

Modeling and Simulation of Electromechanical Transducers in Microsystems using an Analog Hardware Description Language.

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Abstract

The analytical modeling and simulation of conservative electrostatic, electromagnetic and electrodynamic transducers found in microsystems using a non-linear lumped-parameter approach is presented in this paper. A comparison is made between this approach and the linearized equivalent circuit method. All models of transducers are written in HDL-ATM, a proprietary analogue hardware description language (HDL). System-level simulation is performed in the SPICE simulator using behavioral models of the transducers. Finally, a parameter extraction and HDL model generation tool for devices is presented

Introduction

The assembly of low-power devices in a greatly confined space results in strong coupling of physical quantities between the components of a microsystem. Therefore, in addition to component modeling, a system level simulation has to be performed to adequately predict global performance using *macro models* of the devices.

SPICE simulators may be used as lumped parameter (discrete variable) analog solvers to simulate transducers by exploiting electrical-mechanical analogies (also known as the equivalent circuit approach). When instantiated in a netlist with electronics, these models can be used to predict global performance of a microsystem. The ordinary differential equations of a physical device are solved by finding an electrical circuit presenting the same equations, a difficult procedure when modeling non-linear devices presenting behavioral discontinuities. Usually, all components are linearized around an operating (bias) point [1], limiting the validity of these models to small-signal analysis.

VHDL 1076.1, also called VHDL-AMS (VHDL analogue and mixed signal) [2] is of considerable interest

to designers who wish to create behavioral models based on analytical expressions including ordinary differential and algebraic equations (DAEs). The lumped parameters may now be non-linear (functions of efforts, flows or state variables) and the validity of boundary conditions may be verified in these models during run-time. Such models adequately describe device behavior for small and large signals. HDL-ATM of ANACAD Engineering is a proprietary HDL and can be used to build parameterized models of devices where ordinary differential equations (excluding partial derivatives) adequately describe device behavior. HDL-A models of different electromechanical transducers will be presented in this paper.

EDA tools are being adapted to cater for the needs of the microsystem designer. Etching simulators, design rule checkers, electro-thermal simulators and component libraries are appearing [3]. The authors have developed a numerical parameter extraction tool used to characterize microsystem devices simulated using the finite element (FE) method [4]. Behavioral HDL models of the simulated devices are generated to allow simulation of the complete microsystem including electronics. The methodology and software tools will be presented briefly before drawing conclusions.

Lumped parameter modeling of electro-mechanical transducers

In electrical circuits as in bond-graph theory [5], interaction between components occurs by energy exchange through *terminal ports*. An *effort* (or intensive) variable and a *flow* variable is associated with each *port*, the product of which is a power. These variables are also known as power variables in bond-graph theory. The *flow* variable is defined as the time derivative of the *state* (or extensive) variable. Figure 1 shows the ports and variables for a two port electromechanical transducer. By

convention, the *flow* variable is positive when entering a two port, increasing the transducer energy.

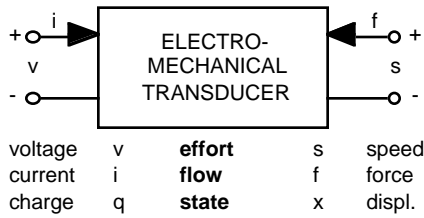


Figure 1 : Two port electromechanical transducer.

The terms *across* and *through* are used in HDL-A to describe the *effort* and *through* variables respectively. The physical domain in question is described as *anature*. Note that the term *port* describes a digital connection (*signal port*) in HDL-A while *pin* denotes an analog connection (*terminal port*). Table 1 below lists associated variables for different domains.

Variable	Mechanical translation	Mechanical rotation	Electrical variables	Hydraulic variables
Effort e	Force F	Torque τ	Voltage V	Pressure P
Flow f	Velocity V	Ang. vel. ω	Current i	Vol. flow rate Φ
Momentum p	Momentum p	Angular momentum P_r	Flux linkage λ	Pressure momentum p_p
State q	Translation x	Angle θ	Charge q	Volume V
Energy W	$\int F dx, \int V dp$	$\int \tau d\theta, \int \alpha dp_r$	$\int V dq, \int i d\lambda$	$\int P dV, \int \Phi dp_p$
Power P	$F(t).V(t)$	$\tau(t). \omega(t)$	$V(t).i(t)$	$P(t). \Phi(t)$

Table 1 : Generalised variables for different physical domains.

