV system can be simulated much faster than the original nonlinear system.

To verify that our macromodel can capture the oscillator amplitude variations correctly, we apply a periodic perturbation current $b(t) = 0.0001sin(\omega_1 t)$ to the node 2 of the oscillator, as shown in Figure 2. We apply different perturbation frequencies ω_1, rebuild the oscillator's output voltage on node 2 using our macromodel and compare with SPICE-level full simulation. The simulation results are shown in Figure 3 and Figure 4. Our macromodel is able to match the full simulation perfectly, with great speedups. The SPICE-level full simulation takes about 6 minutes for a simulation time of 100 cycles; however, it takes only 4 seconds to simulate the same number of cycles using our macromodel. This gives us approximately 120 times speedup on this small oscillator circuit. For larger oscillator circuits with more nonlinear components and complex device model, we expect more significant speedups.

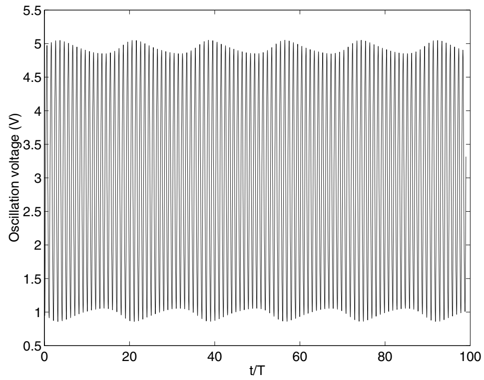


(a) Full simulation

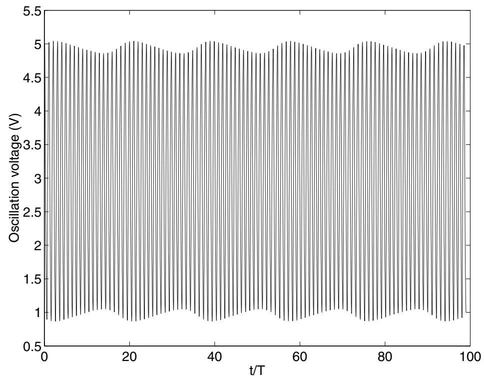


(b) Macromodel

Fig. 3. Output waveform of the 3 stage ring oscillator under perturbation $b(t) = 0.0001sin(1.04\omega_0 t)$.



(a) Full simulation



(b) Macromodel

Fig. 4. Output waveform of the 3 stage ring oscillator under perturbation $b(t) = 0.0001sin(1.06\omega_0 t)$.

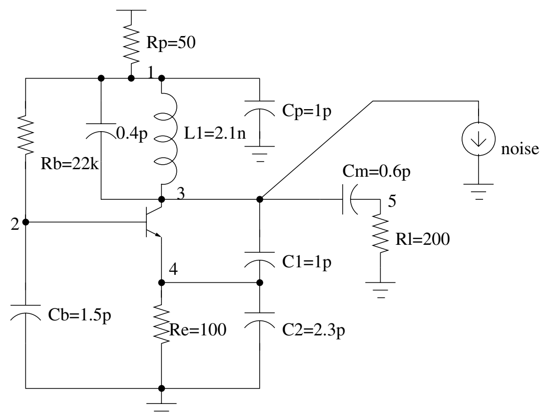


Fig. 5. A 4GHz Colpitts LC oscillator.

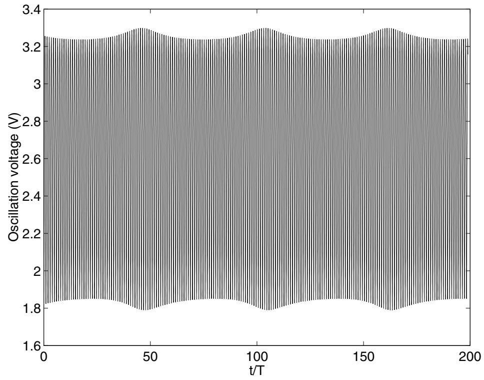
B. 4GHz Colpitts LC Oscillator

Our second test circuit is a 4GHz Colpitts LC oscillator. The circuit and parameters of this oscillator are shown in Figure 5. The oscillator has a system size of 6 and a free-running frequency of $f_0 = 4GHz$.

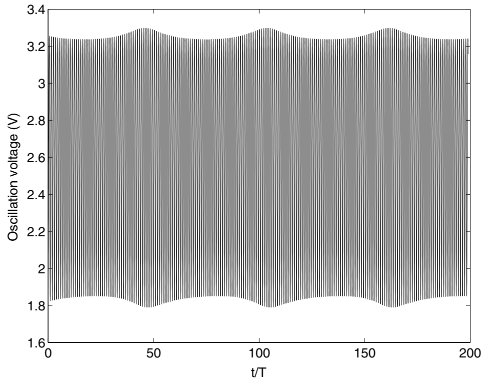
We apply similar method as we describe in the ring oscillator case to calculate the PPV and construct the amplitude macromodel. We evaluate our macromodel with a current perturbation injected into the node 3 of the oscillator, as shown in Figure 5. The perturbation current has an amplitude of $0.1mA$. We simulate the oscillator's output voltage on the node 3 under different perturbation frequencies using our macromodel and compare with SPICE-level full simulation. The simulation results are shown in Figure 6 and Figure 7. Our macromodel is able to match the full simulation with acceptable accuracy. In this case, we obtain about 30 times speedup. This speedup is not as good as the ring oscillator case, because this LC oscillator has only one nonlinear component, hence, its full SPICE-level simulation is very fast.