It is possible to derive two analogies relating the dynamic behavior of electrical and mechanical systems, the force-voltage (FV) and the force-current (FI) analogy. The FI analogy is used here for all models as the mechanical and electrical nets have the same topology.

As with all lumped-parameter systems, the generally applicable assumptions of negligible fringe fields, wavelengths greater than all characteristic dimensions of a device, perfect conductors, etc. are assumed. All the transducers presented are considered conservative. In real transducers, input and output impedances represent losses, and are readily added to the netlist.

Deriving HDL-A behavioral models from transducer internal energy

Analytical expressions for coding behavioral HDL-A models of conservative transducers may be calculated from an existing analytical expression for the internal energy. In conservative systems, the intensive variable of each port may be deduced by deriving the energy in the transducer with respect to the state variable of each port:

1. List the *effort*, *flow* and *state* variables for each port.
2. Express the total energy in the transducer as a sum of the partial (mechanical, thermal, electrostatic, magnetic, fluidic) energies (functions of the *effort*, *flow* and *state* variables).
3. Derive the energy with respect to the *state* variable of each *port* to obtain the respective *effort* variable.
4. Replace time derivatives of *state* variables by the corresponding *flow* variables.

The resulting expressions are then used to link the *effort* and *flow* variables in the relational (or equation) block of the HDL-A model. Four commonly used transducers presented in figure 2 below were selected and HDL-A models were written for each of them.

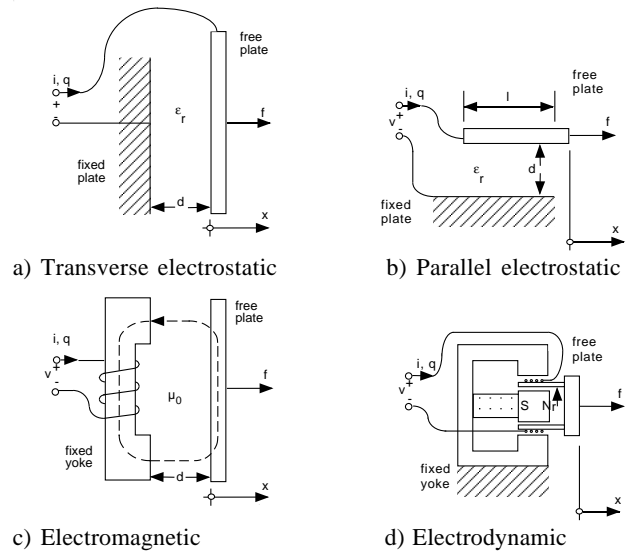


Figure 2 : Electromechanical transducers studied.

It is assumed that A is the active cross-section area and h the depth in all models. Expressions of input impedances and internal energies of the electromechanical transducers studied are presented in table 2 below.

Transducer	Input impedance	Internal energy
a)	$C = \frac{\epsilon_0 \epsilon_r A}{(d+x)}$	$\frac{\epsilon_0 \epsilon_r A}{2(d+x)} V^2$
b)	$C = \frac{\epsilon_0 \epsilon_r h(l-x)}{d}$	$\frac{\epsilon_0 \epsilon_r h(l-x)}{2d} V^2$
c)	$L = \frac{\mu_0 AN^2}{2(d+x)}$	$\frac{\mu_0 AN^2}{4(d+x)} i^2$
d)	$L = \frac{\mu Nr}{2}$	$\frac{\mu Nr}{4} i^2$

Table 2. Impedances and energies of electromechanical transducers in figure 2.

Coding of the HDL-A model is done in the following manner. *Effort* variables are read at ports while the

contributions of *flow* variables are deduced from the derived equations. If necessary, an equation block is added to the model. In all of the electromechanical transducer examples presented, conservation of energy is implicitly satisfied by the two derived *effort* equations. The need for an equation expressing the conservation of energy between the input and output ports is thus alleviated, as SPICE simulators do not verify the conservation of energy between devices in the netlist. Indeed verification of the conservation of energy would benefit the simulation of closed physical systems, which are in practice the most used, and tractable. Table 3 below lists voltages and force contributions of transducers studied.

Transducer	Voltage [V]	Force [N]
a)	$\frac{(d+x)}{\epsilon_0 \epsilon_r A} \int idt$	$\frac{-\epsilon_0 \epsilon_r A}{2(d+x)^2} v^2$
b)	$\frac{d}{\epsilon_0 \epsilon_r h(l-x)} \int idt$	$\frac{-\epsilon_0 \epsilon_r h}{2d} v^2$
c)	$\frac{\mu_0 AN^2}{2(d+x)} \frac{di}{dt}$	$\frac{-\mu_0 AN^2}{4(d+x)^2} i^2$
d)	$\frac{\mu_0 Nr}{2} \frac{di}{dt}$	$2\pi NrBi$

Table 3. Voltages and forces (efforts) derived from energies of transducers.

Listing 1 below is the HDL-A coding for the transverse electrostatic transducer of figure 2a. The procedural (analog) declaration is straightforward; efforts are read, and flows are deduced from the derived equations.

```

ENTITY eletran IS
  GENERIC (A, d, er : analog);
  PIN (a, b : electrical; c, d : mechanical1);
END ENTITY eletran;

ARCHITECTURE a OF eletran IS
  VARIABLE e0, x : analog;
  STATE V, S : analog;
BEGIN
  RELATION
    PROCEDURAL FOR init =>
      e0 := 8.8542e-12;
    PROCEDURAL FOR ac, transient =>
      V := [a, b].v;
      S := [c, d].tv;
      x := integ(S);
      [a, b].i % = e0*er*A/(d + x)*ddt(V);
      [c, d].f % = -e0*er*A*V/(2.0*(d+x)*(d+x));
    END RELATION;
  END ARCHITECTURE a;

```

Listing 1 : HDL-A listing for transverse electrostatic transducer in figure 2a.

Pins a, b are the electrical terminals, pins c, d the mechanical terminals. U is the voltage and V the

translational velocity. A, d and ϵ_r are the electrode surface, gap and relative permittivity values supplied as external parameters to the model. Note that the variables V and S (voltage and speed respectively) are defined as states (variables with history) as they are integrated and derived in the model. No (implicit) equation block is required in this model.

Comparison of linearized lumped parameter equivalent circuit transducer models with HDL-A macro-models.

The transverse electrostatic transducer of figure 2a was modeled both as a linearized equivalent circuit model and as the behavioral HDL-A model presented above. The transducer models were connected to a mechanical resonator (RLC circuit) in a netlist. A voltage source with a finite rise and fall time was used to excite the transducer. The complete system is shown in figure 3 below.

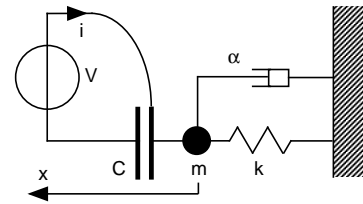


Figure 3 : System composed of electrostatic transducer coupled to a mechanical resonator.

Table 4 lists the values of the different parameters used to simulate the system:

Parameter	Quantity	Value
A	area	1.0E-4 m ²
d	gap	0.15E-3 m
ϵ_r	rel. permittivity	1
m	mass	1.0E-4 kg
k	spring constant	200 N/m
α	damping	40E-3 Ns/m
v_0	dc voltage	10 V
x_0	dc displacement	1.0E-8 m
C_0	dc capacitance	5.8637E-12 F

Table 4. Values of parameters used for the transducer-resonator system in figure 3.

The equivalent circuit transducer model for the system was linearized for a static (dc) operating voltage V_0 which corresponds to an initial displacement x_0 and a capacitance C_0 . This corresponds [1] to a transduction factor Γ of:

$$\Gamma = \frac{\epsilon_0 \epsilon_r A v_0}{(d + x_0)^2} = 3.34675E-9 \text{ N/V}$$

Both transducer models were introduced in the electrical circuit presented in figure 4 below and a variable pulse voltage was applied to the electrodes of the transducer:

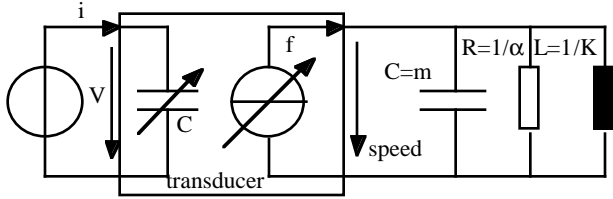


Figure 4 : Electrostatic transducer coupled to mechanical resonator.