VII. Conclusions

We have presented a novel technique to extract simple amplitude macromodels from SPICE-level circuit descriptions of any oscillator circuit. Combining it with the oscillator phase macromodel presented in [9], we can rebuild the output waveforms of any perturbed oscillator. We have tested our macromodel using LC and ring oscillators and provided detailed comparisons against full SPICE-level circuit simulation. Numerical results show our macromodels are able to predict oscillator amplitude and phase variations well in the presence

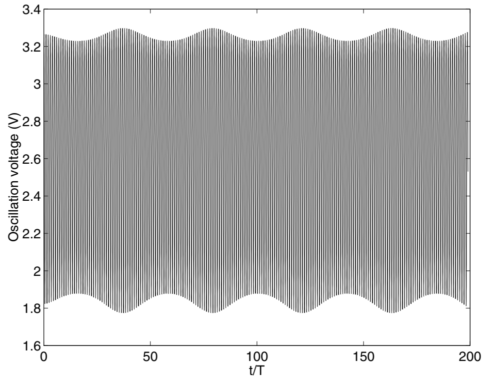


(a) Full simulation

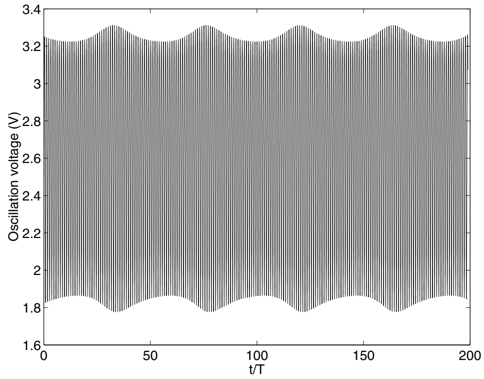


(b) Macromodel

Fig. 6. Output waveform of the Colpitts LC oscillator under perturbation $b(t) = 0.0001 sin(1.02\omega_0 t)$.



(a) Full simulation



(b) Macromodel

Fig. 7. Output waveform of the Colpitts LC oscillator under perturbation $b(t) = 0.0001 sin(1.025\omega_0 t)$.

of perturbations, with great speedups over SPICE-level simulation. Currently, we are working on validating our technique on larger circuits, for which we expect speedups of 3-4 orders of magnitude.

References

[1] L. Nagel. SPICE2: A Computer Program to Simulate Semiconductor Circuits. Electron. Res. Lab., Univ. Calif., Berkeley, 1975.

[2] M. Gardner. Phase-Lock Techniques. Wiley, New York, 1966.

[3] A. Hajimiri and T.H. Lee. A general theory of phase noise in electrical oscillators. IEEE Journal of Solid-State Circuits, 33(2), February 1998.

[4] A. Demir, E. Liu, A.L. Sangiovanni-Vincentelli, and I. Vassiliou. Behavioral simulation techniques for phase/delay-locked systems. In Proceedings of the Custom Integrated Circuits Conference 1994, pages 453–456, May 1994.

[5] A. Costantini, C. Florian, and G. Vannini. Vco behavioral modeling based on the nonlinear integral approach. IEEE International Symposium on Circuits and Systems, 2:137–140, May 2002.

[6] L. Wu, H.W. Jin, and W.C. Black. Nonlinear behavioral modeling and simulation of phase-locked and delay-locked systems. In Proceedings of IEEE CICC, 2000, pages 447–450, May 2000.

[7] M. F. Mar. An event-driven pll behavioral model with applications to design driven noise modeling. In Proc. Behav. Model and Simul.(BMAS), 1999.

[8] X. Lai and J. Roychowdhury. Automated oscillator macromodelling techniques for capturing amplitude variations and injection locking. In IEEE/ACM International Conference on Computer Aided Design, November 2004.

[9] A. Demir, A. Mehrotra, and J. Roychowdhury. Phase noise in oscillators: a unifying theory and numerical methods for characterization. IEEE Trans. on Circuits and Systems-I:Fundamental Theory and Applications, 47(5):655–674, May 2000.

[10] R. Grimshaw. Nonlinear Ordinary Differential Equations. Blackwell Scientific, New York, 1990.

[11] J. Roychowdhury. Reduced-order modelling of time-varying systems. IEEE Trans. Ckts. Syst. – II: Sig. Proc., 46(10), November 1999.

[12] P. Feldmann and R.W. Freund. Efficient linear circuit analysis by pade approximation via the lanczos process. IEEE Trans. on Computer-Aided Design, 14:639–649, May 1995.

[13] A. Odabasioglu, M. Celik, and L.T. Pileggi. PRIMA: passive reduced-order interconnect macromodelling algorithm. In Proceedings of IEEE ICCAD 1997, pages 58–65, November 1997.

[14] R.W. Freund. Reduced-order modeling techniques based on Krylov subspaces and their use in circuit simulation. Technical Report 11273-980217-02TM, Bell Laboratories, 1998.

[15] A. Demir. Phase noise in oscillators: Daes and colored noise sources. In IEEE/ACM International Conference on Computer-Aided Design, November 1998.

[16] A. Demir, D. Long, and J. Roychowdhury. Computing phase noise eigenfunctions directly from steady-state jacobian matrices. In IEEE/ACM International Conference on Computer Aided Design, pages 283–288, November 2000.

[17] A. Demir and J. Roychowdhury. A reliable and efficient procedure for oscillator ppv computation, with phase noise macromodelling applications. IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems, 22(2):188–197, February 2003.