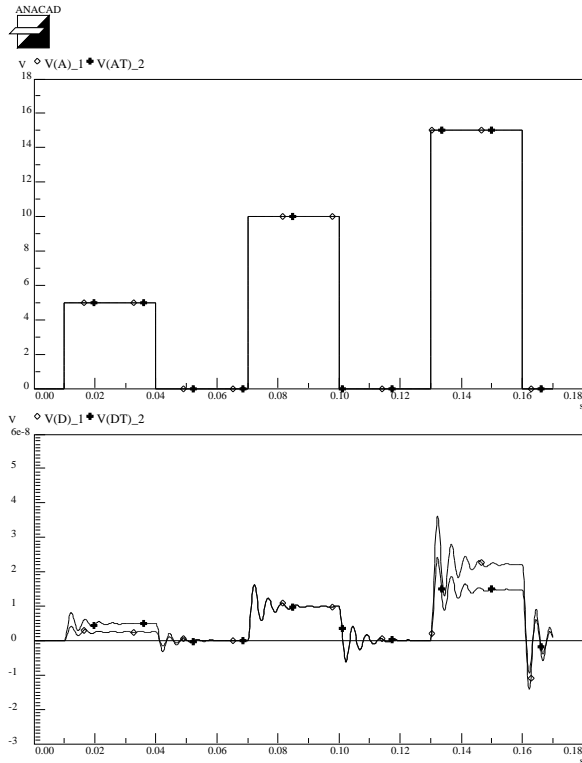


Figure 5: Simulation comparing linear lumped parameter electrostatic transducer and behavioral HDL-A (mathematical) model. Exciting voltage pulses of 5, 10 and 15 V are displayed above. Displacements (represented by voltages D and DT respectively) for the HDL-A and linearised systems are visible below.

The simulation results for the above system using the linearized equivalent circuit transducer and the HDL-A model given in listing 1 are presented in Figure 5. The upper plot shows the signal from the voltage sources of each system. The lower plot shows the displacements (integrals of velocities) of both systems. The displacements converge perfectly for a quasi-static load of

10 V (center of lower graph), which was the linearization point. For a lower exciting voltage (5 V), the linear model overshoots (below at left), and undershoots for a greater voltage (15 V). The dynamic behavior of both systems is primarily defined by the under-critical damping from the choice of the mass, spring-constant and damping-coefficient parameters of the mechanical resonator.

The results from the equivalent circuit model could be improved by using a controlled source ($I = \text{const.} \cdot V1 \cdot V2$), SPICE primitive. Such an approach is however strongly constrained by the given choice of circuit elements whereas the HDL-A description is constrained by the expression syntax and is evidently far more flexible. The drawback is a strong penalty in simulation performance (a factor of 10 was observed) which should in the future be improved by better compilers.

The ELDO simulator performance could be improved by providing bounding and precision controls for the nodes in the netlist of non electrical nature. This would not only help in detecting run-time problems with models (impossible values) but should also improve convergence and speed.

Parameter extraction and model generation from finite element analysis

The FE method is commonly used to solve spatial differential equations to predict micromachined device behavior; this is the device-level of simulation. The ANSYS™ FE software has become a standard in the field as it allows for coupling between mechanical, thermal, electrical, electromagnetic, acoustic and fluidic phenomena. Interaction between devices is limited to correctly defining the FE boundary conditions.

FE and SPICE simulators present analogies concerning the analysis types they can perform: static-dc, harmonic-ac, transient-transient. Ideally one would couple a FE simulator and a SPICE simulator through the ports of a component. For 3D FE models, nodes on terminal ports are surfaces on which the intensive variable is invariant. The *flow* variable could be derived by numerical integration of the *flow* variable density over the surface. The simulation speed of such a system would however be unworkable as each FE iteration may take several hours when dealing with large models and non-linearities.

Prior characterization of transducers using FEs results in efficient behavioral models offering fast system-level simulation, is the approach chosen by the authors. A physical parameter extractor (*PXT*) based on the numerical integration of nodal (and element) degrees of freedom (DOFs) has been developed, and interfaces with ANSYS (Fig. 6).

For static FE analysis, conjugate pairs of *flow*, *intensive* and *state* variables over a terminal port are calculated. An invariant quantity is fixed over a terminal (boundary conditions) and the variant quantity is

calculated using a numerical integration of a DOF density over the terminal. A physical macro-parameter (resistance, capacitance, reactance, displaced volume, electrostatic force etc.) can also be calculated in this manner. By iterating the variation of boundary conditions and extracting the parameter of interest, a piecewise linear behavioral *macro model* is created. A HDL-A model is then generated and implements digital synchronization to ensure sufficient precision during simulation [6] (no rounding errors).

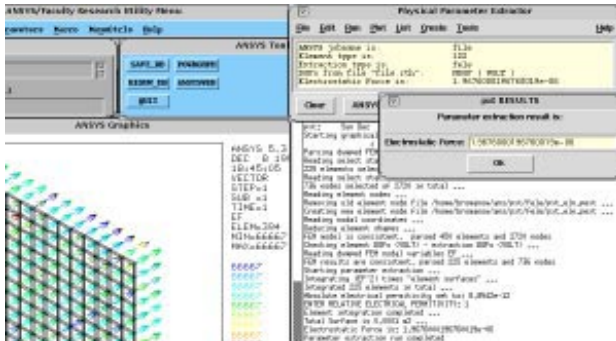


Figure 6. Screen dump detail of the parameter extractor (PXT) used to calculate the force on movable plate of electrostatic transducer in figure 2a.

Harmonic FE analysis produces real and imaginary data of DOFs as discrete functions of frequencies, i.e. the frequency response (amplitude and phase). A polynomial filter is fitted to such a *macro model*, and thus generating a data flow HDL-A model. Models derived from both static and harmonic FE analyses are valid for the dc, ac and transient SPICE analysis domains, and are different representations of the behavior of a given device.

Figure 6 shows PXT being used to calculate the electrostatic force on the movable electrode of the electrostatic transducer of figure 2a simulated previously. ANSYS is used to compute the electric field between the electrodes, the constant electric field is displayed as is the element mesh. PXT uses a numerical version of the following equation to calculate electrostatic forces:

$$f = \frac{1}{2} \int_{\text{surface}} \epsilon E^2 n dS$$

where ϵ is the permittivity of air, E is the electric field and n is the surface normal. The result obtained using the parameters in table 4 and zero displacement ($x=0$) corresponds to the force in table 3. Note that in this example, the fringe field was not modeled. The PXT output log is visible at lower right. By repeating this procedure for different voltages and displacements, a behavioral model is generated. The tool is ideal for modeling devices where complex geometry and strong coupling render analytical approaches impossible.

Conclusion

The methodology presented in this paper is based on macro models describing device behavior using a non-linear lumped parameter approach.

Expressions for the effort-flow relationships defining the behavior of electromechanical transducers can be derived by first expressing the internal energy in the actuator and then deriving it with respect to each of the state variables. Electrostatic, electromagnetic and electrodynamic transducers are studied and coded in HDL-A. A simple simulation demonstrates the large-signal validity of these models contrary to linearized equivalent circuit models. Whereas HDL-A models are constrained by the expression syntax, the equivalent circuit approach is strongly constrained by the choice of circuit elements found in the SPICE library. Another advantage of using HDL-A models is that one avoids the (sometimes) difficult task of finding the equivalent circuit.

Where analytical expressions are not available owing to complex geometry and strong field coupling, physical macro-quantities of interest may be derived from continuous field device FE simulations. The PXT parameter extractor presented is used in static, harmonic and transient FE analysis domains. HDL-A models of the device behavior are automatically generated for dc, ac and transient SPICE domains.

Using either of the presented methods, modeling of transducers and digital and analog electronics, may be performed in a SPICE simulator to predict global system performance. The approach presented results in efficient system-level simulations and allows global optimization of all components in a microsystem.

References

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